Sound Field Assessment by Measurement of Sound Pressure Distribution Close to Machine Noise Source*

(Estimation of Normal Particle Velocity at an Aperture by Single-Plane Sound Pressure Measurement)

Haruo HOUJOH**, Shun-ichi OHSHIMA**
and Kiyohiko UMEZAWA**

For the assessment of machinery noise, we describe a method of estimating the normal particle velocity distribution from a sound pressure distribution measured at a single aperture close to the sound source based on near acoustic holography (NAH). In NAH, it is required that the distance between the surface of the sound source and the measurement aperture is one-tenth of a wavelength. In practice, the distance between the machinery and the measurement aperture is too large for NAH because of the 3-dimensional features of the machine, so this method is investigated under the condition that the sound pressure is measured at about one wavelength from the sound source. The size of the measurement aperture was determined to be $1\text{ m} \times 1\text{ m}$. Through investigation by simulation, it has been shown that the effect of evanescent waves should be taken into consideration. Experimental verification shows that this method can be practically applied in echoic situations.

Key Words: Sound, Measurement, Machinery, Noise, Near-Field Acoustic Holography, Sound Pressure, Particle Velocity, Green's Function, Fourier Transform, Simulation

1. Introduction

Recently, quieter machinery has become necessary, and sound field assessment around such machinery is important for controlling its vibration and noise. For this purpose, various sound field evaluations are available, for example, localization of the sound source, sound intensity distribution measurement, and far-field directivity estimation. These methods require measurement of the two-dimensional distribution of the sound field because of its three-dimensional propagation. In addition, the complex value, i.e., the amplitude and phase, of the sound pressure is necessary to enable processing such as that used for holographic measurement.

The acoustic holography developed by Umezawa

and co-workers^{(1),(2)} yielded the first result of sound source visualization for practical machinery noise (gear sound) by measuring at a plane surface several wavelengths from the objective machinery. This method can be applied in an echoic environment because of its correlation processing feature. However, since the spatial resolution of the reconstructed field is almost equal to the wavelength of the sound, localization of the sound source becomes vague at a low frequency.

The near-field acoustic holography (NAH), which is able to reconstruct the sound source distribution regardless of the resolution, was proposed by Maynard and co-workers^{(3),(4)}. In this method, sound propagation is treated in the Fourier domain. Then, the convolution process due to sound transmission is substituted for the multiplying form. Thus, reconstruction of the sound field is firstly achieved in the Fourier domain by dividing the Fourier-transformed sound pressure by the Fourier-transformed Green's function. And secondly the inverse Fourier transform is applied to present in the physical domain. Although

* Received 6th March, 1997. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 62, No. 602, C (1996), p. 4032-4039 (Received 23rd January, 1996)

** Precision and Intelligence Laboratory, Tokyo Institute of Technology, Nagatsuta-cho 4259, Midori-ku, Yokohama 226, Japan

the spatial resolution does not depend on the wavelength, measurement of a near field is necessary because the 'evanescent wave' component must be acquired. This means that the measurement is valid at a low frequency where the measurement can be carried out relatively close to the source with respect to its wavelength. It is also reported that this method is applicable to the estimation of far-field propagation and sound intensity.

One of the authors showed experimentally that the normal particle velocity distribution over a measurement surface is a valid means of grasping the sound source distribution as well as estimating the far-field directivity⁽⁵⁾. Therefore, it is important that a precise method of estimating normal particle velocity is established on a practical basis.

In each of the methods mentioned above, either a scanning microphone or a microphone array is used to measure the distribution in the sound pressure field. However, no universal conditions of measurement exist because the parameters of wavelength, size of the object sound source, and distance between sound source and measurement aperture are variable, in conjunction with the wavelength.

Generally, the sound field of noise emitted from a machine was measured in an anechoic chamber. However, it is difficult to bring working machinery into the anechoic chamber. Therefore, it is preferable to measure the sound under an *in-situ* condition. In this case, measurement close to the source is expected to avoid the effects of reflective and outgoing waves even in the quasi-echoic environment.

