A Stabilization Method for Nonlinear Time-Delay Systems and Its Dynamics*

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This paper is concerned with a stabilization method for nonlinear time-delay systems. Generally, hunting will often occur due to the time delay. Though the Smith method is known to be able to prevent it, there are many difficulties in its application. In this research, a new stabilization method is proposed, which can prevent hunting and reduce the steady-state error to zero by parameter adjustment. First, the stability analysis of nonlinear time-delay systems is clarified using the circle theorem. Second, in order to analyze the robustness of the new compensator, its robust stability condition is derived for such control systems with a time delay and a saturated-type nonlinearity, and the stability margins under the identified errors of time delay are evaluated. Third, the new compensator is directly represented by a form of z transform, and its stabilization and dynamical characteristics are analyzed using the root locus method. From these studies, it is verified that the new stabilization method is useful for general nonlinear time-delay control systems.

Key Words: Control Device, Time-Delay, Nonlinear Systems, Stabilization, Robust Stability, Root Locus

1. Introduction

In process control systems, one of the problems is the existence of a time delay which prevents the improvement of the dynamics. Because the system with a time delay is of infinite dimension, the analysis is not easy, and especially in a nonlinear time delay system hunting will often occur and it is difficult to obtain stable characteristics. Though the Smith method⁽¹⁾, as a stabilization method for such a time delay system is well known, there are many problems in its practical use, because in constructing the compensating circuit it is necessary to obtain the strict system identification and also the steady-state error generated by a disturbance.

The authors have already proposed a new time delay compensator⁽²⁾, which is different from the

Smith's one, for nonlinear time delay systems such as an electropneumatic valve positioner. This compensating circuit does not include the transfer function of the controlled object, and has a strong robustness to the identified error of the time delay between the object and the compensating circuit. Furthermore, it can reduce the steady-state error to zero. The authors have also clarified the dynamic behavior (especially hunting) of the electropneumatic valve positioner and the pneumatic tube model as a means of signal transmission, both quantitatively and qualitatively⁽³⁾. More recently a digital compensator has been devised on the basis of the new time delay compensator, stabilization experiments performed on the electropneumatic valve positioner with a long pneumatic tube and the validity of the proposed method verified⁽⁴⁾.

In this research, in order to examine both the generality of the time delay compensator and its qualitative characteristics, the controlled object is considered as being of general nonlinear time delay systems of 0 type and 1 type. Since the proposed system with the time delay compensator has infinite poles, there are many difficulties in analysis based on

* Received 13th October, 1997. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 63, No. 608, C (1997), p. 1131-1138 (Received 30th May, 1996)

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the poles and root locus in the continuous system. Accordingly, it is necessary to construct a digital compensator. Here, the system is represented by a discrete time model, and both the dynamic behaviors and robust stability are analyzed based on the system pole locations and the behavior of the characteristic roots. As a result, the validity of the proposed compensator for nonlinear time delay systems is verified.

2. Stabilization of Positioning Process

In order to examine the validity and generality of the proposed time delay compensator, a general nonlinear time delay system is constructed and its validity is studied through the analysis of the frequency response and robust stability.

2.1 Block diagram

The block diagram of the positioning process (0 type) with the new time delay compensator is shown in Fig. 1. The symbols used in this paper are as follows:

- K_q : gain of first-order time-lag element of the compensator
- T_q : time constant of first-order time-lag element of the compensator
- K_b : feedback gain of the compensator
- L_1 : identified time delay of the compensator
- M : linearized coefficient of nonlinear element
- e : input amplitude to nonlinear element
- δ : saturated value of input to nonlinear element
- K_m : gain of controlled object
- T_m : time constant of the controlled object
- L : time delay of the controlled object
- d : disturbance element
- R : input signal
- Y : output signal

