Robust Decoupling Control for Articulated Robot*

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In an articulated robot, coupling forces are generated between the joints. To improve dynamic behavior and reduce the work cycle time, it is necessary to compensate for the force generated between the joints. In this paper, robot dynamics are expressed in a linear form by applying a gain scheduling method and then transformed into a discrete form. A simple control algorithm is proposed which can perform the decoupling and the pole assignment simultaneously by using an observer with a disturbance compensation function. The results, based on experiments with a SCARA-type robot, show that the proposed approach can compensate for the coupling force between joints.

Key Words: Robotics, Decoupling Control, Disturbance Compensation, Pole Assignment

1. Introduction

In articulated robots such as those of the SCARA type, coupling inertia force, Coriolis force and centrifugal force act on each axis. In particular, the coupling force due to acceleration has a large effect on dynamic performance by causing overshoot and affecting positioning time at the end of motion. Successful results have shown that the use of preview controllers for robotic arms is useful for solving this problem⁽¹⁾. In traditional PI servo control (P-action for positioning control and PI-action for velocity control), the coupling effect can be reduced to some extent by integral control of each axis owing to the high reduction ratio in a drive system⁽²⁾. However, there is an increasing demand in industry for achieving greater robot speed and accuracy, which requires reducing the positioning time in particular, which in turn requires developing control methods other than

PI servo control.

Therefore, robust control based on disturbance estimation has recently become an area of major interest⁽³⁾⁻⁽⁸⁾. This approach deals with the nominal plant without coupling and time-variant terms and is superior to conventional PI servo control in improving response time to commands. It compensates for the variation in all parameters beyond the nominal values including coupling and time-variant terms using a disturbance observer. Integral control is accomplished to the stepwise disturbance, provided that the sampling period is short in comparison with the change in disturbance. However, it cannot sufficiently compensate for the effect due to the inertia term (i.e., the acceleration term) which instantaneously changes the control system, thus affecting the transient response. This approach therefore shows a limitation in obtaining high positioning performance.

Several decoupling control methods have been proposed so far which can compensate for the coupling force between joints without having a large effect on output⁽⁹⁾⁻⁽¹²⁾. A compensator is generally used in the control loop to improve robustness. However, most of these methods require a more

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complex construction than our system. These proposed techniques also require a large amount of calculation time due to the increase in system dimensions. Although microprocessor performance is increasing with continued progress in electronics, they still produce operational errors due to integer calculation, which acts on the control system as a disturbance. By overcoming such problems, researchers in robotics will be able to more actively pursue practical applications based on human-centered control⁽¹³⁾.

The objective of the present work is to develop a decoupling method with a practical decoupling control system which solves the above-mentioned problems, and at the same time to reduce the positioning time as much as possible by assigning the poles to obtain the desired transfer function for each axis. A simple control algorithm used together with the gain scheduling method is proposed to achieve this goal from the practical viewpoint of dealing with positioning instead of complicated time-variant nonlinear robot dynamics. The calculated torque method generally requires accurate parameter identification and calculation. In a disturbance observer approach, the difference between the nominal and actual parameter values affects the control performance. Although the present work also requires parameter identification, the dynamics are represented by two simplified parameters, P and R , to realize a linear plant model.

The disturbance compensation function is implicitly involved in the velocity observer and the desired control algorithm, given that the low-dimensional controller is constructed using only the input torque and the positional output to the robot. Although this compensation algorithm performs a decoupling control for the multivariable system, it basically requires the same amount of calculation as the disturbance observer in which the model is treated as a linear system. The proposed control algorithm has three simultaneous roles: disturbance compensation, decoupling and pole assignment.

2. Modeling of Plant and Disturbance

The dynamic equation for an n -axis articulated robot is generally expressed by the angular coordinate $\theta(t)$ as

$$J\ddot{\theta}(t) + D\dot{\theta}(t) + E(\theta, \dot{\theta}, \ddot{\theta}) = Ku(t), \quad (1)$$

$$E(\theta, \dot{\theta}, \ddot{\theta}) = \{J(\theta) - J\} \ddot{\theta}(t) + \{D(\theta, \dot{\theta}) - D\} \dot{\theta}(t) + f, \quad (2)$$

where $J(\theta)$ is an $n \times n$ inertia matrix and $D(\theta, \dot{\theta})$ is an $n \times n$ matrix expressing viscous damping, Coriolis force, and centrifugal force. J and D are the corresponding $n \times n$ definite matrices (representative nominal matrices). K is a diagonal gain matrix for the motor, amplifier and driving mechanism, and f is

an n -dimensional vector specifying Coulomb friction and gravity. The n -dimensional vector $u(t)$ is the control input signal applied to the amplifier.

