Simulation of Trabecular Surface Remodeling based on Local Stress Nonuniformity*

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In this study, to investigate mechanical remodeling of cancellous bone from the viewpoint of trabecular surface remodeling due to trabecular level mechanical stimuli, a rate equation for trabecular surface remodeling is proposed based on the uniform stress hypothesis, which suggests that nonuniformity in the local stress distribution on the trabecular surface is the driving force for remodeling. A proposed local rate equation for trabecular surface remodeling is applied to a computational simulation of morphological changes, in which the trabeculae are modeled as an assemblage of two-dimensional boxel elements and their morphological changes are simulated by removal/addition of the elements from/to the trabecular surface. The basic features of the proposed rate equation and simulation method are investigated through remodeling simulations applied to the basic trabecular-level structure, cancellous-bone-level structure and the proximal femur. In all cases, the trabecular orientation, thickness and connectivity are adapted to the mechanical environment, which demonstrates the capability of the proposed rate equation for computational prediction of the adaptation phenomenon and supports the hypothesis regarding the optimality of trabecular architecture as a mechanical load bearing structure.

Key Words: Biomechanics, Bone Remodeling, Cancellous Bone, Trabecular Surface Remodeling, Adaptation

1. Introduction

Bone has mechanical functions to provide support for the body and to serve as a protective covering for the organs, and its morphological formation and remodeling are affected by its mechanical environment⁽¹⁾. The functional adaptation mechanism for bone has long been a subject of interest in bone mechanics and has been investigated ever since an understanding of the trabecular architecture of cancellous bone as a mechanical structural system was developed^{(2),(3)}. Based on experimental observations⁽⁴⁾⁻⁽⁷⁾ from cellular to tissue levels, mathe-

matical models of bone remodeling have been proposed, and computational simulations have been carried out for quantitative predictions of bone formation and resorption⁽⁸⁾⁻⁽¹⁹⁾. In these bone-remodeling simulations, the cancellous bone is modeled as a continuous material, and the mechanical stimulus that regulates the remodeling is related to continuum-level variables, such as macroscopic stress, strain and strain energy density.

Because the morphological changes in trabecular bone are due to microscopic resorption and formation on the trabecular surface by cellular activities⁽²⁰⁾, the local stimulus at the cellular level is considered to be the primary factor that affects bone remodeling⁽²¹⁾⁻⁽²⁴⁾. Thus, in modeling and simulation of mechanical bone remodeling, it is essential to relate the morphological changes in the trabeculae to the cellular-level mechanical environment^{(7),(15),(19)}. However, the mechanisms in which the regulation of microscopic mechanical conditions relate to the microscopic morphological changes in the trabeculae and how the microscopic morphological changes result in the changes of macroscopic trabecular architecture have not yet been clarified.

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In this study, to investigate the relation between cellular-level mechanical stimuli regulating the trabecular surface remodeling and the macroscopic trabecular architecture, a surface remodeling rate equation considering the microscopic mechanical environment at the trabecular level is proposed based on the uniform stress hypothesis which assumes that the local stress distribution on the trabecular surface is a driving force of cellular remodeling activities. In addition, the proposed local rate equation for trabecular surface remodeling is applied to a two-dimensional finite element simulation of morphological changes under a changing mechanical environment. Finally, the capabilities of the proposed surface remodeling rate equation and the direct simulation method are investigated through remodeling simulations on trabecular, cancellous and load bearing system levels.

2. Model of Trabecular Surface Remodeling

From the mechanical point of view, a model of trabecular surface remodeling is proposed to express the morphological changes in the trabeculae as a local stress regulation process.

2.1 Resorption and formation by surface remodeling

Trabecular surface remodeling in cancellous bone has a cycle of five successive stages of quiescence,

Fig. 1 Trabecular surface remodeling in hierarchical time scale

activation, resorption by osteoclasts, reversal and formation by osteoblasts⁽²⁵⁾, as illustrated in Fig. 1. The relative difference in amount between downward erosion by resorption⁽²⁶⁾ and upward refilling of a cavity by formation⁽²⁰⁾ in one cycle, moving in the direction perpendicular to the trabecular surface, determines the local movement of the trabecular surface. Repetitive cycles lead to macroscopic changes in the trabecular architecture in cancellous bone.