The transfer function of the new time delay compensator derived from Fig. 1 is expressed as follows:

$$D_1(s) = \frac{K_q}{1 + T_q s - K_q K_b e^{-L_1 s}} \quad (1)$$

Fig. 1 Block diagram of control system with new time-delay compensator

This time-delay compensator has two features. One is that there is a filter of first-order time-lag element in it, and the other, different from the Smith's one, is that it does not include transfer function of the controlled object and only the time delay L_1 is necessary to take a value being close to that of the time delay L of the controlled object. It has been made clear using this compensator the controlled object can work steadily in the nonlinear time delay system of saturated type such as an electropneumatic valve positioner. In this study, in order to examine the generality of the new time delay compensator and its qualitative characteristics, the transfer function of the controlled object is given as 0 type such as,

$$G_{m1}(s) = \frac{K_m}{T_m s + 1} \quad (2)$$

2.2 Stability criterion by circle theorem

To ascertain the effectiveness of the time delay compensator of Eq. (1), the circle theorem⁽⁵⁾, which is comparatively applicable in practical use, is introduced to distinguish the stability of the system in Fig. 1. The relation between input and output of nonlinear element is given by

$$m = N(e) \quad (3)$$

As shown in Fig. 2, there exists a nonlinear characteristic in the first and the third quadrant. When this characteristic is in the region between the lines $m = k_2 e$ and $m = k_1 e$, the following equation must be satisfied so as to realize asymptotic stability in the large of the feedback control system.

$$Re \left[\frac{D_1(j\omega) G_{m1}(j\omega)}{1 + k_1 D_1(j\omega) G_{m1}(j\omega)} + \frac{1}{k_2 - k_1} \right] \geq 0 \quad (4)$$

As the nonlinear element is of saturated-type, the following equation is satisfied,

$$0 \leq \frac{N(e)}{e} \leq M \quad (5)$$

Using Eqs. (1) and (2), the condition of asymptotic stability in the large in Eq. (4) can be obtained as

Fig. 2 Characteristics of nonlinear element

follows:

$$\left[K_q K_m \frac{a \sin(\omega L) - b \sin(\omega L)}{a^2 + b^2} \right] \geq -\frac{1}{M} \quad (6)$$

where,

$$\begin{aligned} a &= 1 - K_q K_b \cos(\omega L_1) \\ &\quad - T_m \{ T_q \omega + K_q K_b \sin(\omega L_1) \} \\ b &= T_m \omega \{ 1 - K_q K_b \cos(\omega L_1) \} \\ &\quad + T_q \omega + K_q K_b \sin(\omega L_1) \end{aligned} \quad (7)$$

The stability margin of the system with the new time delay compensator is determined by Eq.(6) and the parameters are given as

$$\begin{aligned} T_q &= 2.5, \quad K_b = 1.0/K_q, \quad L = L_1 \\ K_m &= 2.0, \quad T_m = 0.1 \end{aligned} \quad (8)$$

The result is shown in Fig. 3. Here, the same approach is applied to PI control ($T_i=10$) to examine the stability margin. From Fig. 3, it can be seen that the stable region in the case with the time delay compensator is expanded significantly compared to that of PI control. Also, it is clear that the larger the linearized coefficient of nonlinear element M , the smaller is the stable region. Figure 4 is an example of response characteristics when the time delay is $L=L_1=1.0$ s. The solid line shows the case with a time delay compensator and the dotted line that of PI controller (gain $K_p=1.0$, integral time $T_i=1.0$). $L=1.0$ s stands for about 200 m of the pneumatic tube (in this case, the inner diameter 6.0×10^{-3} m and the input pressure 0.1 MPa), and it is difficult to consider a time delay above this value. Thus, it is sufficient from the point of view of time delay compensation. From Fig.

Fig. 3 Stability region

Fig. 4 Simulation

4, in the case of PI control, hunting occurs with a period of 2.4 s and we cannot obtain a stable control, whereas by the addition of the compensator, a stable control without steady-state error can be realized⁽⁴⁾.

2.3 Frequency characteristics

The effectiveness of the proposed compensator has been ascertained through the investigation of the frequency characteristics. The controlled object is represented by Eq.(2) and the parameters used are set as:

$$\begin{aligned} K_p &= 1.0, \quad T_i = 1.0, \\ K_q &= 1.0, \quad T_q = 2.5, \quad K_b = 1.0, \quad L = L_1 = 1.0 \text{ s} \\ K_m &= 2.0, \quad T_m = 0.1, \quad M = 1.0 \end{aligned} \quad (9)$$

Figures 5 and 6 show the frequency characteristics of overall transfer function in Fig. 1. The solid line is the case with time delay compensator and the broken line is that of PI controller. From Fig. 5, in the case with time delay compensator, because the gradient of gain is larger than -20 db/dc in the low frequency area and the gain can take a large value near the frequency of 0 rad/s, the characteristics of target-tracing, the elimination of disturbance, and low sensitivity can be realized and the steady-state error can be reduced to zero. Moreover, in the high frequency range the gain can be reduced to a low value to ensure

