Nonlinear Forced Oscillation of a Rotating Disk Excited at a Point Fixed in Space*

(Combination Tones near Primary Resonance Points)

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Nonlinear forced oscillation of a rotating circular disk excited at a point fixed in space is considered. For theoretical analysis, modal equations are derived from the governing partial differential equations. Then, based on these modal equations, the characteristics of oscillation induced by a harmonic excitation are investigated. Numerical calculation is conducted for a typical case in which the excitation frequency is near the primary resonance points of the forward and backward traveling wave modes. It is shown that near the primary resonance point of the backward traveling mode, the forward traveling mode oscillation occurs in addition to the backward traveling mode oscillation, and conversely, near the primary resonance point of the forward traveling mode, the backward traveling mode oscillation occurs in addition to the forward traveling mode oscillation. An experimental analysis is also conducted, which confirms the validity of the theoretical analysis.

Key Words: Vibration of Continuous System, Nonlinear Vibration, Forced Vibration, Rotating Disk, Combination Tone, Travelling Wave

1. Introduction

Rotating disks are used in various machines such as floppy disks, hard disks, and circular saw blades. In these machines, rotating disks are often constrained or excited at point fixed in space, and caused to vibrate.

The problems of vibrations of a rotating disk constrained at a point fixed in space were treated by Iwan and Moeller⁽¹⁾, Benson and Boggy⁽²⁾, Honda et al.⁽³⁾ and Kodera⁽⁴⁾. As similar problems, Ono et al.^{(5),(6)}, and Jiang et al.⁽⁷⁾ dealt with the problem of floppy disks with read/write head. Chonan et al.⁽⁸⁾, and Hutton et al.⁽⁹⁾ investigated the problem of circular saw blades. Sueoka et al.⁽¹⁰⁾ and Yoshitake et al.⁽¹¹⁾

studied self-excited vibration of rotating disk subjected to a concentrated frictional force.

On the other hand, vibrations of a rotating disk excited at a point fixed in space are less well investigated. Honda et al.⁽¹²⁾ treated linear forced vibration of a rotating disk. In particular nonlinear problems have scarcely been investigated.

In this study, we treat nonlinear forced oscillations of a rotating disk excited a point fixed in space. We study the oscillations near the primary resonance points of the forward and backward traveling modes. We found in preliminary experiments that near the resonance point of the backward travelling mode, the forward travelling mode oscillation occurs in addition to the backward travelling mode, and conversely, near the resonance point of the forward travelling mode, the backward travelling mode oscillation occurs in addition to the forward travelling mode. We show that these oscillations are a kind of combination tone induced by an external force and the force synchronous with the disk rotation.

For theoretical analysis, modal equations are first derived from the Kármán-type equations which govern the motion of a rotating disk. Then, these equations are analyzed by the harmonic balance method. Experimental analysis is conducted to confirm the

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validity of the theoretical analysis.

2. Fundamental Equations

2.1 Equation of motion

A disk of thickness h , outer radius a , inner radius b , rotates with angular velocity ω . The polar coordinate system (r, θ) is fixed in the middle plane of the disk in its equilibrium state, with origin coinciding with the center of the disk. The Kármán-type equations of the rotating disk are as follows⁽¹³⁾.

$$\left. \begin{aligned} \rho h \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + D \nabla^4 w \\ + \frac{1}{2} \rho h \omega^2 \left(r^2 \nabla^2 w + 2r \frac{\partial w}{\partial r} \right) - N(w, F) = q \\ \nabla^4 F - 2(1-\nu) \rho h \omega^2 = -\frac{1}{2} E h N(w, w) \end{aligned} \right\} \quad (1)$$

where

$$\begin{aligned} D &= \frac{E h^3}{12(1-\nu^2)} \\ N(w, F) &= \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \theta^2} \right) + \frac{\partial^2 F}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right. \\ &\quad \left. + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) - 2 \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right) \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial F}{\partial \theta} \right) \end{aligned}$$

In these equations, w , F and q are the deflection of the disk, the stress function determining the disk force, and the excitation force, respectively. The quantities ρ , E and c are the density, the Young's modulus and the damping coefficient, respectively.