A majority of industrial robot work such as assembling is generally implemented by point-to-point control, and positioning time is a crucial problem in addition to decoupling control. To solve this problem, the control algorithm is simplified based on the following practical considerations:

(1) The acceleration term $J(\theta)$ in Eq. (2) has a large effect at the end of motion. This term is a time-variant coupling inertia which is time-variant to the robot manipulator angle (position). In other words, the effect due to inertia is not serious when a constant velocity having a small acceleration acts on the manipulator. The final position is, however, often altered depending on the application conditions and as a result the coupling term changes largely subject to that position. To increase the accuracy of this decoupling control, it is therefore necessary to change the system parameters depending on the position from the practical standpoint. In this case, the timing of parameter changes is of great importance and, at the same time, large instantaneous parameter changes have a significant effect on the system dynamics. Thus the gain scheduling method in which the necessary system parameters are gradually altered in accordance with the position is applied throughout the overall range of manipulator operations.

(2) The effect of the time-variant nonlinear velocity term $D(\theta, \dot{\theta})$ in Eq. (2) is very small at the end of motion, thus generating no serious loss of positioning accuracy. This term is therefore processed as a nominal value throughout the manipulator working range. The error due to this treatment in the case of velocity changes can be suppressed using the disturbance compensator with an integrator and regarding this error as a stepwise disturbance.

(3) Gain scheduling method: Based on these considerations, the gain scheduling method was applied. For a discrete set of angular $\{\theta_i\}$ such as $\{0^\circ, 10^\circ, 20^\circ, 30^\circ, \dots\}$, the corresponding inertia values J_i (i.e., J at i) can be obtained in advance as deterministic values. A set of control parameters at each θ_i is then determined by using J_i and the nominal values D , i.e., $\{\theta_i\} \rightarrow \{J_i, D\} \rightarrow \{\text{set of control parameters}\}$.

In positioning control of a certain angular θ , the control law u is calculated using the control parameters chosen from $\{J_i, D\}$ at $\theta \approx \theta_i$. Sufficient accuracy is expected with the increase in the number of θ_i .

The error due to approximation by the time-invariant model can be practically resolved by applying this gain scheduling technique. The terms $\{J(\theta) - J\}$ and $\{D(\theta, \dot{\theta}) - D\}$ in Eq. (2) represent errors at

a certain position θ , thus $E(\theta, \dot{\theta}, \ddot{\theta})$ is regarded as a disturbance $\eta(t)$ with the additional force f .

The state equation of Eq. (1) is

$$\dot{X}(t) = AX(t) + B\{u(t) + \eta(t)\} \quad (3)$$

$$y(t) = CX(t) \quad (4)$$

$$\eta(t) = -KE(\theta, \dot{\theta}, \ddot{\theta}), \quad (5)$$

where $X(t) = [\theta^T, \dot{\theta}^T]^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -J^{-1}D \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ J^{-1}K \end{bmatrix} \quad C = [I \quad 0].$$

By using the Laplace transformation, the relatively right prime fraction of Eqs. (3)–(5) yields^{(14),(15)}

$$y(s) = R(s)P^{-1}(s)\{u(s) + \eta(s)\}, \quad (6)$$

where $P(s)$ and $R(s)$ are $n \times n$ polynomial matrices. Equation (6) represents the transfer function matrix of the system. The control algorithm is implemented in the microprocessor by the software. Equation (1) is therefore expressed in a discrete form using the Z -transformation with the sampling period τ and the time $t = k\tau$ ($s = \delta/\tau$, $\delta = z - 1$) as follows, provided that the control input $u(k)$ is held constant over the sampling period.

$$y(k) = R(\delta)P^{-1}(\delta)\{u(k) + \eta(k)\} \quad (7)$$

$$P(\delta) = I\delta^2 + P_1\delta + P_0 \quad (8)$$

$$R(\delta) = R_1\delta + R_0, \quad (9)$$

where $R(\delta)$ is the non-singular matrix and I denotes the unit matrix. In robot positioning control, P_0 , R_1 are zeros and P_1 , R_0 are non-diagonal matrices.