Fig. 5 Gain diagram

Fig. 6 Phase diagram

stability and the non-vibrating conditions can be satisfied. Especially, as L is large, it is necessary to reduce the gain of transfer function of open-loop to below 1 (0 db) because the phase-delay becomes large even in the low frequency range. From Fig. 6, since the gain characteristic of overall transfer function with this compensator is that, for $\omega \geq 0.6$, the gain is below 0 db (below 1), the condition mentioned above is satisfied. Furthermore, from Fig. 6, though the phase delay is affirmed compared to PI control, so much delay cannot be considered in the frequency range required in the control performance, and so in the design of the control system the sufficient gain margin can be taken with Fig. 5. When the new time delay compensator is used, the gain margin is 8.33 db, the phase margin is 58.13 deg, which can satisfy the demands of design specification in process control.

3. Stabilization of Non-positioning Process

The effectiveness of the proposed time delay compensator to positioning process has been confirmed theoretically and experimentally^{(4),(11)}. However, it has not been applied to non-positioning process. Here, this question will be considered. Applying the time delay compensator represented by Eq.(1) to non-positioning process, its stable region tends to be narrow compared to that of P control only. Adjusting the parameters of the compensator of Eq. (1), which is valid for a positioning process, it is shown that this compensator is also effective for a non-positioning process.

3.1 Improvement of time delay compensator by parameter adjustments

The block diagram of a saturated nonlinear time delay system with improved time delay compensator is shown in Fig. 7. The transfer function $G_{m2}(s)$ of the controlled object is represented as,

$$G_{m2}(s) = \frac{K_m}{s(T_m s + 1)} \quad (10)$$

The normal time delay compensator expressed as

Fig. 7 Block diagram with improved time-delay compensator

Eq.(1) is constructed by the positive local feedback loop. In this case, the compensator is constructed from the negative feedback loop. However, on account of no alteration in the structure of compensator, the compensator can be improved without a basic adjustment if the feedback gain K_b is set to a negative value. The transfer function of the improved time delay compensator is obtained as,

$$D_2(s) = \frac{K_q}{1 + T_q s + K_q K'_b e^{-L_1 s}} \quad (11)$$

where $K'_b = -K_b$. In order to verify the validity of the improved time delay compensator by adjusting the parameters, the stability analysis is performed by a vector locus. An example of the analytical result is shown in Fig. 8. The parameters of controlled object are given as,

$$K_m = 2.0, \quad T_m = 0.1, \quad M = 1.0, \quad L = L_1 = 1.0 \text{ s} \quad (12)$$

The solid line is the vector locus of the system with the improved time delay compensator and the dotted line is that of a normal time delay compensator. From Fig. 8, in the case of normal compensator, the vector locus and the amplitude one, $-1/N(z)$ of saturated nonlinear element have a stable intersection, and the limit cycle occurs at this point. On the other hand, the vector locus of the improved compensator has no intersection with the amplitude one and turns out to be asymptotically stable. From this fact, it can be considered that hunting can be avoided using the improved time delay compensator. Also, comparing the stable region with that of P control, its effectiveness is shown in Fig. 9.

3.2 Dynamical experiments

To verify whether the dynamics has been improved by the addition of time delay compensator in Eq. (11), simulation studies have been carried out with the transient response of a non-positioning system. The parameters used in the simulation are given as

Fig. 8 Stability analysis based on Nyquist method

$$\begin{aligned}
 K_p &= 1.0, \\
 K_q &= 1.0, \quad T_q = 8.0, \quad K_b' = 5.0, \quad L = L_1 = 1.0 \text{ s} \\
 K_m &= 2.0, \quad T_m = 0.1, \quad M = 1.0
 \end{aligned}
 \quad (13)$$

Fig. 9 Stability region

Fig. 10 Simulation

Fig. 11 Gain diagram

Fig. 12 Phase diagram

An example of experimental results is shown in Fig. 10. The solid line stands for the case with improved time delay compensator and the dotted line is that of P control. In the case of P control, hunting occurs and stable control characteristics cannot be obtained. However, when the time delay compensator is added, the response reaches the steady state in about 7.8 s, and good control characteristics without steady-state error can be obtained.