If the sound is emitted from a planar surface, sound transmission can be treated as a boundary condition scheme at the surface. Machinery, however, is composed of various parts, and its surfaces have three-dimensional complex expansion. Therefore, the measurement aperture cannot be placed close to the objective sound source because of geometrical interference with the complex front surface of the machinery.

In this study, a method of estimating normal particle velocity based on the NAH is examined and an optimal estimation method is proposed. This method enables the normal particle velocity distribution on the aperture to be determined from the measured single plane sound pressure distribution in the identical plane. Thus, a microphone array can be used for measurement instead of an array of intensity probes for particle velocity which is larger than usual microphone and disturbs the sound field and is more costly because it requires a pair of matched microphones at each grid point of the probe array. The final goal of this study is to establish an instantaneous

measurement which can be practically applied to various machines under an operating condition.

The conditions of measurement are decided under practical considerations as follows: the frequency is about 1 kHz, the distance between the sound source and measurement aperture is about one wavelength, the size of the measurement aperture is 1 m by 1 m, and the number of discrete measurement points is 16 by 16.

In this paper, a method of estimating the normal particle velocity under the conditions mentioned above is proposed, and the treatment of the finite region in the calculation is considered. Computational simulations and the results of experimental verification are shown.

2. Calculation Method for Normal Particle Velocity Distribution

2.1 Theory of normal particle velocity calculation in an infinite plane

An infinite measurement plane is defined perpendicular to the z -axis at $z=z_M$ in the Cartesian coordinate system, and the sound pressure distribution over the plane is expressed as $P_M(x, y, z_M)$ in terms of complex value. Point sources are assumed to exist on the plane $z=0$, called the source plane. Then, P_M is written as a two-dimensional convolution integral,

$$P_M(x, y, z_M) = \iint_{-\infty}^{\infty} P_0(x', y', 0) \times G(x-x', y-y', z_M) dx' dy', \quad (1)$$

where $G(x, y, z_M)$ is Green's function for a free field with an infinite boundary, and $P_0(x, y, 0)$ is the sound pressure distribution on the source plane ($z=0$).

Applying a Fourier transform to P_M , and in Eq. (1), we obtain

$$\hat{P}_M(k_x, k_y, z_M) = \hat{P}_0(k_x, k_y, 0) \cdot \hat{G}(k_x, k_y, z_M), \quad (2)$$

where $\hat{P}_M$, $\hat{G}$ and $\hat{P}_0$ are Fourier transforms as a function of spatial frequencies k_x , k_y , and k_z , respectively. Therefore, the Fourier-transformed sound pressure distribution on an arbitrary plane parallel to the measurement plane at $z(>0)$, $\hat{P}_1(x, y, z)$, is calculated from $\hat{P}_M$ as

$$\hat{P}_1(k_x, k_y, z) = \hat{P}_M(k_x, k_y, z_M) \cdot \frac{\hat{G}(k_x, k_y, z)}{\hat{G}(k_x, k_y, z_M)}. \quad (3)$$

Since the normal particle velocity U_z is related with the sound pressure as

$$U_z = \frac{1}{j\omega\rho} \cdot \frac{\partial}{\partial z} P, \quad (4)$$

where P is sound pressure, ω is angular frequency, and ρ is density of medium. U_z can be obtained by substituting the inverse Fourier transform of Eq. (3) into Eq. (4) as

$$U_z(x, y, z) = \frac{1}{j\omega\rho} \cdot \mathbf{F}^{-1} \left[\hat{P}_M(k_x, k_y, z_M) \right]$$

$$\times \frac{\frac{\partial}{\partial z} \hat{G}(k_x, k_y, z)}{\hat{G}(k_x, k_y, z_M)}, \quad (5)$$

where F^{-1} expresses the inverse Fourier transform. Equation (5) yields the normal particle velocity distribution over the plane placed at z from the sound pressure $\hat{P}_M$ over the measurement aperture.

The normal particle velocity over the measurement plane can be obtained by applying $z=z_M$ in Eq. (5).