Introducing the nondimensional quantities

$$\left. \begin{aligned} r' &= \frac{r}{a}, \kappa' = \frac{b}{a}, t' = \frac{t}{a^2 \sqrt{\rho h / D}}, c' = \frac{a^2}{\sqrt{\rho h / D}} c, \\ \omega' &= \omega a^2 \sqrt{\rho h / D}, w' = \frac{w}{h} \sqrt{12(1-\nu^2)}, \\ F' &= \frac{F}{D}, q' = \frac{q a^4}{D h} \sqrt{12(1-\nu^2)} \end{aligned} \right\} \quad (2)$$

into Eq. (1) and omitting the primes yields:

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + \nabla^4 w \\ + \frac{1}{2} \omega^2 \left(r^2 \nabla^2 w + 2r \frac{\partial w}{\partial r} \right) - N(w, F) = q \\ \nabla^4 F - 2(1-\nu) \omega^2 = -\frac{1}{2} N(w, w) \end{aligned} \right\} \quad (3)$$

The case is considered in which the disk is fixed at the inner edge, and free at the outer edge. Then the boundary condition at the inner edge ($r=\kappa$) is as follows:

$$\left. \begin{aligned} w &= 0, \frac{\partial w}{\partial r} = 0 \\ \frac{\partial^2 F}{\partial r^2} - \frac{\nu}{r} \frac{\partial F}{\partial r} - \frac{\nu}{r^2} \frac{\partial^2 F}{\partial \theta^2} - \frac{1}{2} (1-\nu^2) \omega^2 r^2 &= 0 \\ \frac{\partial^3 F}{\partial r^3} + \frac{1}{r} \frac{\partial^2 F}{\partial r^2} - \frac{1}{r^2} \frac{\partial F}{\partial r} + \frac{2+\nu}{r^2} \frac{\partial^3 F}{\partial r \partial \theta^2} \\ - \frac{3+\nu}{r^3} \frac{\partial^2 F}{\partial \theta^2} - (1+\nu) r \omega^2 &= 0 \end{aligned} \right\} \quad (4)$$

The boundary condition at the outer edge ($r=1$) is as

follows:

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial r^2} + \frac{\nu}{r} \frac{\partial w}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 w}{\partial \theta^2} &= 0, \\ \frac{\partial}{\partial r} (\nabla^2 w) + \frac{1-\nu}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial^2 w}{\partial \theta^2} \right) &= 0 \\ \frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \theta^2} - \frac{1}{2} r^2 \omega^2 &= 0, \\ \frac{1}{r^2} \frac{\partial F}{\partial \theta} - \frac{1}{r} \frac{\partial^2 F}{\partial r \partial \theta} &= 0 \end{aligned} \right\} \quad (5)$$

To solve Eq. (3), stress function F is considered to consist of the component relating centrifugal force G and the other component H , as follows:

$$F = G + H \quad (6)$$

To obtain G , neglecting $N(w, w)$ in the second equation of Eq. (3) and replacing F by G , we have

$$\nabla^4 G - 2(1-\nu^2) \omega^2 = 0 \quad (7)$$

We suppose that G is axis symmetry. Solving Eq. (7) by with the boundary conditions (4) and (5) yields

$$G = \omega^2 \left(\frac{1-\nu}{32} r^4 + \frac{1}{16} c_1 r^2 + \frac{1}{8} c_2 \log r \right) \quad (8)$$

where

$$\begin{aligned} c_1 &= \frac{(1+\nu)(3+\nu) + \kappa^4(1-\nu^2)}{\kappa^2(1-\nu) + (1+\nu)}, \\ c_2 &= \frac{\kappa^2(1-\nu)(3+\nu) - \kappa^4(1-\nu^2)}{\kappa^2(1-\nu) + (1+\nu)} \end{aligned}$$

Substituting Eq. (8) into Eq. (6) and Eq. (3) successively yields the equations governing w and H , as follows:

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + \nabla^4 w - \omega^2 L(w) - N(w, H) &= q \\ \nabla^4 H &= -\frac{1}{2} N(w, w) \end{aligned} \right\} \quad (9)$$

where

$$\begin{aligned} L(w) &= \frac{1}{8r} \frac{\partial}{\partial r} \left[\left\{ -(3+\nu)r^3 + c_1 r + \frac{c_2}{r} \right\} \frac{\partial w}{\partial r} \right] \\ &\quad + \frac{1}{8} \left\{ -(1+3\nu) + \frac{c_1}{r^2} - \frac{c_2}{r^4} \right\} \frac{\partial^2 w}{\partial \theta^2} \end{aligned}$$

In Eq. (9), $\omega^2 L(w)$ is the term induced by centrifugal force, and $N(w, H)$ is the nonlinear term induced by large deflection.