3.3 Frequency response characteristics

Just as in the case of the positioning system, the frequency response of overall transfer function in a non-positioning system has been investigated and the results are shown in Figs. 11 and 12. The parameters used here are set as in Eq. (13).

From Figs. 11 and 12, it can be said that the compensator can lower the gain to low value in the high frequency range and therefore is desirable for robust stability.

4. Dynamical Analysis by Root Locus

The transient response characteristics of the control system can be determined by the disposition of poles of the closed-loop transfer function and the desired output of the system. Given the poles of the closed-loop transfer function, each mode of the controlled output can be derived and the response can be evaluated. However, as expressed in Eq. (14), the system with new time delay compensator includes a time delay in the state and possesses infinite poles in general.

$$\begin{aligned}
 \dot{x}(t) &= Ax(t) + Bx(t-L_1) + Cx(t-L) + Dr(t-L) \\
 y(t) &= Ex(t)
 \end{aligned}
 \quad (14)$$

Therefore, it is difficult to perform dynamical analysis by poles and root locus, if not applying Pade approximation to the continuous system. Here, to explore the quantitative aspects of the time delay compensator, the system is represented by a discrete-time model and the generality of the compensator is studied through the poles of z plane and the position of its characteristic roots. According to the result described above, the dynamical behaviors of the control system with time delay compensator are investigated.

4.1 The root locus of positioning system

The block diagram of the discrete system is shown in Fig. 13. In this figure, the pulse transfer function of digital compensator is expressed as $D(z)$. Letting the control system be designed as 0 type, the pulse transfer function of controlled object is written, from zero-order holder and Eq. (2), as

$$G_m(z)z^{-n} \equiv G_{m1}(z) = K_m \frac{1-p}{z-p} z^{-n} \quad (15)$$

Fig. 13 Block diagram of discrete system

Fig. 14 Root locus with P controller

Fig. 15 Root locus with new compensator

where $p=e^{-h/T_m}$, $L=n \cdot h$, and h is a sampling period.

The discrete time characteristic equation is calculated numerically using the DKA method⁽⁶⁾ and then the dynamical behaviors in Fig. 13 are analyzed by a root locus. The parameters of controlled object are as follows and the time delay is set as $L=L_1=0.5$ s:

$$\begin{aligned} T_q &= 1.5, \quad K_b = 1.0/K_q, \quad K_m = 2.0, \\ T_m &= 0.1, \quad M = 1.0 \end{aligned} \quad (16)$$

The root locus using P control is shown in Fig. 14 and that with compensator is shown in Fig. 15, where the sign $\circ$ means zero and $\times$ means pole. The stabi-

Fig. 16 Robust stability analysis by root locus ($\Delta L = -0.3$) Fig. 17 Robustness of response ($\Delta L = -0.3$)

zation condition in the z plane is the same as for the case when poles and roots of pulse transfer function exist inside a unit circle.

From Fig. 14, in the case of P control, when there is a small increase in the gain of controller $K_p \geq 0.7$, the root will diverge to out of the unit circle and the system becomes unstable. On the other hand, from Fig. 15 as the new time delay compensator is added, we obtain $K_p \leq 2.7$, and is larger than in the case of P control.

Next, there is a difficulty in identifying the time delay in the compensator and its value changes easily. Due to this, the time delay system is required to be robust to the identified error of the time delay. The root locus and transient response are examined in this system, when the identified error of time delay arises. Given the time delays $L=0.5$ s and $L_1=0.8$ s, the root locus is shown in Fig. 16 and the transient response in Fig. 17. From Fig. 16, even for an identified error, $\Delta L = -0.3$ s of time delay, the adjusting range of gain can be designed widely to make the system stable. While the gain becomes larger, the roots move not only to out of the unit circle but also towards the real axis or the origin, and then it can be guessed that this is the reason to stabilize the system in the case of gain being large. According to Fig. 17, it can be seen that even if

there is an identified error of time delay, good response can be obtained.