2.2 Adaptation to the finite aperture measurement

Equation (5) is derived theoretically when the measurement plane is infinite and continuous. In practice, however, finite and discrete operation is required for obtaining the particle velocity based on Eq. (5). Therefore a discrete Fourier transform (DFT) is applied in the operation, which leads to the assumption of the wraparound sound field caused by periodically located point sources, i.e., imaginary sources. Figure 1 shows the unexpected phenomenon due to these imaginary sources, which is equivalent to a process of circular convolution. This means that the sound field over the real measurement aperture should be affected by imaginary sources with respect to theoretical matching. To avoid this effect, zero padded aperture which consists of the measurement aperture and surrounding 'guard band' of zeros, is applied to calculation. By this zero padded aperture, imaginary sources are kept away from the measurement aperture⁽³⁾.

Although the effect of imaginary sources is avoided by incorporation of the guard band, it still exists since Green's function is defined in the infinite space. The NAH places the measurement plane quite close to the source and uses a measurement plane which is large enough to decay sufficiently at the perimeter. In contrast to the situation described above, we limit the area of measurement aperture to 1 m square, and the distance between the sound source and the measurement aperture to about one wavelength away, from the practical point of view. This condition causes the amplitude of pressure to decay insufficiently at the perimeter of the measurement aperture, and may lead to a considerable circular convolution effect. There-

Fig. 1 Effect of the circular convolution

fore, in this study, not only the infinite Green's function, but also the truncated Green's function whose contribution is limited to twice the width of the measurement aperture are considered, and the sampled aperture of DFT is four times the area of the measurement aperture, as shown in Fig. 2.

The characteristics of infinite and truncated Green's functions are examined in the next section, and estimation results of normal particle velocity by means of each Green's function are shown in section 3.

2.3 Green's function for DFT

In this section, each Green's function, introduced in the previous section, is discussed with respect to its characteristics from the viewpoint of Fourier transformation.

2.3.1 Green's function defined over an infinite region Green's function of free space specified by Dirichlet's boundary condition is known as

$$G = \frac{-z(jkr+1)\exp(-jkr)}{2\pi r^3}, \quad r = \sqrt{x^2 + y^2 + z^2}, \quad (6)$$

where r is the distance between points in space, and k is the wave number. The Fourier-transformed equation from Eq. (6) is

$$\hat{G}(k_x, k_y, z) = \begin{cases} \exp(-jz\sqrt{k^2 - k_x^2 - k_y^2}), & k_x^2 + k_y^2 \leq k^2 \\ \exp(-z\sqrt{k_x^2 + k_y^2 - k^2}) & k_x^2 + k_y^2 > k^2 \end{cases}, \quad (7)$$

which has two cases depending upon the value of spatial frequency in the x and y directions, k_x and k_y , one inside the circle of radius k (called k -circle⁽³⁾), and the other outside. The inner component of the k -circle presents the planar wave component which propagates to the far field, while the outer component of the k -circle presents the spherical wave component, called the evanescent wave, which is effective in the near field and whose effect decays exponentially as the sound wave travels away.

The transfer function, which determines the relationship between sound pressure distribution and normal particle velocity in the Fourier domain, is derived by substituting Eq. (7) under the infinite condition into Eq. (5). The derived transfer function in the k -space, or Fourier domain, can be expressed as

Fig. 2 Schematic of definite area of Green's function

$$\frac{\partial}{\partial z} \hat{G}(k_x, k_y, z_M) = \begin{cases} -j\sqrt{k^2 - k_x^2 - k_y^2}, & k_x^2 + k_y^2 \leq k^2 \\ -\sqrt{k_x^2 + k_y^2 - k^2}, & k_x^2 + k_y^2 > k^2 \end{cases} \quad (8)$$

Since this equation is independent of z , the measurement aperture can be placed at an arbitrary position regardless of the distance.

Figure 3(a) shows the absolute value of Eq.(8) in the k -space, and shows that it increases as k increases outside the k -circle. It is clear that the contribution of the higher spatial frequency component of sound pressure is significant. In the figure, the sampled aperture of the transfer function of Eq.(8) is twice the measurement aperture in Fig. 2.

