Robust Identification and Control Design for Truck Cab Model Device*

Zhongyang GUO**, Itsuro KAJIWARA**
and Akio NAGAMATSU**

The increasing attention on the applications of advanced robust control theories to automotive motivates the further investigation of truck cab model device for active vibration control. A balanced state-space model is necessary when designing a robust controller. Hence, the state-space frequency domain identification method is used to construct a model for model-based robust control design. Then H_∞ control theory and μ synthesis are readily to be used in order to calculate a controller for vibration suppression. The ν -tool is exploited to evaluate the quantitative margin for robust stability and performance. Finally, the closed-loop system achieved the satisfactory control results.

Key Words: State-Space Modeling, H_∞ Control, μ Synthesis, Robustness Measure.

1. Introduction

With the advent of microprocessors, the real implementation of advanced control algorithms have become feasible. In automotive industry, the increasingly stringent laws ask for lower automotive vibration and noise. Therefore, the active and adaptive control methods are proposed and applied to many aspects of automotive controls⁽¹⁾. The passive vibration isolation of dynamic conjunction reaches its limit since the complex operation conditions of vehicle and road conditions can not be predicted precisely. Now the advanced robust control theories are available and make the active control solution tractable. A state-space model of control object is essential for robust control design. Here we use the State-Space Frequency Domain identification (SSFD)⁽²⁾ method and Eigensystem Realization Algorithm (ERA)⁽³⁾ method to obtain the state-space representation of the truck cab model. The SSFD method is first to curve-fit the frequency response function (FRF) by the factorization approach, then to construct the *Markov* parameter using the factorization coefficients. Hereafter, a balanced minimum realization of the system state-

space parameters can be produced by using ERA method. This modeling approach consists of linearized curve-fitting and *Markov* parameter construction as well as the ERA method. As we know, the best model identification in correlating with experiment data does not mean the best robust control design. The model identification criterion influences the model-based control design and robust control design does have requirement to the model. Therefore, the modeling approach in this study is congruent to the robust control design because the identification results include a quantitative error norm bound of the reduced order model⁽⁴⁾. This error norm bound can be used as the robust weighting reference to evaluate the unstructured model uncertainty since it is interpreted as the neglected dynamics of high frequencies. Therefore, using the sub-optimal solution of two-Riccati equations⁽⁵⁾, an H_∞ controller can be easily calculated.

If the considered system is subject to structured uncertainties, μ synthesis technique can be applied to design a controller in guaranteeing the robust stability and robust performance. The ν tool⁽⁶⁾ can be used to assess the allowable uncertainties while maintaining a certain level of robust control performance. The remains of this paper are organized as the follows. The second section contains the truck cab modeling to demonstrate the identification results. The third section includes the robust control design and

* Received 1st May, 1997

** Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8550, Japan

implementation as well as control analysis. The basic description of ν tool and robustness evaluation are in the fourth section. The conclusion remarks are in the final section.

2. Truck Cab Modeling

2.1 State-space identification

The state-space representation of a linear discrete-time system is

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k). \end{aligned} \quad (1)$$

Where k is the sampling indicator, $x(k) \in \mathcal{R}^{n \times 1}$ is the state vector, $u(k) \in \mathcal{R}^{r \times 1}$ the input vector, $y(k) \in \mathcal{R}^{m \times 1}$ the output vector; A, B, C, D are the system matrix, input matrix, output matrix and direct-influence matrix, respectively. Matrices A, B, C, D are referred to as state-space parameters or state-space model representation. The relation between the state-space parameters and the FRF $G(\omega_i)$ is

$$\begin{aligned} G(\omega_i) &= C(e^{j\omega_i \Delta t} I_n - A)^{-1} B + D \\ &= D + CBe^{-j\omega_i \Delta t} + CABe^{-j2\omega_i \Delta t} \\ &\quad + CA^2Be^{-j3\omega_i \Delta t} + \dots \\ &= \sum_{k=0}^{\infty} Y_k e^{-jk\omega_i \Delta t} \triangleq [A \ B \ C \ D] \end{aligned} \quad (2)$$

where, $Y_0 = D$, $Y_k = CA^{k-1}B$, Δt is the sampling time of the discrete-time system in seconds, and Y_k are the *Markov* parameters.