4.2 Root locus of non-positioning system

In the same way as section 4.1, the root locus of the non-positioning system is drawn. The pulse transfer function of controlled object in Fig. 13 is derived from a zero-order hold and Eq.(10) as

$$G_m(z)z^{-n} \equiv G_{m2}(z) \\ = K_m \frac{(h - T_m + T_m p)z + T_m - T_m p - ph}{(z-1)(z-p)} z^{-n} \quad (17)$$

where, $p = e^{-h/L}$, and $L = n \cdot h$.

Using the same method as for the positioning system, the dynamical behavior of the non-positioning system is analyzed by root locus. The parameters of controlled object used here are indicated as follows and the time delay is given as $L = L_1 = 0.6$ s,

$$\begin{aligned} T_q &= 8.0, \quad K_b' = 5.0, \quad K_m = 2.0, \\ T_m &= 0.1, \quad M = 1.0 \end{aligned} \quad (18)$$

The root locus in P control is shown in Fig. 18 and the one with time delay compensator in Fig. 19. Taking note of the motion of the characteristic roots near to (1,0) which influences most of the response, it

Fig. 18 Root locus with P controller

Fig. 19 Root locus with new compensator

can be understood that by applying the compensator in Fig. 19, the locus of characteristic roots is wide and the gain becomes $K_q = 5.0$ when the root is out of the unit circle. This means that the stable range of gain is wider, compared with $K_p = 1.32$ in Fig. 18. Owing to this fact, the selectable range of gain can be designed wider than that of P control.

5. Application to Pneumatic Robot

To verify the effectiveness of the improved compensator expressed as Eq.(11) for non-positioning system, the experiments are performed using an one-link robot with pneumatic motor. Figure 20 illustrates the outline of the experimental apparatus⁽⁷⁾.

The pneumatic tube is represented by the transfer function $G_L(s)$ of first-order time-lag element and time delay e^{-Ls} , and the pneumatic robot $G_m(s)$ by a type-1 system⁽⁸⁾. In this case, from the viewpoint of preventing explosion the driving part of the compensator and the electropneumatic transducer are combined by a long pneumatic tube. Such a pneumatic robot is a non-positioning system and, when the pneumatic tube exceeds a certain length, hunting will occur due to the time delay of pneumatic pressure signal and cannot generate stable action. Accordingly, the proposed compensator is applied to realize a stable control action. Let the pneumatic tube be 40 m, in which the time delay derived from the experiment is $L = 0.12$ s⁽⁴⁾. Therefore, the parameters of the compensator in Eq.(11) are determined by

$$\begin{aligned} K_p &= 0.2, \quad T_i = 10.0, \\ K_q &= 1.2, \quad T_q = 20.0, \quad K_b' = 2.5, \\ K_v &= 9.72, \quad L_1 = 0.12 \end{aligned} \quad (19)$$

One example of the experimental results using the pneumatic robot of Fig. 20 is shown in Fig. 21. The desired angles of the manipulator are given as 20 deg, and 30 deg. The solid lines are examples of transient responses obtained using the proposed compensator and the dotted ones are those for the PI controller with the desired angle of 20 deg.

Fig. 20 Structure of experimental equipment

Fig. 21 Experimental result

As shown in Fig. 21, when the PI controller is used the rise time increases and the settling time becomes long due to the large overshoot. Also, when the gain increases, hunting will occur and stable control characteristics cannot be obtained. On the other hand, when the proposed compensator is applied, good transient responses can be obtained. From these results, it is shown that this compensator is effective for practical use in the stabilization of non-positioning systems.

6. Conclusions

In this paper, in order to prevent the hunting of nonlinear time delay systems, a time delay compensation method is proposed as a new stabilization method and its validity is confirmed through simulations and experiments. The following conclusions are obtained:

(1) To clarify the generality of the new time delay compensator, a general nonlinear time delay system represented by type-0 system was constructed and the range of tolerant error for identified error of time delay was indicated.

(2) It has been confirmed that even for a controlled object of a type-1 time delay system, a good response without steady-state error can be obtained by adjusting the parameters of the compensator.

(3) In order to examine the qualitative analysis, the system was represented as a discrete model, and moreover, the gain of stable margin and the dynamical behavior of the system with time delay compensator were studied by poles and location of characteristic roots in the z plane.

(4) This compensator was applied to a pneu-

matic robot represented as a type-1 system with time delay, and the effectiveness of this method was verified through experiments.

From these results, an efficacy of the new stabilization method in practical use which has been proposed for the stabilization of nonlinear time delay control systems, is verified.

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