2.3.2 Area-limited Green's function The Fourier transform of the area-limited Green's function cannot be obtained analytically, so the truncated transfer function in the k -space is estimated numerically by the DFTs of Green's function and its partial derivative assuming $z = z_M$. Figure 3(b) shows the absolute value of the truncated transfer function in the k -space. The resolution in the k -space is twice as

(a) Theoretical, infinite result

(b) Proposed, finite result

(c) Comparison of (a) and (b) within k -circle (Eq.(8))

—○— : (a) —▲— : (b)

Fig. 3 Absolute value of transfer function in Fourier domain

fine as the case of Fig. 3(a) because the size of the area of interest is twice the theoretical one.

The shape of the transfer function inside the k -circle is similar to the shape of the theoretical transfer function shown in Fig. 3(a). However, large differences exist outside the k -circle, as shown in the two results. In Fig. 3(b), irregularities appear at the rim of the k -circle.

We assume that the distance between the sound source and the measurement aperture is about one wavelength, thus it is expected that the component of the evanescent wave, or the component outside the k -circle in the Fourier domain, does not contribute significantly. Furthermore, there is good agreement between the two transfer functions in the region inside the k -circle. Then, the third case of an alternative transfer function which is equal to Fig. 3(a) inside the k -circle but zero outside the k -circle is also investigated.

3. Examination by Simulation

3.1 Simulation method

The proposed method for estimation of normal particle velocity from sound pressure distribution is investigated by numerical simulation. Figure 4 shows the geometrical configuration of the measurement. The size of the measurement aperture is determined from practical considerations; the size of the aperture L is 1 m square, and is divided into 16 by 16 measurement points. The sound source is assumed to be a point source and its wavelength is $\lambda = 0.25$ m (about 1.3 kHz), located on the origin shown in Fig. 4. The distance between the point source and the measurement aperture is equal to the wavelength, $d = \lambda$.

The estimation procedure is as follows. The sound pressure distribution in the measurement aperture in terms of the point source is defined as

$$P(r) = \frac{jk\rho cQ}{4\pi r} \exp(-jkr), \quad (9)$$

$$r = \sqrt{x^2 + y^2 + z^2}.$$

Then, the normal particle velocity distribution is calculated over the same plane through DFT and inverse DFT processes using each of the three kinds of transfer functions presented previously. The estimated result is compared with the theoretical normal

Fig. 4 Configuration of measurement in simulation

particle velocity given by

$$U_z(r) = \frac{\dot{Q}}{4\pi r^2} (1 + jkr) \exp(-jkr). \quad (10)$$

The results are displayed in two charts which show the amplitude and phase distribution over the measurement plane. Although the estimated normal particle velocity displayed is 1 m by 1 m, and has 16 by 16 measurement points, a two-dimensional DFT was

Fig. 5 Theoretical particle velocity distribution (top: amplitude, bottom: phase)

Fig. 6 Error distribution of estimated particle velocity based on theoretical infinite transfer function

achieved with 32×32 and 64×64 points, for the cases described in sections 2.3.1 and 2.3.2, respectively.

Figure 5 shows the theoretical normal particle velocity distribution, where the distance between the point source and the measurement aperture is one wavelength. The amplitude is referenced to the maximum value of amplitudes within the estimated aperture as 0 dB.

3.2 Effect of the different transfer functions

The three kinds of simulated normal particle velocity obtained from the three corresponding transfer functions, i.e., infinite and analytical transfer function, truncated and numerical one, and analytical transfer function neglecting the evanescent wave component are compared. In this section, to confirm the results, the distribution of deviations of the estimated aperture with respect to both amplitude and phase are evaluated from theoretical values.

Figure 6 shows the distribution of deviations, or errors, caused by estimation of normal particle velocity by means of the infinite transfer function. The upper chart shows amplitude deviations in the two-dimensional plane, while the middle chart shows phases. The lower chart shows the deviation values on the A-A' line indicated in the upper and middle charts. Figures 7 and 8 also show the deviations caused by estimation by means of the truncated transfer function and the transfer function neglecting the evanescent wave, respectively.