In z -domain, Eq. (2) is written as

$$G(z) = C(zI_n - A)^{-1}B + D, \quad z = e^{j\omega \Delta t}. \quad (3)$$

2.1.1 Linear curve-fitting The SSFD method is first to curve-fit the FRF data in using a finite order matrix fraction then to construct the *Markov* parameters. It is known that the transfer function matrix can be expressed by a left matrix-fraction description⁽²⁾; then Eq. (3) can be written as

$$G(z) = R^{-1}(z)S(z) \quad (4)$$

Where both $R(z)$ and $S(z)$ are matrix polynomials:

$$\begin{aligned} R(z) &= I_m + R_1 z^{-1} + \dots + R_p z^{-p} \quad (R_i \in \mathcal{R}^{m \times m}) \\ S(z) &= S_0 + S_1 z^{-1} + \dots + S_p z^{-p} \quad (S_i \in \mathcal{R}^{m \times r}). \end{aligned} \quad (5)$$

This factorization is not unique. For convenience one can choose the orders of both polynomials to be equal (e.g. $= p$). Pre-multiplying Eq. (4) by $R(z)$ one has

$$R(z)G(z) = S(z), \quad (6)$$

which can be rearranged to become

$$\begin{aligned} G(z) &= -R_1 G(z) z^{-1} - \dots - R_p G(z) z^{-p} \\ &\quad + S_0 + S_1 z^{-1} + \dots + S_p z^{-p} \end{aligned} \quad (7)$$

Note that with $G(z)$ and z given, Eq. (7) is a linear equation. Because $G(z)$ is known at $z = e^{j\omega_i \Delta t}$, ($i = 1, \dots, l$), there are a total of l equations available. Then based on these equations from Eq. (7), the least square solutions to $R(z)$ and $S(z)$ in Eq. (6) is available.

2.1.2 Construction of *Markov* parameters

After obtaining the solutions of $R(z)$ and $S(z)$, it is now necessary to construct the system *Markov* parameters. Combining Eq. (2), Eq. (6) can be written as

$$\left(\sum_{i=0}^p R_i z^{-i} \right) \left(\sum_{k=0}^{\infty} Y_k z^{-k} \right) = \sum_{i=0}^p S_i z^{-i} \quad (8)$$

From this relation, the following equations are derived by equating terms of like powers and recalling Eq. (5).

$$\begin{aligned} Y_0 &= S_0 \\ Y_k &= S_k - \sum_{j=1}^p R_j Y_{k-j} \quad \text{for } k=1, \dots, p \\ Y_k &= - \sum_{j=1}^p R_j Y_{k-j} \quad \text{for } k=k+1, \dots, \infty \end{aligned} \quad (9)$$

Note that there are only p independent *Markov* parameters excluding Y_0 ; all the other parameters are linear combinations (with matrix coefficients) of this independent set. Therefore, the *Markov* parameters represent a signal containing a maximum of $m \times p$ states where m is the number of outputs.

2.1.3 State-space model realization

With the system *Markov* parameters via system realization theory (i.e. the balanced realization of state-space models from ERA), the state-space model is calculated. In this process, the minimum order system is realized by the finite *Hankel* singular value truncation. It is assumed that the realized minimum order system is presented as $[A, B, C, D]_r = G_r(z)$ and before model reduction the system is $[\hat{A}, \hat{B}, \hat{C}, \hat{D}]_N = \hat{G}(z)$, provided that its non-zero *Hankel* singular values are $[\sigma_1 > \sigma_2 > \dots > \sigma_N]$. For structural analysis and controller designs, experimental data from structural testing are used to either verify an analytical model or directly determine a mathematical model. The ERA is an algorithm which computes a mathematical model directly from experimental data. In this part, the basic ERA formulations are briefly included. The ERA minimum realization is a balanced realization which has been frequently used for model reduction, and further in robust control design. The ERA algorithm uses the singular value decomposition of a *Hankel* matrix to determine a system model under test. Consider the discrete-time state-space model with m -dimensional output as given by Eq. (10) below. Let the input u be the q -dimensional impulse input at an initial time.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \\ y(0) &= D \end{aligned} \quad (10)$$

Where k is the sampling indicator; $[A, B, C]$ is system matrices and D is the initial influence matrix. The time domain description is given by *Markov* parameters Y_k in Eq. (2), i.e. impulse response functions.

often used to adjust the controller gain, here $W_2 = 0.000625$. The weighting functions are listed in Eqs. (14) and (15), and demonstrated in Fig. 4.