Based on the cyclical nature of this process, a trabecular surface remodeling rate equation is expressed considering two different hierarchical time scales of t and T as shown in Fig. 1, where t denotes the time on the cellular activity scale, and T denotes a time longer enough than ΔT of the characteristic period of one cycle in the remodeling turnover process. Let $\dot{m}(=\partial m/\partial t)$ denote the rate of surface movement by remodeling in the outer direction perpendicular to the surface; that is, $\dot{m}=0$: quiescence, $\dot{m}<0$: resorption, and $\dot{m}>0$: formation. Averaging $\dot{m}$ over the period ΔT for one cycle of remodeling, trabecular surface remodeling rate $\dot{M}(=\partial M/\partial T)$ is defined as

$$\dot{M} = \frac{1}{\Delta T} \int_{\Delta T} \dot{m} dt. \quad (1)$$

In the following sections, the expression of $\dot{M}$ is discussed for surface remodeling as a local stress regulation process.

2.2 Local mechanical factors stimulating remodeling

Regulation of the osteoclastic bone resorption and the osteoblastic bone formation at local sites has been postulated to occur by two mechanisms, that is, systematic regulation by hormones and local regulation by, for instance, growth factors⁽²¹⁾⁻⁽²⁴⁾. We assume that the former relates to the quantitative volumetric changes in bone and that the latter relates to the morphological changes in trabecular architecture. To discuss how the microscopic mechanical conditions relate to the trabecular surface remodeling and how the macroscopic trabecular architecture emerges from the local surface remodeling, we employed local mechanical conditions as a local mechanical stimulus regulating the trabecular surface remodeling based on the fact that some cell-to-cell networks exist^{(27),(28)}. Thus, we consider the positive scalar function σ as a mechanical stimulus that bone cells can sense, and assume that nonuniformity is a driving force for remodeling. To express the nonuniformity of the scalar function in space, the differential form^{(13),(17)} using the Laplacian operator ∇^2 or the integral form with suitable weight functions can be used. In this study, we use the latter integral form considering the

stresses $\sigma_c(\mathbf{x}_c)$ and $\sigma_r(\mathbf{x}_r)$ at point $\mathbf{x}_c$ and its neighbor point $\mathbf{x}_r$. Averaging σ_r with the weight function $w(l)$, the representative stress σ_d is determined as

$$\sigma_d = \int_S w(l) \sigma_r dS / \int_S w(l) dS, \quad (2)$$

where S denotes the trabecular surface, $l = |\mathbf{x}_r - \mathbf{x}_c|$, and the weight function $w(l)$ takes a non-zero value at the neighbor point ($l < l_L$). As the driving force of local trabecular remodeling, we use the relative value of σ_c to σ_d , that is defined as

$$\Gamma = \ln\left(\frac{\sigma_c}{\sigma_d}\right). \quad (3)$$

This function Γ , which expresses the convexity of the stress distribution, takes a positive value with convex stress distribution and a negative one with concave distribution.

2.3 Trabecular surface remodeling rate equation toward uniform stress distribution

For a mechanical load bearing structure, the uniform stress condition is one optimal criterion leading to uniform strength. Based on the uniform stress hypothesis in the remodeling equilibrium, we propose a rate equation for trabecular surface remodeling as a stress regulation process toward uniform stress distribution on the surface. Using the stress convexity Γ in Eq. (3), the remodeling rate equation is written as

$$\dot{M} = F(\Gamma) = \begin{cases} \geq 0 & (\Gamma > 0): \text{Formation} \\ \leq 0 & (\Gamma < 0): \text{Resorption} \end{cases} \quad (4)$$

to express the formation at the site with convex distribution of σ and resorption at the site with concave distribution of σ , where the function F determines the feature of the rate equation. In this equation, a local remodeling rate is determined using only local mechanical conditions evaluated by Γ without using the systematic global stresses prescribed *a priori*, such as goal or optimal stress.