Now let us define the disturbance model for each axis in the general form of a polynomial equation with t ,

$$\eta(t) = \eta_0 + \eta_1 t + \eta_2 t^2 + \dots + \eta_d t^d, \quad (10)$$

where η_i and d denote an unknown coefficient and an integer ($= 1, 2, 3, \dots$), respectively. If the sampling period is sufficiently short in comparison with the change in disturbance, Eq. (10) can be regarded as a stepwise disturbance at each sampling period, thus d in Eq. (10) is practically zero. $\eta(t)$ is then expressed in the discrete form as

$$\eta(\delta) = \frac{\eta_0}{\delta}. \quad (11)$$

3. Derivation of the Control Algorithm

To decouple the robot dynamics and to assign poles for obtaining the desired transfer function, the designed transfer function $m(\delta)$ is

$$m(\delta) = \frac{m_0}{\delta^2 + m_1\delta + m_0}, \quad (12)$$

i.e., the desired transfer function matrix from reference input $v(k)$ to output $y(k)$ is

$$y(k) = N(\delta)M^{-1}(\delta)v(k), \quad (13)$$

where

$$M(\delta) = I(\delta^2 + m_1\delta + m_0), \quad N(\delta) = Im_0$$

$M(\delta)$ is a characteristic polynomial matrix of the model $m(\delta)$ and it also stands for the poles in the

overall control system. $N(\delta)M^{-1}(\delta)$ must be diagonal if $y(k)$ is to be decoupled. The control law is determined so that the original system of Eq. (7) is transferred into the decoupled system of Eq. (13) by choosing an adequate matrix of $N(\delta)M^{-1}(\delta)$ (The disturbance $\eta(k)$ is omitted here and will be discussed in section 4). $M(\delta)$ must be chosen so as to avoid the mechanical resonance point in the plant.

As the maximum order of each axis is 2 and that of the disturbance is $d+1=1$, we can define an arbitrary asymptotically stable monic polynomial $q(\delta)$ with the order of $(2-1)+(d+1)=2$ by

$$q(\delta) = \delta^2 + q_1\delta + q_0. \quad (14)$$

Let $Q(\delta)$ be an $n \times n$ polynomial matrix with a diagonal element $q(\delta)$ such that

$$Q(\delta) = Iq(\delta), \quad (15)$$

where $q(\delta)$ represents the characteristic polynomial of an observer which is designed independently of the pole assignment in the control system. $Q(\delta)$ must be chosen by taking the response to the disturbance and noise into account. Let $K(\delta)$ and $H(\delta)$ be a first-order- $n \times n$ polynomial matrix and a second-order- $n \times n$ polynomial matrix with the same order as $Q(\delta)$, respectively, i.e.,

$$K(\delta) = K_1\delta + K_0 \quad (16)$$

$$H(\delta) = H_2\delta^2 + H_1\delta + H_0. \quad (17)$$

Then, for the third-order arbitrary polynomial matrix $F(\delta)$, the polynomials $K(\delta)$ and $H(\delta)$ are determined to satisfy Eq. (18) by the parameter comparison approach,

$$K(\delta)P(\delta) + H(\delta)R(\delta) = F(\delta). \quad (18)$$

By letting $K_i = Iq_i$ ($i=0, 1, \dots, d$), the disturbance $\eta(\delta)$ becomes zero as shown in the following section.

Let us choose $F(\delta)$ in Eq. (18) to be

$$F(\delta) = Q(\delta)\{P(\delta) - GM(\delta)N^{-1}(\delta)R(\delta)\}, \quad (19)$$

$$G = N(\delta)R^{-1}(\delta).$$

To assign poles in the system, let us consider the following control law,

$$u(k) = Q^{-1}(\delta)\{K(\delta)u(k) + H(\delta)y(k) + Gv(k) + F\delta v(k)\}. \quad (20)$$

$F\delta v(k)$ in Eq. (20) represents a feedforward term which is applied to improve the response characteristic to the command. This term is omitted to avoid confusion in the following discussion because it has nothing to do with the decoupling, pole assignment and disturbance compensation. By using Eqs. (7) and (20), we obtain

$$y(k) = R(\delta)\{Q(\delta)P(\delta) - K(\delta)P(\delta) - H(\delta)R(\delta)\}^{-1}Q(\delta)Gv(k). \quad (21)$$