$$W_1 =$$

$$\frac{1.6418 \times 10^7}{s^4 + 49s^3 + 4.07 \times 10^4 s^2 + 1.1158 \times 10^6 s + 3.2836 \times 10^8} \quad (14)$$

$$W_3 = \frac{0.5s + 5.975}{s + 15.685 \times 10^7} \quad (15)$$

w is the disturbance to the closed-loop system, which is input from x_1 as road surface disturbance. An H_∞ controller with order 11 is produced ($\gamma = 0.05$) and it satisfies the constraint of Eq. (16).

$$\left\| \begin{matrix} \gamma W_1 T_{z_1 w} \\ W_2 T_{z_2 w} \\ W_3 T_{z_3 w} \end{matrix} \right\|_\infty \leq 1 \quad \text{i.e.} \quad \|T_{zw}\|_\infty \leq 1 \quad (16)$$

Fig. 4 Robust weighting functions

Fig. 5 Controller implementation

The controller stabilized the closed-loop system. The experimental implementation for the controller on the device is demonstrated in Fig. 5, where the response of x_3 is given out.

The vibration suppressions at the mode peaks are 5 dB, 20 dB and 17 dB, respectively. As we know, H_∞ control guarantees the stability of the closed-loop system, but the quantitative evaluation for the robust performance hitherto is not feasible for the method itself. However, the μ synthesis and the ν measure are the alternatives.

4. Robustness Evaluation

4.1 μ/ν tool for robustness analysis

4.1.1 Introduction The μ synthesis and analysis are known to be an effective tool for assessing robust performance of LTI model subject to structured uncertainties. Yet what the μ analysis provides is a bound β on the uncertainty under which stability and H_∞ performance level of α is guaranteed. The aim of μ analysis is to study robust stability or performance in the face of parametric uncertainties and neglected dynamics. The closed-loop subjected to model uncertainties is first transformed into the standard interconnection structure $M(s) - \Delta$ of Fig. 6, where all uncertainties are gathered in the structure model uncertainty Δ .

For the sake of simplicity, if one considers real parametric and full block uncertainties, Δ is defined as

$$\Delta = \text{diag}\{[\delta_1 I_{r_1}, \dots, \delta_s I_{r_s}, \Delta_1, \dots, \Delta_F] : \delta_i \in \mathcal{R}, \Delta_j \in \mathcal{C}^{m_j \times m_j}\} \quad (17)$$

here

$$\sum_{i=1}^s r_i + \sum_{j=1}^F m_j = n.$$

The real scalar δ_i represent the normalized parametric uncertainties, or $\delta_i \in [-1, 1]$ represents the range of variations if the i -th uncertainty parameter is within the prespecified dispersions. Introducing the unit hypercube D in space of structured uncertainties δ_i and Δ_j :

$$D = \left\{ [\delta, \Delta] = [\delta_1, \dots, \delta_s, \Delta_1, \dots, \Delta_F] : \begin{matrix} \delta_i \in \mathcal{R}, |\delta_i| \leq 1; \\ \Delta_j \in \mathcal{C}, \bar{\sigma}(\Delta_j) \leq 1 \end{matrix} \right\} \quad (18)$$

μ analysis enables us to compute the maximal amount of structured uncertainties before destabilization of the closed-loop, by computing the maximal value $\theta_{\max}$

Fig. 6 Standard interconnection

of θ such that the interconnection structure of Fig. 6 is stable for all parametric uncertainties inside θD .

In practice, the SSV (Structured Singular Value), i.e. $\mu[M(j\omega)]$ is usually computed as a function of frequency ω , and $\theta_{\max}$ is obtained as

$$\theta_{\max} = \max_{\omega} (\theta) = \min_{\omega} [\bar{\sigma}(M), \Delta \in \theta D, \det(I - M\Delta) = 0]. \quad (19)$$

Note that $\theta_{\max}$ can be interpreted as the size of the smallest parameter uncertainty for which a closed-loop pole is obtained on the imaginary axis at $j\omega$, so that $\theta_{\max}$ represents the size of the smallest uncertainty for which a closed-loop pole is obtained on the imaginary axis.

For notational convenience, we leave out the ω dependence in the following, so that M denotes the complex matrix $M(j\omega)$ at frequency ω .