2.2 Modal equations

Firstly, we seek w in the form of the superposition of the modes of linearized system, as follows:

$$w(r, \theta, t) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} X_{mn}(t) \Phi_{mn}(r) e^{im\theta} \quad (10)$$

where Φ_{mn} is the modal function with m nodal diameters and $(n-1)$ nodal circles, X_{mn} is an unknown. Modal function Φ_{mn} of a rotating disk were obtained in the authors' previous paper⁽¹⁴⁾. In the following, we normalize Φ_{mn} by the condition

$$\int_{\kappa}^1 r \Phi_{mn}^2 dr = 1 \quad (11)$$

Secondly, we seek H in the following form:

$$H(r, \theta, t) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} Y_{mn}(t) \Psi_{mn}(r) e^{2im\theta} \quad (12)$$

where Ψ_{mn} is the eigenfunction of the eigenvalue problem which is given by the equation:

$$\nabla^4 H = \mu^4 H \quad (13)$$

with the same boundary condition as the original problem. We normalize Ψ_{mn} by the condition

$$\int_{\kappa} r \Psi_{mn}^2 dr = 1 \quad (14)$$

Substituting Eq. (10) and (12) into Eq. (9), applying the Galerkin method yields

$$\left. \begin{aligned} \ddot{X}_{mn} + c\dot{X}_{mn} + p_{mn}^2 X_{mn} \\ + \sum_{i,l=-\infty}^{\infty} \sum_{i',l'=1}^{\infty} \alpha_{mni' l'} X_{ii'} Y_{l'} = Q_{mn} \\ Y_{l'} = \sum_{j,k=-\infty}^{\infty} \sum_{j',k'=1}^{\infty} \beta_{l'jj' kk'} X_{jj'} X_{kk'} \end{aligned} \right\} \quad (15)$$

Eliminating $Y_{l'}$ in Eq. (15) yields

$$\ddot{X}_{mn} + c\dot{X}_{mn} + p_{mn}^2 X_{mn} + \sum_{i,j,k=-\infty}^{\infty} \sum_{i',j',k'=1}^{\infty} \epsilon_{mni' jj' kk'} X_{ii'} X_{jj'} X_{kk'} = Q_{mn} \quad (16)$$

where

$$Q_{mn} = \frac{1}{2\pi} \int_0^{2\pi} \int_{\kappa} q(r, \theta, t) \Phi_{mn} e^{-im\theta} r dr d\theta$$

and p_{mn} is the natural frequency of the (m, n) -th mode, and finally α, β, ϵ in Eq. (15) and (16) are the coefficients of nonlinear terms.

3. Oscillations Near the Primary Resonance Points

3.1 Analysis by the harmonic balance method

We consider the oscillation near the primary resonance points excited at a point fixed in space. When a disk rotates, the disk is often subjected to periodic force synchronous with the rotation because of excentricity or inclination of the disk. So, we consider that the external force consists of two components, a harmonic force independent of the disk rotation, and a periodic force synchronous with the disk rotation, as follow:

$$q(r, \theta, t) = \delta(r - r_0) \delta(\theta + \omega t) \times \left\{ \frac{1}{2} q_1 (e^{i\omega t} + e^{-i\omega t}) + \frac{1}{2} q_2 (e^{i\omega t} + e^{-i\omega t}) \right\} \quad (17)$$

where δ is delta function, q_1 is the amplitude of the external force, and q_2 is the amplitude of the force synchronous with the disk rotation.

If oscillations of the rotating disk are observed at a point fixed in space, (m, n) -th modes appear in the form of forward and backward traveling modes with natural frequency $p_{mn} + m\omega$ and $p_{mn} - m\omega$, respectively. So, the frequencies of the primary resonance points of the forward and backward traveling modes are $p_f = p_{mn} + m\omega$ and $p_b = p_{mn} - m\omega$, respectively⁽¹²⁾.