Substitution of Eqs. (18) and (19) into Eq. (21) yields

$$\begin{aligned} y(k) &= R(\delta)\{Q(\delta)GM(\delta)N^{-1}(\delta)R\}^{-1}Q(\delta)Gv(k) \\ &= R(\delta)R^{-1}(\delta)N(\delta)M^{-1}(\delta)G^{-1}Q^{-1}(\delta)Q(\delta)Gv(k) \\ &= R(\delta)R^{-1}(\delta)N(\delta)M^{-1}(\delta)v(k) \\ &= N(\delta)M^{-1}(\delta)v(k), \end{aligned} \quad (22)$$

algorithm was implemented using a RISC chip (Hitachi: SH7032) which can deal with 16-bit integer data. The motor drive amplifier with current feedback control operates the robot which has a harmonic drive with high reduction gear rates (1/50 to 1/200) for high-speed control. This is done using the RISC chip signal.

The control algorithm was applied to the first axis (θ_1 axis) and the second axis (θ_2 axis) of the robot manipulator which typically produces the coupling force between two axes because of the horizontal two-link mechanism construction. The velocity term (second term in Eq. (1)) is small at the end of motion and has no effect on the positioning time. It is thus considered to be a constant value. The error caused by this assumption is compensated for by the disturbance compensator as a constant disturbance applied at every sampling period of τ . This approximation may effectively reduce positioning time. The acceleration term (inertia matrix), on the other hand, has a large effect on positioning time. Hence, decoupling control is especially required at the end of the motion. The inertia term varies with the θ_2 axis. The control algorithm is therefore designed by applying the gain scheduling approach based on θ_2 measurements. The calculation time for the algorithm is approximately 40 μ s. However, taking other data processing times into account, the sampling period was set at 0.625 ms.

The parameters in Eq. (7) were identified using the least squares method and the results obtained for the θ_2 axis are shown in Table 1. The desired poles

were assigned at 22 rad/s and 400 rad/s for the plant model, and at 220 rad/s for the second-order observer. The control law for the θ_2 axis was then determined as shown in Table 2. The parameters and control law for the θ_1 axis can also be obtained in the same manner as the θ_2 axis. In a practical gain scheduling control, for example, the parameters at $\theta_2=0^\circ$ in Table 2 were used for the θ_2 axis motion of $0^\circ \leq \theta_2 \leq 15^\circ$. For $15^\circ < \theta_2 \leq 30^\circ$, the parameters at $\theta_2=20^\circ$ were used.

6.1 Experiment I

First, the effectiveness of the decoupled system was examined by moving a single axis only. For quantitative evaluation, the experimental results were compared with those obtained using the traditional PI servo control in each axis. For this control, the parameters are experimentally determined as follows: proportional gain in the positioning control = 20, rate time in the velocity control = 10 ms, and proportional gain in the velocity control = 0.018 (joint 1) and 0.010 (joint 2). The results for a thirty-degree motion are shown in Figs. 4 and 5. Figure 4(a) illustrates the velocity pattern of the θ_1 axis motion at a certain fixed position of the θ_2 axis. The corresponding positional error in the θ_2 axis is given in Fig. 4(b), where the plots are made from the error between the reference position and the actual position obtained from the pulse number of the encoder. Using the traditional method, the θ_2 axis for the zero command

Fig. 3 Structure of the experimental system

Table 1 Parameters for θ_2 axis

θ_2	$P(\delta)$		$R(\delta)$	
0°	$\delta^2 + 0.00128 \delta$	-0.00059δ	$+0.00164$	-0.00076
	-0.00093δ	$\delta^2 + 0.00355 \delta$	-0.00099	$+0.00377$
30°	$\delta^2 + 0.00165 \delta$	-0.000623δ	$+0.00163$	-0.00062
	-0.00080δ	$\delta^2 + 0.00343 \delta$	-0.00085	$+0.00364$
60°	$\delta^2 + 0.00200 \delta$	-0.00041δ	$+0.00165$	-0.00034
	-0.00047δ	$\delta^2 + 0.00322 \delta$	-0.00050	$+0.00342$
90°	$\delta^2 + 0.00256 \delta$	$+0.00000 \delta$	$+0.00188$	$+0.00000$
	$+0.00000 \delta$	$\delta^2 + 0.00313 \delta$	$+0.00000$	$+0.00332$