4.1.2 Definition of μ and ν Consider now the case of mixed uncertainties and split the structured uncertainty Δ into two structured uncertainties Δ_1 and Δ_2 . Consequently, Δ_i is a block diagonal matrix, which contains real scalar δ_i (representing the parametric uncertainties) and full complex blocks (representing either neglected dynamics or performance blocks). Introducing the unit ball $B\Delta$ as:

$$B\Delta = \{\Delta = \text{diag}(\Delta_1, \Delta_2) | \bar{\sigma}(\Delta) \leq 1\} \quad (20)$$

Then the μ and the related quantity $\nu(M)$, can now be defined as:

$$\mu(M) = \frac{1}{\min[\theta | \exists \Delta \in \theta B\Delta \text{ with } \det(I - M\Delta) = 0] \text{ or } \mu(M) = 0 \text{ if no } (\theta, \Delta) \text{ exists}} \quad (21)$$

$$\nu(M) = \frac{1}{\min[\theta | \exists \Delta = \text{diag}(\Delta_1, \theta \Delta_2) \text{ with } \Delta_i \in B\Delta_i \text{ and } \det(I - M\Delta) = 0]} \quad (22)$$

When computing μ , the unit ball $B\Delta$ is expanded (or shrunk) by factor θ until the matrix $I - M\Delta$ becomes singular for a structured uncertainty Δ inside $\theta B\Delta$. When computing the skewed SSV, namely ν , the unit ball $B\Delta_2$ (in the space of structured uncertainty Δ) is expanded (or shrunk) by factor θ , but the structured uncertainty Δ_1 remains inside the unit ball $B\Delta_1$. Here quantity ν provides an answer to the following question: given guaranteed H_∞ performance level of α , determine the largest uncertainty bounded by β , or given β , determine the smallest α with the property that for any uncertainty bounded by β , an H_∞ performance level of α is guaranteed. Therefore the ν tool gives out a robustness measure for the underlying control system.

4.1.3 Mixed ν upper bound First introduce the hermitian structure matrix G associated with uncertainty $\Delta = \text{diag}(\Delta_1, \Delta_2)$ and the scaling matrices $\underline{D}_i$ associated with uncertainties Δ_i . Then D_1 & D_2 are

defined as:

$$D_1 = \begin{bmatrix} \underline{D}_1 & 0 \\ 0 & 0 \end{bmatrix} \quad D_2 = \begin{bmatrix} 0 & 0 \\ 0 & \underline{D}_2 \end{bmatrix} \quad (23)$$

So that $\tilde{D} = D_1 + D_2$ is a scaling matrix associated with uncertainty Δ . Considering then two hermitian matrices H_A and H_B , with $H_B \geq 0$, $\eta(H_A, H_B)$ and $\lambda(H_A, H_B)$ are then defined as:

$$\eta(H_A, H_B) = \sup[\rho \in \mathcal{R} : \lambda(H_A - \rho H_B) \geq 0] \\ \lambda(H_A, H_B) = \sup[\rho \in \mathcal{R} : \bar{\lambda}(H_A - \rho H_B) = 0] \quad (24)$$

Result: A ν upper bound is given by:

$$\nu(M) \leq \nu_{UB}(M) = \min_{D_1, D_2, G} \sqrt{\max[0, \eta(H, D_2)]} \quad (25)$$

where

$$H = M^*(D_1 + D_2)M - D_1 + j(BM - M^*G).$$

Remark: $\eta(H_A, H_B)$ is almost always equal to the generalized eigenvalue $\lambda(H_A, H_B)$, except possibly when $\eta(H_A, H_B) = +\infty$. In the same way as the mixed μ upper bound of Ref. (9), this mixed upper ν bound can thus be computed in practice using recent methods for solving linear matrix inequalities (LMIs).

4.2 μ Synthesis for truck cab model

With the features of μ/ν tools, here the structured model uncertainties can be considered and robust performance evaluation can be assessed. The multiplicative uncertainty weight function W_2 is chosen for this problem to cover high frequency model uncertainty, where the neglected high frequency dynamics are mainly concentrated. The performance weight W_1 indicates that the closed-loop should reject disturbances at low frequencies for low frequency vibration suppressions. The weighting functions are listed in Eqs. (26) and (27) as well as shown in Fig. 7.

Fig. 7 Robust weighting functions

$$W_1 = \frac{5.03 \times 10^7}{s^4 + 48s^3 + 4.4 \times 10^4 s^2 + 1.185 \times 10^6 s + 3.355 \times 10^8} \quad (26)$$

$$W_2 = 0.015 \times \frac{s+100}{s+1000} \quad (27)$$

The μ/ν control design scheme is shown in Fig. 8.