3. Simulation Method of Trabecular Surface Remodeling

Using a two-dimensional finite element method, the simulation method of trabecular surface remodeling is proposed in this section.

3.1 Finite element discretization of trabecular architecture

Trabecular architecture in a two-dimensional rectangular region, as shown in Fig. 2(a), is discretized using square crossed-triangles finite elements with regular mesh divisions as shown in Fig. 2(b). The trabecular surface movement by remodeling is expressed by adding and removing square elements on the surface in response to the mechanical conditions determined by a finite element analysis. The element size should be smaller than the dimension of the basic multicellular unit, so that, in the remodeling simula-

Fig. 2 Discretization of trabeculae by finite elements

Fig. 3 Surface remodeling for discretized trabecular bone

tion, trabecular width is discretized with at least 3 elements as an initial shape. The trabecular bone part is assumed to be a homogeneous and isotropic elastic material with Young's modulus $E = 20$ GPa and Poisson's ratio $\nu = 0.3$, and the marrow is not considered to be a cavity in the finite element analysis.

3.2 Nonuniformity of stress on trabecular surface

The stress convexity Γ on the trabecular surface as the driving force of surface remodeling in Eq. (3) is determined for the discretized trabecular elements. For the surface element c , Γ_c is expressed as

$$\Gamma_c = \ln\left(\frac{\sigma_c \sum_i^N w(l_i^c)}{\sum_i^N w(l_i^c) \sigma_i}\right), \quad (5)$$

where N is the total number of trabecular surface elements, σ_i is the stress at the surface element i at the distance l_i^c , illustrated in Fig. 3. The weight function $w(l)$ is assumed, as a simple case, as

$$w(l) = \begin{cases} 1 - \frac{l}{l_L} & (0 \leq l < l_L) \\ 0 & (l \leq l_L) \end{cases}, \quad (6)$$

representing the monotonic linear decrease with l to zero at $l = l_L$ as shown in Fig. 3. Thus, the cell at $\mathbf{x}_c$ is assumed to be able to sense the mechanical conditions in the neighboring region of $l \leq l_L$.

3.3 Rate of surface movement for a discrete trabecular element

Trabecular surface movement is expressed by adding and removing the finite element on the trabecular surface as shown in Fig. 3, so that the rate of surface movement per one simulation step takes the discrete value of $\dot{M}_c = 1, 0, -1$ (elem./step). However, since $\dot{M}$ naturally takes a continuous value

Fig. 4 Probability function of remodeling rate

in time, as illustrated in Fig. 1, we introduce the continuous probability function $P_{\dot{M}_c}$ in the range of $\Gamma_l \leq \Gamma_c \leq \Gamma_u$ as is shown in Fig. 4, where Γ_l and Γ_u denote the lower and upper threshold values, respectively. The regions $\Gamma_l \leq \Gamma_c \leq 0$ and $0 \leq \Gamma_c \leq \Gamma_u$ are interpolated by sine functions. With the progress of remodeling, the nonuniformity of the surface stress becomes small, that is, $|\Gamma_c|$ approaches zero. The probability of the remodeling $|P_{\dot{M}_c}|$ also approaches zero, which represents the dead zone or lazy zone of remodeling⁽⁹⁾ around the equilibrium state ($\dot{M}_c=0$).

3.4 Remodeling simulation procedure

The trabecular surface remodeling simulation is carried out in the following procedures.

(1) Define the region for analysis, the initial shape of trabecular architecture, and the mechanical constants of the trabeculae as noted in Sec. 3.1.

(2) Analyze the stress σ of the trabecular surface elements by a finite element procedure under the given boundary conditions.

(3) Evaluate the surface stress convexity Γ_c in Eq. (5) for all surface elements as in Sec. 3.2.

(4) Determine the rate of surface movement $\dot{M}_c$, by putting the evaluated value of Γ_c into the relation in Fig. 4 as in Sec. 3.3, and add or remove a surface element.