In Fig. 6, although large deviations exist only on the edge of the estimated aperture, good estimation is achieved inside the aperture. In Fig. 7, deviation tends to fluctuate over the whole aperture. In Fig. 8, deviations are observed over the whole aperture with monotonous trend.

In the case of Figs. 7 and 8, compared with the case of Fig. 6, the evanescent wave component in the transfer function is not correctly accounted for. Therefore, it is clear from these figures that regardless of the distance between the sound source and measurement aperture, the infinite transfer function, which can describe the evanescent wave component, should be adopted.

The cause of the large deviation at the edge of the aperture in Fig. 6 is the immediate step to zero at the zero padding boundary on the measurement plane involving the sound pressure distribution. To reduce this deviation, the measurement aperture should be expanded, and the zero padding boundary kept away from the estimation aperture.

Figure 9 shows the results of estimation using expanded measurement apertures. Figure 9(a) is the result for a row of measurement points added at each of the four sides of the measurement aperture, 18×18

Fig. 7 Error distribution of estimated particle velocity based on numerical area-limited finite transfer function

Fig. 8 Error of estimated particle velocity using transfer function, except for the evanescent wave

(a) Expanded to 18×18

(b) Expanded to 20×20

Fig. 9 Effect of expanded measurement region

Fig. 10 Schematic of effect of distance between point source and measurement region of 'd'

points in total and $1.125 \text{ m} \times 1.125 \text{ m}$ in size, and Fig. 9 (b) is that where two rows of measurement points are added at each side, 20×20 and $1.25 \text{ m} \times 1.25 \text{ m}$. It is clarified that the deviation at the four sides of the estimation aperture caused by the zero padding boundary is swept away from the estimation region. Thus it is preferable for the expanded measurement aperture to be used to ensure reliable estimation over the estimation region. The size of the measurement aperture is selected to be 1.25 times the estimation aperture in this paper.

3.3 Effect of the location of point source

As shown in section 3.2, the infinite transfer function of Eq.(8) is independent of the distance between the sound source and the measurement aperture. Namely, this estimation method is suitable for estimating the actual sound field produced by various sources located at various depths from the measurement plane. In this section, first, the effects of the depth of the point source is investigated. Then, the effect of the lateral position is examined.

3.3.1 Effect of the depth of the point source from the measurement aperture Figure 10 shows the measurement configuration in the simulation. The distance d , depth, in Fig. 10 is varied, and the normal

particle velocity is estimated over the measurement aperture. Figure 11 shows the deviation of estimated results: (a) $d=0.5\lambda$, (b) $d=1.0\lambda$, (c) $d=1.5\lambda$, (d) $d=2.0\lambda$. The result of the nearest case, Fig. 11(a), exhibits phase deviation at the edge of the aperture, which is due to the large rate of phase distribution variation in the measurement aperture. In contrast, the deviation and its fluctuation decrease as the depth (d) increases.

3.3.2 Effect of lateral position Figure 12 shows the deviations of results estimated about the offset point source from the z -axis: (a) 0.25 m (1.0λ) offset from the z -axis, (b) 0.5 m (2.0λ) offset. The distance between the point source and the measurement aperture is λ . In the case of Fig. 12(a), good agreement is observed between estimated and theoretical particle velocity. In the case of Fig. 12

(a) $d = 0.5\lambda$

(b) $d = 1.0\lambda$

(c) $d = 1.5\lambda$

(d) $d = 2.0\lambda$

Fig. 11 Results of effect of distance between point source and measurement region

(b), however, deviations exist at the other side of the point source about the point of origin. Amplitude exhibits deviation of about 1 dB, while phase deviates by about 10 degrees. From this result, in practice, the sound source should be located near the z -axis with a small offset to obtain better information regarding propagation from the source. This means that the measurement aperture should not be positioned such that the sound source is located in front of the edge. There is, however, an allowable range around the center of the plane.