Then through D - K iteration, a 15 order μ controller K_μ in accounting for real parametric uncertainty is obtained. The achieved $\mu=0.622$ ($\gamma=0.728$) indicates that the closed-loop robust performance is achieved, see μ plot in Fig. 9. To test the robust stability, small pieces of mass are detached from the test device (45 g from m_2 and m_3 , 51 g from m_1) in accounting for the real physical parameter uncertainty. Therefore, the designed μ controller is implemented on the changed system and the experiment result shows that a stable control can be achieved and corresponding to the nominal model, the vibration suppressions are still obvious at mode peaks with 7 dB, 17 dB and 9 dB, respectively, in Fig. 10.

4.3 ν Robustness evaluation

In above part, the μ synthesis controller is experimentally implemented on the perturbed plant. Now we conduct the quantitative analysis of the robust stability and performance for this closed-loop system.

4.3.1 Robust performance analysis Consider a plant subject to a structured model perturbation Δ_1 .

Fig. 8 μ/ν Control scheme

Fig. 9 μ Plot

A fictitious complex Δ_2 , here standing for robust performance, is traditionally added and an augmented perturbation $\Delta = \text{diag}(\Delta_1, \Delta_2)$ is considered. We would like to verify that the case in Fig. 6,

1. System is internally stable for $\|\Delta\|_\infty \leq 1$ and $\|F_i(M, \Delta)\|_\infty \leq \alpha$ for all such Δ iff $\sup_{\omega} \nu(M(j\omega)) < \alpha$; or
2. system is internally stable for $\|\Delta\|_\infty \leq \beta$ and $\|F_i(M, \Delta)\|_\infty \leq 1$ for all such Δ iff $\sup_{\omega} \nu(M(j\omega)) < 1/\beta$.

Here F_u and F_l are the linear fractional transformations for the upper Δ and lower Δ cases, respectively.

But here we are more precisely interested in computing the best H_∞ robust performance level α such that the system is stabilized by the controller K_μ for all uncertainties Δ_1 of size less than or equal to 1. In our practice of this paper, for the size of Δ_1 being one-dimensional, Δ can only be real or complex. Now by maintaining Δ_1 inside its unit ball, and maximizing the size of Δ_2 , the best H_∞ robust performance, i.e. minimal value of α is then obtained for all uncertainties Δ_1 of size less than or equal to 1. Then the ν -tool computes the allowable best H_∞ robust performance level α for real parametric uncertainties to be: $\alpha_r=0.6343$ at $\omega_r=182$ rad/s. Obviously, this value is better than that achieved in D - K iterations and is the best H_∞ robust performance level for such a system subject to normalized uncertainty, stabilized by the obtained controller K_μ .

4.3.2 Robust stability analysis The μ analysis provides the largest square in the parameter space such that the stability of the closed-loop is guaranteed inside this space. However the ν -tool provides the largest rectangle in the parameter space such that the stability of the closed-loop is guaranteed inside this rectangle. Now simplifying our problem by considering the case where Δ_1 corresponds to real parametric uncertainty in the stability of structural mass m_i , $i=1, 2, 3$. For the closed-loop system stabilized by the obtained controller K_μ , its mixed ν upper bound

Fig. 10 μ synthesis implementation

computed by ν -tool is 0.6407 and the stability margin is 1.561. Hence, for the nominal structure mass m_i then the maximally allowable mass variation:

$$m_a = m_i[1 \pm (1 - 1.561)] = m_i[0.439, 1.561].$$

However, the real structure mass variation can not reach the extreme bound since the actual model uncertainty is not purely contributed only by real mass perturbation and it is also very difficult to be practically implemented. But for the analysis here it supplies a quantitative feeling in considering the model uncertainty size. Based on the μ/ν control design, the robustness measure indicates the possibility of controller implementation and attributes some engineering significance.

5. Concluding Remarks

1. The State-space frequency domain identification involving ERA minimum realization formulates the effective experimental state-space modeling method for control design. Especially, the merit of quantitative model error bound description, directly relates the identification problem to robust control design.

2. The H_∞ control design for the vibration control of truck cab model is effective and the stable H_∞ control with fine performance is achieved.

3. The robustness analysis for this truck cab model control system proves that the designed control system has strong robustness and the experimental implementation is stable for the truck cab model system perturbed by physical parameter.

4. The quantitative robust measure attributes the features of robust stability and robust performance analysis for parametric model uncertainties. Further, it makes sense in controller implementation.

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