(5) If remodeling equilibrium is not attained, then return to procedure (2).

Procedures (2) to (5) are one step of the simulation. In this simulation, the equivalent stress is used as the scalar function σ of the stress, and the finite element analysis assumes the plane-strain condition. The volumetric change in the region due to remodeling is controlled to remain constant for every step by shifting the equilibrium point of Γ_c in Fig. 4, assuming that the quantitative volumetric change is caused by global systematic regulation and not by local regulation.

4. Remodeling at Trabecular Level

To investigate the primary features of the morphological changes of trabeculae at the microstructural level regulated by the proposed remodeling rate equation, the remodeling simulation is carried out for structural units of trabeculae⁽²⁹⁾ under compressive loading.

Fig. 5 Boundary condition for trabecular-level-structure model

4.1 Trabecular model under compressive loading

Single or a few trabeculae are modeled as shown in Figs. 5 and 6. Rigid plates are attached at upper and lower surfaces and a uniformly distributed compressive loading of 2.0 MPa is applied to the upper rigid plate. The whole area, including the rigid plates, are discretized by 40×40 square finite elements as shown in Fig. 5, in which each element is $12.5 \mu\text{m}$ in length; and the whole region is $500 \mu\text{m} \times 500 \mu\text{m}$. The parameter l in the weight function $w(l)$ of Eq. (6) is set to $200 \mu\text{m}$, equal to the length of 16 elements. The threshold values of remodeling are set to Γ_u , $\Gamma_l = \pm 0.05$. In this section, three characteristic patterns of the trabecular structure, Models A to C, whose initial shapes are shown in the left row in Fig. 6, are simulated under the same boundary conditions.

4.2 Morphological changes of trabeculae

Morphological changes in trabeculae and the regulation of the distribution of equivalent stress σ by surface remodeling under compressive loading are shown in Fig. 6, where each column shows the progression from the original shape (left) to the final equilibrium shape (right).

For Model A of a trabecular structure that originally has a Z shape, as shown in Fig. 6(a), the center trabecula is subject to a bending moment. The trabecular surface near both ends with relatively higher equivalent stress σ grows, while that in the middle with lower stress resorbs in a few initial steps. At the 4th step, the compressed surface still has high stress, whereas on the growing tensed surface, the equivalent stress σ decreases and the surface remodeling process turns to resorption by approximately the 20th step. Gradually, the stress distribution on the trabecular surface is regulated to become uniform, the trabecula reorientates to the loading direction, and then reaches its equilibrium shape at the 40th step.

For Model B of a trabecular structure that originally has an X shape, as shown in Fig. 6(b), the upper

Fig. 6 Morphological changes of compressed trabecular structure

and lower corners of the crossing region have lower stress, and its right and left corners have higher stress. The original X shape changes to an H shape by remodeling at the 4th step. The horizontal trabecula between two vertical trabeculae becomes thinner at the 8th step and then disappears. Finally, it changes to two parallel trabeculae at the remodeling equilibrium at the 20th step.

For Model C of a trabecular structure that originally has a Y shape, as shown in Fig. 6(c), the surface at the lower corners of the branch grows, whereas the surface at the upper corner resorbs due to the nonuniform stress distribution on the surface. The original Y shape changes to a V shape at the 4th step. These two trabeculae move closer to each other by remodeling at the 16th step. Finally, the remodeling reaches an equilibrium state at the 32th step before they fuse together due to the existence of the lazy zone around the remodeling equilibrium.

4.3 Discussion

The remodeling progression and the equilibrium shape shown in Fig. 6 are the results governed by the proposed local rate equation for trabecular surface remodeling. Depending on the initial shape, even if the same boundary conditions and model parameters are

applied to the simple compressed trabecular structure, they change their shapes to different equilibrium structures. If there is no lazy zone in the remodeling equilibrium, then the trabecular structure that originally has a Y shape as in Fig. 6(c) has two potential alternatives for remodeling. One is the fusion of two trabeculae to a single trabecula as in Fig. 6(a), and the other is separation into a parallel trabecular structure as in Fig. 6(b). In all cases, the trabeculae change their shape to orientate in the loading direction, that is, the adaptation to support the mechanical loading.