Table 2 Control parameters for θ_2 axis

θ_2	G		K_1		H_2		H_1		H_0	
0°	+2.37838	+0.47568	-0.26248	-0.00059	-65.13282	-13.10967	-4.08435	-0.81687	-0.04471	-0.00894
	+0.62118	+1.03529	-0.00093	-0.26020	-17.11968	-28.03038	-1.06674	-1.77790	-0.01168	-0.01946
30°	+2.31579	+0.39368	-0.26211	-0.00063	-63.28341	-10.84984	-3.97687	-0.67607	-0.04353	-0.00740
	+0.53835	+1.03529	-0.00080	-0.26032	-14.83688	-28.03027	-0.92451	-1.77790	-0.01012	-0.01946
60°	+2.14634	+0.21463	-0.26175	-0.00041	-58.50665	-5.91520	-3.68588	-0.36859	-0.04035	-0.0040
	+0.31059	+1.03529	-0.00047	-0.26053	-8.55964	-28.03007	-0.53337	-1.77790	-0.00584	-0.01946
90°	+1.83333	+0.00000	-0.26119	+0.00000	-49.79632	+0.00000	-3.14836	+0.00000	-0.03447	+0.00000
	+0.00000	+1.03529	+0.00000	-0.26063	+0.00000	-28.02998	+0.00000	-1.77790	+0.00000	-0.01946

$$q(\delta) = \delta^2 + 0.27490 \delta + 0.01880$$

shows large variations of motion (Fig. 4(b)), while the decoupled system provides sufficient compensation for the coupling forces and shows a small fluctuation amplitude in the θ_2 axis. Figure 5(b) shows the result for the θ_2 axis drive only, with the velocity command of Fig. 5(a) at a certain fixed position of the θ_1 axis. Both control methods can reduce the coupling force effects but the proposed method shows smaller fluctuation amplitudes than the PI control.

The experiments on a ninety-degree motion were

accomplished in the same way, and satisfactory results were obtained as shown in Figs. 6 and 7.

6.2 Experiment II

Next, experiments were conducted for both axis drives with the velocity commands given in Fig. 8(a) for a thirty-degree motion and Fig. 9(a) for a ninety-degree motion. It is clearly shown in Figs. 8(b), 8(c), 9(b) and 9(c) that the proposed approach shows fewer irregularities at the beginning and end of motions, especially θ_2 axis motions, thus indicating a

Fig. 4 θ_1 axis motion at the fixed θ_2 axis ($0^\circ \rightarrow 30^\circ$)

Fig. 6 θ_1 axis motion at the fixed θ_2 axis ($0^\circ \rightarrow 90^\circ$)

Fig. 5 θ_2 axis motion at the fixed θ_1 axis ($0^\circ \rightarrow 30^\circ$)

Fig. 7 θ_2 axis motion at the fixed θ_1 axis ($0^\circ \rightarrow 90^\circ$)

(a) position pattern of θ_1 and θ_2 axes (b) positional error of θ_1 axis (c) positional error of θ_2 axisFig. 8 Both axis motions ($0^\circ \rightarrow 30^\circ$) (a) position pattern of θ_1 and θ_2 axes (b) positional error of θ_1 axis (c) positional error of θ_2 axisFig. 9 Both axis motions ($0^\circ \rightarrow 90^\circ$)

shorter positioning time in comparison with the PI control.

The robustness may be emphasized by the following results.

1) The positional error is finally zero even though Coulomb friction is present during the manipulator motion.

2) The positional error converges to zero without losing manipulator stability although the error due to the system model is shown, for example, by ignoring the velocity term.

These experimental results suggest the effectiveness of the proposed method. However, further investigation is needed to reduce the initial positional error shown in Figs. 8 and 9. One possibility is to apply deadbeat control by increasing the system gain. In this case, the noise would also be amplified and the system might finally lose stability. This problem remains to be considered.

7. Conclusion

We proposed a simple and practical decoupling control algorithm together with a reduction in positioning time in the point-to-point control motion. This algorithm can perform the decoupling and the pole assignment simultaneously by using an observer with disturbance compensation. One advantage of the algorithm is the small amount of calculation even though it deals with a multivariable system with a disturbance compensator. Only twenty cycles of products and summations are required to implement the algorithm. Parameter calculations are mostly done by comparing the coefficients between polynomials. Furthermore, in this approach, all parameters for the disturbance compensation, decoupling control and pole assignment are obtained at the same time. This enables a CAD system to be constructed easily for control. Although the SCARA-

type robot has a high reduction ratio for the drive mechanism, the proposed method can greatly reduce the overshoot at the end of the motion compared with PI servo control for each axis, thus enabling the reduction of positioning time.

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