Now, the case is considered in which the excitation frequency Ω is near the primary resonance points of the backward traveling modes, i.e.,

$$\Omega \approx p_b (= p_{mn} - m\omega) \quad (18)$$

is valid. In this case, the relation

$$\Omega + 2m\omega \approx p_f (= p_{mn} + m\omega) \quad (19)$$

holds. Also, the case is considered that the excitation frequency Ω comes near the primary resonance points of the forward traveling modes, i.e.,

$$\Omega \approx p_f (= p_{mn} + m\omega) \quad (20)$$

is valid. In this case, the relation

$$\Omega - 2m\omega \approx p_b (= p_{mn} - m\omega) \quad (21)$$

holds. In generally, combination tones are expected to occur in a nonlinear vibratory system with the natural frequency p , when it is excited by two external force with frequencies Ω_1 and Ω_2 satisfying

$$j\Omega_1 + k\Omega_2 \approx lp \quad (j, k, l \text{ is integer}) \quad (22)$$

The relations of Eq. (19) and (21) fall on this case, so combination tones are expected to occur near the primary resonance points of the forward and backward travelling modes.

As an example, the primary resonance point of the backward travelling mode of order $(1, 1)$ is considered. In this case, it is assumed that the behavior of the system is governed mainly by the backward and forward traveling modes of order $(1, 1)$, and hence the unknown quantities other than X_{11} and X_{-11} in Eq. (16) can be neglected. Then, Eq. (16) is reduced to

$$\left. \begin{aligned} \ddot{X}_1 + c\dot{X}_1 + p^2 X_1 + \epsilon X_1^2 X_{-1} \\ = \frac{1}{2} Q_1 \{ e^{i(\Omega+\omega)t} + e^{-i(\Omega-\omega)t} \} \\ + \frac{1}{2} Q_2 \{ e^{2i\omega t} + 1 \} \\ \ddot{X}_{-1} + c\dot{X}_{-1} + p^2 X_{-1} + \epsilon X_{-1}^2 X_1 \\ = \frac{1}{2} Q_1 \{ e^{i(\Omega-\omega)t} + e^{-i(\Omega+\omega)t} \} \\ + \frac{1}{2} Q_2 \{ 1 + e^{-2i\omega t} \} \end{aligned} \right\} \quad (23)$$

where

$$\begin{aligned} X_1 &= X_{11}, X_{-1} = X_{-11}, p = p_{11}, \\ \epsilon &= \epsilon_{11, -11, 11, 11} + \epsilon_{11, 11, -11, 11} + \epsilon_{11, 11, 11, -11} \\ Q_1 &= q_1 r_0 \Phi_{11}(r_0)/2\pi, Q_2 = q_2 r_0 \Phi_{11}(r_0)/2\pi \end{aligned}$$

To solve Eq. (23) by the harmonic balance method, the solution is put in the form:

$$\left. \begin{aligned} X_1 &= \frac{1}{2} R e^{i((\Omega+\omega)t-\gamma)} + \frac{1}{2} S e^{-i((\Omega+\omega)t-\delta)} \\ &+ \frac{1}{2} P_0 e^{-i(\Omega-\omega)t} + \frac{1}{2} P_1 e^{2i\omega t} + \frac{1}{2} P_2 \\ X_{-1} &= \frac{1}{2} R e^{-i((\Omega+\omega)t-\gamma)} + \frac{1}{2} S e^{i((\Omega+\omega)t-\delta)} \\ &+ \frac{1}{2} P_0 e^{i(\Omega-\omega)t} + \frac{1}{2} P_1 e^{-2i\omega t} + \frac{1}{2} P_2 \end{aligned} \right\} \quad (24)$$

where

$$P_0 = \frac{Q_1}{p^2 - (\Omega - \omega)^2}, P_1 = \frac{Q_2}{p^2 - 4\omega^2}, P_2 = \frac{Q_2}{p^2}$$