5. Remodeling at Cancellous Bone Level

To investigate the morphological changes of trabecular architecture at the cancellous bone level by the proposed remodeling rate equation, the remodeling simulation is carried out for the square region of cancellous bone in the field of uniform stress.

5.1 Square region model of cancellous bone

Consider a two-dimensional square region of cancellous bone under a uniform stress field in which a sufficient number of the trabeculae exist. This region can be considered an infinitesimal area of the cancellous bone under the conditions of the continuum

assumption.

The initial morphology of the model for cancellous bone, circular trabeculae are randomly pasted in the square region as shown in Fig. 7(a). This initial cancellous bone has an isotropic trabecular structure that can be quantitatively evaluated from the measurement of the fabric ellipse⁽³⁰⁾ as shown in Fig. 7(b) determined by the mean intercept length. The square region of the cancellous bone is surrounded by three-layered dummy elements with Young's modulus $E_d (= E/10) = 2.0$ GPa and Poisson's ratio $\nu_d = 0.3$ to apply uniformly distributed external loads as a boundary condition in the uniform stress field with the corresponding normal and shearing stresses. The whole square region is discretized using 150×150 finite elements. The side of one element is $60 \mu\text{m}$ in length, that is, the side of the square region is 9 mm . The parameter l_L is set to $l_L = 600 \mu\text{m}$ ($= 10$ elems.), and the threshold values for the remodeling are set to $\Gamma_u, \Gamma_l = \pm 0.5$.

5.2 Changes of trabecular architecture

Consider the two-dimensional square region of cancellous bone in the stress field with the principal stresses σ_1 and σ_2 , in which the angle between the principal direction of σ_1 and the horizontal axis is denoted by θ_p , as shown in Fig. 8. Two cases of principal stresses are considered: (a) $\sigma_1 = -1.0$, $\sigma_2 = -2.0$ MPa (compression-compression) and (b) $\sigma_1 = 1.0$, $\sigma_2 = -2.0$ MPa (tension-compression) in which the principal stresses σ_1 and σ_2 have the same magnitudes with (a) same signs and (b) opposite signs.

Fig. 7 Cancellous-level structure model

Fig. 8 Boundary conditions for rectangular cancellous bone

The progression of remodeling in the cancellous square region and its fabric ellipses, that are caused by the rotation of the principal direction of stress from $\theta_p = 0^\circ$ to 45° at intervals of 15° , are shown in Fig. 9, in which a and b denote the principal radii of the fabric ellipses. For each direction of θ_p , the changes in the trabecular architecture for 8 steps by remodeling is calculated. In these cases, it is confirmed that 15 steps of remodeling are enough to reach an equilibrium state.

For the case of the same signs for the compression-compression principal stresses at $\theta_p = 0^\circ$, the originally isotropic trabecular architecture changes its morphology to the anisotropic architecture aligning along the direction of the principal stress σ_2 , which is indicated by the principal direction of the fabric ellipse, as shown in Fig. 9(a). The thickness of the trabeculae aligning along the direction of σ_2 increases, whereas that along the direction of σ_1 decreases. Following the rotation of the principal stress direction from $\theta_p = 0^\circ$ to $\theta_p = 15^\circ, 30^\circ$ and 45° , the trabeculae gradually reorientate, which results in the rotation of the principal direction of the fabric ellipse, as shown in the lower column of Fig. 9(a). Because of the disappearance of the trabeculae aligning along the principal direction of σ_1 , the principal radius a becomes longer, whereas b does not significantly change, so the aspect ratio of the fabric principal radii, a/b , increases from 1.02 to 1.46.