4. Experimental Verification

The actual sound field produced by loud speakers was measured and the proposed method was applied to verify the estimation method. Figure 13 shows the sound source and measurement configuration. The speaker box is 320 mm cube, and incorporates the two speakers whose diameter is 80 mm. A sinusoidal wave of 1375 Hz is emitted from the two speakers driven with a power ratio of two to one, in phase. The distance between the center of the box and the measurement aperture is equal to one wavelength. The specification of the measurement aperture and the

(a) 0.25 (1.0λ) offset from z -axis

(b) 0.5 (2.0λ) offset from z -axis

Fig. 12 Results of effect of traversed point source

Fig. 13 Experimental configuration

estimated aperture is equivalent to that of the simulation. Basically, the measurement environment has no special facilities for acoustic absorption except for a urethane foam absorber on the floor and glass wool on the ceiling near the measurement aperture. A scanning microphone is used for measurement of the distribution of sound pressure over the x - y plane and for measurement of particle velocity by shifting in the z direction to give spatial differentiation. The measurement was validated using a phase reference signal to eliminate environmental fluctuation during the measurement.

Figure 14 shows the normal particle velocity distributions estimated by the method presented in this paper; the upper figure shows the amplitude, and the lower one shows the phase. Figure 14(a) shows the normal particle velocity estimated from the difference between the sound pressures on the measurement points 10 mm apart in the direction, that is, two apertures of sound pressure 10 mm apart are measured. Figure 14(b) shows the normal particle velocity estimated from the sound pressure distribution by the proposed method, that is, sound pressure was measured over an expanded single aperture. The deviations of both amplitude and phase between Figs. 14(a) and (b) exist only along the edge of the estimated aperture of Fig. 14(b). Nevertheless, on the whole, good estimation is achieved.

The number of sound pressure measurement points is 512 for the direct particle velocity estimation under condition (a), and 400 in a wider area for Fourier domain estimation, (b). Therefore, proposed method can reduce the number of pressure measurement points while maintaining good estimation results by measuring sound in a wider area. This effect shortens the acquisition time. In addition, when multipoint simultaneous sound pressure measurement becomes possible through the use of a two-dimensional microphone array, normal particle velocity

(a) Result of two-microphone method

(b) Result of proposed method

Fig. 14 Experimental results of particle velocity

estimation can be achieved with less disturbance than when using an intensity probe array which is much more complex in construction. Therefore, the proposed method is considered to be a practical method for estimating normal particle velocity.

5. Conclusion

In this paper, a method of estimating normal particle velocity from a single plane sound pressure distribution measurement has been proposed, and proved to be a practical method by numerical simulation and experimental verification. It was found that the proposed method requires the expansion of the measurement aperture of sound pressure, and requires the theoretical Green's function defined in the k -space and containing the evanescent wave component in spite of the distance of one wavelength between the

sound source and the measurement aperture. However, the method can reduce the number of measuring points.

References

- (1) Ueha, S., Fujinami, M., Umezawa, K. and Tsujiuchi, J., Imaging of Acoustic Radiation Sources with Acoustical Holography, *Optica Acta*, Vol. 23, No. 2(1976), p. 107-114.
- (2) Umezawa, K. and Houjoh, H., The Sound Radiated from Gears (1st report, Behaviour by Means of Acoustical Holography), *Bulletin of the JSME*, Vol. 24, No. 195(1981), p. 1651-1657.
- (3) Maynard, J.D., Williams, E.G. and Lee, Y., Nearfield Acoustic Holography: I. Theory of generalized holography and the development of NAH, *JASA.*, Vol. 78(1985), p. 1395-1413.
- (4) Veronesi, W.A. and Maynard, J.D., Nearfield Acoustic Holography II. Holographic Reconstruction Algorithms and Computer Implementation, *JASA.*, Vol. 81(1987), p. 1307-1322.
- (5) Umezawa, K., Houjoh H. and Kitano, T., Estimation of Plate Vibration and its Far Field Sound by Means of Near Field Sound Measurement, *Bulletin of the JSME*, Vol. 29, No. 252(1986), p. 1874-1880.