To solve Eq. (23), Eq. (24) are substituted into Eq. (23), and the harmonic balance method is applied. The resulting equations are then separated into real and imaginary parts to yield

$$\begin{aligned}
2(\Omega + \omega)\dot{R} &= -c(\Omega + \omega)R \\
&\quad - \frac{1}{4}\varepsilon P_2^2 S \sin(\gamma - \delta) \\
&\quad - \frac{1}{2}\varepsilon P_0 P_1 P_2 \sin \gamma + Q_1 \sin \gamma \\
2(\Omega + \omega)R\dot{\gamma} &= -\{p^2 - (\Omega + \omega)^2\}R \\
&\quad - \frac{1}{4}\varepsilon(R^2 + 2S^2 + 2P_0^2 + 2P_1^2 + 2P_2^2)R \\
&\quad - \frac{1}{4}\varepsilon P_2^2 S \cos(\gamma - \delta) \\
&\quad - \frac{1}{2}\varepsilon P_0 P_1 P_2 \cos \gamma + Q_1 \cos \gamma \\
2(\Omega + \omega)\dot{S} &= -c(\Omega + \omega)S \\
&\quad - \frac{1}{4}\varepsilon P_2^2 R \sin(\delta - \gamma) \\
&\quad - \frac{1}{2}\varepsilon P_0 P_1 P_2 \sin \delta \\
2(\Omega + \omega)S\dot{\delta} &= -\{p^2 - (\Omega + \omega)^2\}S \\
&\quad - \frac{1}{4}\varepsilon(S^2 + 2R^2 + 2P_0^2 + 2P_1^2 + 2P_2^2)S \\
&\quad - \frac{1}{4}\varepsilon P_2^2 R \cos(\delta - \gamma) - \frac{1}{2}\varepsilon P_0 P_1 P_2 \cos \delta
\end{aligned} \quad (26)$$

To obtain steady-state solutions of Eq. (26), derivatives with respect to time are put to zeros. Solving the resulting equations and substituting the obtained solutions into Eqs. (24) and (10), successively, yields the deflection of the rotating disk in the form:

$$\begin{aligned}
w &= \Phi_{11}[R \cos\{(\Omega + \omega)t + \theta - \gamma\} \\
&\quad + S \cos\{(\Omega + \omega)t - \theta - \delta\} \\
&\quad + P_0 \cos\{(\Omega - \omega)t - \theta\} \\
&\quad + P_1 \cos(2\omega + \theta) + P_2 \cos \theta]
\end{aligned} \quad (27)$$

Equation (27) expresses the deflection observed at the rotating coordinate (r, θ) . To transform it into that observed at the fixed coordinate in space (r, ϕ) , we replace θ in Eq. (27) by $(\phi - \omega t)$. The result is

$$\begin{aligned}
w &= \Phi_{11}[R \cos\{\Omega t + \phi - \gamma\} \\
&\quad + S \cos\{(\Omega + 2\omega)t - \phi - \delta\} \\
&\quad + P_0 \cos(\Omega t - \phi) + P_1 \cos(\omega t + \phi) \\
&\quad + P_2 \cos(\omega t - \phi)]
\end{aligned} \quad (28)$$

The above equation shows that R is the amplitude of the backward traveling wave with frequency Ω , and S the amplitude of the forward traveling wave with frequency $(\Omega + 2\omega)$. Furthermore, R is the amplitude of forced oscillation induced by external force, and S the amplitude of combination tone caused by non-linear coupling.

The stability of steady-state solutions is determined by giving a slight disturbance to a steady-state solution, and examining its variation with respect to time.

The case near the primary resonance point of the forward travelling mode can be analyzed similarly. In this case, it is shown that the backward travelling mode oscillation with frequency $(\Omega - 2\omega)$ occurs in addition to the forward travelling mode oscillation

$$\begin{aligned}
\kappa &= 0.21, \quad \nu = 0.3, \quad p_{11} = 5.743, \quad \omega = 2.0, \quad \varepsilon = 2.788 \\
r_0 &= 0.8, \quad q_1 = 0.01, \quad q_2 = 40.0, \quad c = 0.001
\end{aligned}$$

Fig. 1 Response and phase curves obtained theoretically

with frequency Ω .

This phenomenon can occur at any rotating speed, because the relations (19) and (21) are independent of ω .