For the case of the opposite signs for the tension-compression principal stresses at $\theta_p = 0^\circ$, the originally isotropic trabecular architecture also changes its morphology to anisotropic architecture aligning along the direction of the principal stress σ_1 and σ_2 , as shown in Fig. 9(b). However, the trabecular architecture obtained obviously differs from that for the case with the opposite signs for the principal stress in Fig. 9(a). For the case of the same signs, shown in Fig. 9(a), the trabeculae interconnect at rounded corners, whereas, they interconnect at right angles for the case with opposite signs as shown in Fig. 9(b). As a result, the trabeculae in the direction of σ_1 do not disappear as much in Fig. 9(b), and the ratio of the thicknesses of the trabeculae becomes about 1:2, corresponding to $|\sigma_1| : |\sigma_2|$. Following the rotation in the direction of the principal stress from $\theta_p = 0^\circ$ to $\theta_p = 15^\circ, 30^\circ$ and 45° , trabeculae gradually reorientate in the same manner as in the case with the same signs. The aspect ratio of the fabric principal radii changes from 1.02 to 1.30.

5.3 Discussion

As investigated in Sec. 4, a single trabecula under compressive loading rotates to align along the loading direction as a result of the regulation of the stress on

Fig. 9 Morphological changes of cancellous structure

the trabecular surface to become uniform. From the viewpoint that the cancellous-bone model in a square region discussed in this section is the assemblage of such trabeculae with originally isotropic architecture, characteristic features of the simulation results are discussed here.

Basic features of the evolution of trabecular architecture can be explained in the context that each trabecula reorients in the loading direction. Trabeculae change their morphology to align along the direction of the principal stress. This reorientation of the trabecular architecture is reasonable for the bone tissue structure as a load bearing system. The evolution equation proposed by Cowin⁽¹²⁾ also leads to a reorientation of the trabecular architecture that supports the hypothesis by Wolff⁽³⁾ in which the principal stress axes coincide with the principal trabecular direction at the remodeling equilibrium.

As the boundary conditions change and the

remodeling progresses with time, the principal radii of the fabric tensor gradually elongate, implying that the number of trabeculae decreases by remodeling even if the total volume is preserved. The reason for this change is that the proposed remodeling model permits the disappearance of the trabeculae, however, there is no mechanism to increase the number of trabeculae. Because the proposed rate equation only considers the evolution of the surface movement facing the marrow, the new bifurcation and nucleation of the trabeculae that increases the number of connections within the trabecular architecture cannot be expressed in a two-dimensional case.

Depending on the sign of the principal stresses σ_1 and σ_2 , the pattern of trabecular architecture at the remodeling equilibrium are different from each other as shown in Figs. 9(a) and (b). The difference between the architecture with roundish corners in Fig. 9(a) and that with right-angled corners in Fig. 9(b)

is because the remodeling rate equation employs the equivalent stress σ as a mechanical stimulus that always has a positive value. When the principal stresses σ_1 and σ_2 have the same signs as illustrated in Fig. 10(a), the stress in the direction of σ_2 becomes convex and that in the direction of σ_1 becomes concave, which leads to growth and resorption, respectively, and results in the roundish corners of the trabecular architecture. In contrast, if they take the opposite signs as in Fig. 10(b), the stress σ in both directions of σ_1 and σ_2 becomes convex, leading to growth that results in the interconnection of trabeculae at right angles. This result is due to the evaluation of the mechanical stimulus as a positive definite value, so the same results are expected if other positive values, such as strain energy density,

Fig. 10 Illustrative normal stress distribution

are used as the mechanical stimulus for the rate equation in trabecular surface remodeling. In actual cancellous bone, orthogonality of the trabecular architecture can be observed in the frontal section of the proximal femur⁽³⁾ and in the sagittal plane of patella⁽³¹⁾, in which the principal stresses may have opposite signs.

6. Remodeling of Proximal Femur

A remodeling simulation using the proposed model is carried out for a human proximal femur as a typical mechanical load bearing system with trabecular architecture.

6.1 Proximal femur model

The proximal femur is discretized in the two-dimensional rectangular region of 350×200 square finite elements. As the initial trabecular architecture, the pattern of randomly pasted circular trabeculae is used as shown in Fig. 11. Due to a limitation in computational memory, the resolution of the discretization becomes lower and the dimension of each trabecula becomes relatively large compared to that of the proximal femur. Thus, the following simulation and discussion are restricted to the qualitative investigation of morphological changes by remodeling.

With fixed conditions at the most distal nodes in the diaphysis, the joint reaction force and the hip abductor force are applied to the femoral head and the trochanter major, respectively. To represent the loading history, three loading cases⁽¹⁰⁾ are used as listed in Table 1. The first loading case (a), shown in Fig. 12 (a), is the condition for the one-legged stance phase of a gait. The other two loading cases, shown in Figs.

Fig. 11 Proximal femur model

Fig. 12 Morphological changes of trabecular architecture in proximal femur for single-loading conditions

Table 1 Loading conditions on femoral model

	Joint reaction force	Hip abductor force	Frequency
(a) One-legged	2317 N, 24°	703 N, 28°	6000/day
(b) Abduction	1158 N, -15°	351 N, -8°	2000/day
(c) Adduction	1548 N, 56°	468 N, 35°	2000/day

12(b) and (c), are selected to represent the extreme ranges of the motion of (b) abduction and (c) adduction.

These external loadings are applied as distributed forces generated using a sine function. The parameter l_c in the weight function $w(l)$ in Eq. (6) is set to 4 mm and is equal to the length of 10 elements. The threshold values for remodeling are set to Γ_u , $\Gamma_l = \pm 0.5$. In this simulation, only the cancellous region is assumed to remodel and the shape of the outer cortical bone does not change.

6.2 Remodeling under single loading

Predicted trabecular architecture under the three single-loading conditions listed in Table 1 are shown in Fig. 12 at the 8th step in the remodeling simulation. Trabecular architecture obtained by remodeling in response to loading from a single direction for the one-legged stance in Fig. 12(a) shows remarkable major systems of compressive trabeculae in the femoral head and neck, extending from the joint contact area to the medial cortical diaphysis, and that of the tensile trabeculae, extending from the trochanter major to the lateral cortical diaphysis. Subtle systems of the arcuate tensile and compressive trabeculae, which are perpendicular to the major systems, are predicted. However, the medullary cavity is not predicted that is condensed with the compressive trabeculae from the inferior femoral neck region to the lateral cortical diaphysis due to fixed boundary conditions. Except for the diaphyseal region, the predicted results show a natural pattern of the trabecular architecture.

The trabecular architecture predicted for the loading condition during abduction is shown in Fig. 12(b). Due to the increase in the bending moment at the femoral neck, the remarkable arcuate system of the tensile trabeculae is predicted from the femoral head to the lateral cortical diaphysis. The major system of the compressive trabeculae extended from the joint contact area curves into the medial cortical diaphysis. The medullary cavity is predicted in this case.

For the loading condition during adduction, the predicted trabecular architecture is shown in Fig. 12(c). In the femoral head and neck, a remarkable system of compressive trabeculae from the joint contact area to the trochanter major is predicted. In the diaphyseal region, the medullary cavity is occupied

Fig. 13 Morphological changes of trabecular architecture in the proximal femur for a multiple-loading condition

with the orthogonal curvilinear system of trabeculae due to the bending moment, as seen in Michell's structure⁽³²⁾ for the shear bracing of a long cantilever.

6.3 Remodeling under multiple loadings

To represent the daily loading history applied to the proximal femur in different directions and magnitudes of external loadings, a multiple-loading condition⁽¹⁰⁾ as listed in Table 1 is assumed. At each remodeling step, the value Γ as a driving force of remodeling is determined by averaging Γ_a , Γ_b and Γ_c corresponding to each loading condition (a) to (c) with weight depending on the frequencies. The trabecular architecture obtained by remodeling under the multiple-loading condition at the 8th step is shown in Fig. 13.