3.2 Numerical examples

To see the characteristics of the oscillations, numerical calculations are conducted. Figure 1 shows an example of the response curves obtained for values of parameter given in the figure. In it, solid and dotted lines indicate stable and unstable oscillations, respectively.

The figure shows that the forward and the backward traveling wave always occur simultaneously. It also shows that two peaks appear in the resonance curves.

4. Experimental Analysis

4.1 Experimental method

To check the validity of the theoretical analysis, we conducted experimental analysis using a thin steel disk. A schematic diagram of the experimental apparatus used in this study is shown in Fig. 2.

An excitation force was given by two electro-magnets put on either side of disk and by flowing

Fig. 2 Schematic diagram of experimental apparatus

Fig. 3 Natural frequencies of rotating disk

electric current into them alternately. The force synchronous with the disk rotation was not given intentionally, but was induced by eccentricity inclination of the disk.

To measure its vibrations, four sensors were placed at intervals of 90 degrees apart in the circumferential direction. To extract data concerning circumferential modes, the signals obtained from the sensors were processed by operating amplifiers.

The dimensions of the disk are as follows:

$$a=225 \text{ mm}, b=48 \text{ mm}, h=1.0 \text{ mm}$$

(a) Time history

(b) Spectrum

Fig. 4 Time history and spectrum of oscillation obtained experimentally

4.2 Experimental results

To measure the natural frequencies of the disk, we gave an impact to the disk when it was rotating. The measured natural frequencies of modes of order (1, 1) are shown in Fig. 3. In the figure, $\circ$ indicates the measured values, and broken lines the theoretical values. It is seen that the natural frequencies are shifted from p by $\pm\omega$, and the difference of the natural frequencies of forward and backward traveling modes is always 2ω .

Setting the rotating speed at 10 Hz (600 rpm), and the excitation frequency close to the primary

(a) The component with frequency Ω

(b) The component with frequency $(\Omega + 2\omega)$

Fig. 5 Time history of the components with frequencies Ω and $(\Omega + 2\omega)$

resonance point of backward mode, an oscillation as shown in Fig. 4(a) was observed. This figure shows the time history obtained by four sensors. Fig. 4(b) shows the results of its spectrum analysis. It is seen in Fig. 4(b) that the oscillation includes a component of frequency $(\Omega + 2\omega)$ in addition to the component of excitation frequency Ω . Figures 5(a) and 5(b) show the components of frequencies Ω and $(\Omega + 2\omega)$ extracted from the original time history, respectively. It is seen in Fig. 5 that the components of frequencies Ω and $(\Omega + 2\omega)$ are the backward and forward traveling waves of order (1, 1), respectively (See dotted line in the figure).

Figure 6 shows the response curves obtained experimentally. It is seen in Fig. 6 that the forward and backward traveling waves occur always simultaneously, and that two peaks appear in these response curves.

The experiment near the primary resonance point of forward travelling mode was also conducted, and similar results were obtained. Experiments at other speeds were also conducted, and it is shown that the above-mentioned phenomenon can occur at any speed we conducted.

From above, it can be concluded that the experimental results confirm the validity of the theoretical

Fig. 6 Response curves obtained experimentally

analysis qualitatively.

5. Conclusion

As a typical case of nonlinear forced oscillations of rotating disk excited at a point fixed in space, the oscillation near the primary resonance points of forward and backward traveling modes has been considered. The obtained results can be summarized as follows:

(1) To analyze the oscillations near the primary resonance points of forward and backward traveling modes, it is needed to take account of forces synchronous with the disk rotation.

(2) Near the primary resonance point of the backward traveling modes, the forward traveling mode oscillation occurs in addition to the backward traveling mode oscillation. Similarly near the primary resonance point of forward traveling modes, the backward traveling mode oscillation occurs in addition to forward traveling mode oscillation.

(3) The oscillation is a kind of combination tone induced by the external force and the force synchronous with the disk rotation.

(4) In the response curves of this oscillation, two peaks appear.

An experimental analysis has been also conducted and the validity of the theoretical analysis has been confirmed.

Finally we mention that we studied oscillations of a rotating disk excited at a point fixed in space near the resonance points other than those in this paper and found that various types of oscillations occur^{(15),(16)}.

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