Since the loading condition (a) for the one-legged stance is assumed with 60% frequency, similar trabecular architecture as shown in Fig. 12(a) is obtained, that is, the major systems of the compressive and tensile trabeculae and the arcuate trabeculae are organized. However, due to the multiple directions of loadings, the trabecular orientations scatter slightly and the trabeculae do not interconnect at right angles. In the femoral head, for example, trabecular orientations as in Figs. 12(a) to (c) differ depending on the direction of the joint reaction force, whereas the trabecular orientation under multiple loadings in Fig. 13 shows the scattering and corresponding to all multiple loadings. In the femoral neck, the arcuate system of tensile trabeculae, which is remarkable under condition (b) in Fig. 12(b), shows ambiguous architecture in Fig. 13. In the trochanter major, the system of the tensile trabeculae that is not significantly different in Figs. 12(a) to (c), also shows remarkable structure in Fig. 13.

6.4 Discussion

The trabecular architectures obtained for the single-loading conditions show the characteristic patterns that align along the loading direction discussed in Sections 4 and 5. In the vicinity of the joint contact area in the femoral head, the trabeculae align along the loading direction in each case as shown in Fig. 12 to support single-directional loading, implying that this region is uniaxially compressed. In the femoral neck region in Fig. 12(b) and in the diaphyseal region in Figs. 12(a) and (c), two directional trabecular patterns exist that interconnect with each other at right angles, which implies that these regions are in the two-axial stress field where principal stresses have opposite signs as discussed in Sec. 5 for Fig. 9(b). Thus, each trabecular architecture in Fig. 12 adapts itself to each single-loading condition. However, one pattern of the trabecular architecture can not adapt to other loading conditions, so the results obtained in Figs. 12(b) and (c) are dissimilar to those observed in the actual proximal femur. The predicted trabecular architecture for multiple-loading conditions does not show the remarkable parallel and orthogonal patterns of the trabecular architecture, but shows a trabecular network pattern with a scattered orientation.

In the diaphyseal region, the medullary cavity is occupied with condensed trabecular architecture that cannot be observed in actual diaphysis. This is due to the fixed boundary conditions at the most distal section that is cut off in the half length used in the model. Except for this diaphyseal region, the predicted trabecular architecture for the multiple loadings qualitatively coincides with the actual pattern in the proximal femur.

Understanding the trabecular architecture in cancellous bone as a mechanical structural system^{(2),(3)} implies that there is correlation between trabecular orientation and the local direction of loading, such as the principal stress direction. The alignment of the trabeculae in the parallel or orthogonal manners indicate the existence of some optimal structure as a load bearing structural system, such as the frame-structure called Michell's structure⁽³²⁾. The proposed remodeling rate equation regulates the stress distribution on the trabecular surface to become uniform, which causes the reorientation of the trabeculae aligning along the local principal direction of the stress. As the results, the trabecular architecture obtained adapts to its mechanical environment, showing a good coincidence with the actual trabecular architecture. These comparisons demonstrate the capability of the proposed remodeling rate equation to express the adaptation phenomenon of trabecular architecture by

remodeling.

7. Conclusions

In this study, we proposed a rate equation for trabecular surface remodeling as a regulation process of the nonuniform stress distribution on the trabecular surface toward uniform distribution. This rate equation regulates the nonuniformity of the stress at the local site as a driving force of the remodeling, which does not employ the goal or optimal stress prescribed *a priori* as a global constraint of the systematic regulation.

Using a finite element method for the discretized trabecular architecture, trabecular surface remodeling simulations were conducted for the trabecular-level structure, cancellous-bone-level structure and proximal femur as a load bearing system. Results show the morphological changes of trabeculae in response to mechanical conditions by changing trabecular orientation, thickness and connectivity. The trabecular architecture of cancellous bone in the proximal femur obtained from the simulation shows good agreement with that of the actual femur.

These simulation studies lead to the conclusions that the proposed rate equation for trabecular surface remodeling to seek the local uniform stress distribution on the trabecular surface at the remodeling equilibrium has one capability to support the hypothesis of the optimality of trabecular architecture as a mechanical load bearing structure. And, the proposed simulation method can be applied to the prediction of the remodeling phenomena in the clinical aspects for the change of trabecular architecture as a mechanical adaptation by remodeling.

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