Nonstationary Oscillations of a Nonlinear Rotor during Acceleration through the Major Critical Speed* (Influence of Internal Resonance)

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The Jeffcott rotor is the most widely used theoretical model in the analysis of rotor systems. This model satisfies the condition of $1: (-1)$ internal resonance, and we clarified the effect of this internal resonance on nonlinear steady-state oscillations at the major, twice the major and three times the major critical speeds in previous papers. In this study, we investigated nonstationary oscillations during acceleration through the major critical speed in the Jeffcott rotor and other rotor systems which almost but not exactly satisfy the $1: (-1)$ internal resonance relation, and investigated the influence of internal resonance. The following are clarified theoretically and experimentally. (a) Nonstationary oscillations of the Jeffcott rotor and other rotor systems whose natural frequencies almost satisfy the $1: (-1)$ relation are influenced by the internal resonance. (b) It is easier to pass the major critical speed of a rotor system with internal resonance than that of a rotor system with no internal resonance. (c) When the system has slight discrepancies among critical speeds, that is, almost satisfies the relation of internal resonance, the rotor can pass the major critical speed very easily with small amplitude even if the acceleration is small.

Key Words: Vibration of Rotating Body, Nonlinear Vibration, Critical Speed, Nonstationary Response, Internal Resonance

1. Introduction

Many studies have appeared on the nonstationary oscillations of rotor systems. However, reports on nonlinear rotor systems are few in number.

Mitropol'skii analyzed nonstationary oscillations of a nonlinear rotor during acceleration through the major critical speed by the asymptotic method he developed⁽¹⁾. Evan-Iwanowski analyzed nonstationary oscillations through the critical speeds of various combination resonances by the asymptotic method^{(2),(3)}. We developed the complex Fast Fourier Transform (complex-FFT) method utilizing a digital signal processing technique and investigated nonstationary oscillations through the major critical speed⁽⁴⁾, the critical speeds of subharmonic resonances^{(5),(6)} and the critical speed of a combination resonance⁽⁷⁾. In these studies, we clarified the physical

characteristics of nonstationary oscillations by drawing trajectories and separatrices in the phase plane and the phase space. We also investigated the accuracy of the complex-FFT method by comparing its results with those of the asymptotic method⁽⁸⁾. However, all of these studies are related to systems with no internal resonance, that is, the cases the critical speeds appear with no duplication.

The Jeffcott rotor model is shown in Fig. 1(a). This model has often been used in the analysis of nonlinear resonances^{(9),(10)}. Natural frequencies of this model are shown in Fig. 2(a), in which p_f and p_b are the natural frequencies of a forward and a backward whirling modes, respectively. In the previous paper, we pointed out that the Jeffcott rotor satisfies the internal resonance relation $p_f: p_b = 1: (-1)$ and clarified that various phenomena called internal resonance phenomena appear at the major, twice the major and three times the major critical speed⁽¹¹⁾.

In this paper, nonstationary oscillations of the Jeffcott rotor and other rotor systems with nonlinear spring characteristics during acceleration through the major critical speed are theoretically and

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experimentally investigated. The influences of internal resonance on the characteristics of nonstationary oscillations are clarified. We also demonstrate that a system with internal resonance can pass the major critical speed much easier than a system with no internal resonance.

2. Equations of Motion

As shown in Fig. 1 (a), the Jeffcott rotor is a 2-degree-of-freedom (2 DOF) deflection model whose natural frequencies are independent of the rotational speed ω . In such a system, critical speeds of the harmonic resonance $[+p_f]^{(12)}$, the backward harmonic resonance $[-p_b]^{(13)}$, and the super-combination resonance $[(p_f - p_b)/2]^{(14)}$ coincide precisely. Here, for example, the notation $[(p_f - p_b)/2]$ means the resonance that occurs when the relationship $(p_f - p_b)/2 = \omega$ holds. To investigate the transition from the state with no internal resonance to that with internal resonance, we use the 2 DOF inclination model shown in Fig. 1 (b) in which the gyroscopic moment works and natural frequencies change as a function of the rotational speed ω . The equations of motion governing nonstationary oscillations in the dimensionless form are given as follows⁽⁵⁾:

$$\left. \begin{aligned} \ddot{\theta}_x + i_p \ddot{\Psi} \theta_y + i_p \dot{\Psi} \dot{\theta}_y + c \dot{\theta}_x + \theta_x + N_{\theta x} \\ = (1 - i_p) \tau (\dot{\Psi}^2 \cos \Psi + \ddot{\Psi} \sin \Psi), \\ \ddot{\theta}_y - i_p \ddot{\Psi} \theta_x - i_p \dot{\Psi} \dot{\theta}_x + c \dot{\theta}_y + \theta_y + N_{\theta y} \\ = (1 - i_p) \tau (\dot{\Psi}^2 \sin \Psi - \ddot{\Psi} \cos \Psi), \end{aligned} \right\} \quad (1)$$

where θ_x and θ_y are the components of the shaft inclination angle θ at the rotor position, i_p is the ratio of the polar moment of inertia to the diametral moment of inertia of the rotor, c is the damping coefficient, $N_{\theta x}$ and $N_{\theta y}$ are the nonlinear terms in the restoring forces in the x - and y -directions, τ is the dynamic unbalance of the rotor and t is time. The phase angle Ψ , which represents the angle between the direction of the dynamic unbalance τ and the x -axis in the x - y plane, is given as follows when the angular acceleration λ is constant:

(a) The Jeffcott rotor model (b) 2 DOF inclination model

Fig. 1 Analytical models

$$\Psi = \frac{1}{2} \lambda t^2 + \omega_0 t + \Psi_0, \quad \omega = \dot{\Psi} = \lambda t + \omega_0, \quad (2)$$

where ω_0 and Ψ_0 are the initial angular velocity and the initial angle, respectively.

The natural frequency diagram is shown in Fig. 2 (b). The natural frequencies p_f and p_b change with the rotational speed ω and the rates of these changes depend on the parameter i_p relating the magnitude of the gyroscopic moment. Therefore, when the value of i_p is not small, critical speeds of nonlinear resonances separate from each other and each resonance occurs independently⁽¹¹⁾. When the value of i_p is equal to zero, Eq. (1) gives the equations of motion of the Jeffcott rotor where the critical speeds of the harmonic resonance and the backward harmonic resonance coincide as shown in Fig. 2 (a).

The expressions of nonlinear terms $N_{\theta x}$ and $N_{\theta y}$ are derived from the potential energy V of the system as follows⁽¹²⁾:

$$V = V_0 + V_N, \quad N_{\theta x} = \frac{\partial V_N}{\partial \theta_x}, \quad N_{\theta y} = \frac{\partial V_N}{\partial \theta_y}. \quad (3)$$

Here, V_0 and V_N are the potential energies corresponding to linear and nonlinear restoring forces, respectively. If we consider that $N_{\theta x}$ and $N_{\theta y}$ consist of the second and the third power terms of θ_x and θ_y , V_0 consists of the second power terms and V_N consists of the third and the fourth power terms. In rotor systems, as shown in the previous paper⁽¹²⁾, the polar coordinate expression (θ, φ) is more suitable than the rectangular coordinate expression (θ_x, θ_y) . By substituting the transformation

$$\theta_x = \theta \cos \varphi, \quad \theta_y = \theta \sin \varphi \quad (4)$$

into Eq. (3), we have V_N in the polar coordinate expressions as follows:

$$\begin{aligned} V_N = & (\varepsilon_c^{(1)} \cos \varphi + \varepsilon_s^{(1)} \sin \varphi + \varepsilon_c^{(3)} \cos 3\varphi \\ & + \varepsilon_s^{(3)} \sin 3\varphi) \theta^3 \\ & + (\beta_c^{(0)} + \beta_c^{(2)} \cos 2\varphi + \beta_s^{(2)} \sin 2\varphi + \beta_c^{(4)} \cos 4\varphi \\ & + \beta_s^{(4)} \sin 4\varphi) \theta^4. \end{aligned} \quad (5)$$

(a) The Jeffcott rotor model (b) 2 DOF inclination model

Fig. 2 Natural frequencies

From this expression, we can classify nonlinear forces $N_{\theta x}$ and $N_{\theta y}$ into the components $N(n)$. The notation $N(n)$ represents the component whose magnitude of potential energy changes n times while φ changes from 0 to 2π with keeping θ constant. If the nonlinear restoring forces consist of the terms up to the third order of the angle θ , we have nonlinear components $N(0) - N(4)$. The component $N(0)$ is the isotropic nonlinear component, and the components $N(n)$ ($n=1-4$) are the anisotropic nonlinear components.

The symbol $\varepsilon_c^{(1)}$, $\varepsilon_s^{(1)}$ and $\varepsilon_c^{(3)}$, $\varepsilon_s^{(3)}$ in Eq. (5) are the coefficients representing the magnitudes of asymmetric nonlinear components $N(1)$ and $N(3)$, respectively, and $\beta^{(0)}$, $\beta_c^{(2)}$ and $\beta_s^{(2)}$, and $\beta_c^{(4)}$ and $\beta_s^{(4)}$ are the coefficients representing the magnitudes of symmetric nonlinear components $N(0)$, $N(2)$ and $N(4)$, respectively.

In practical systems, components with complex potential distribution, such as $N(3)$ and $N(4)$, rarely appear. When the system is well-assembled, the isotropic nonlinear component $N(0)$ appears predominantly. However, when there is a discrepancy between the center lines of the bearings due to misalignment, the anisotropic nonlinear component $N(1)$ appears in addition to $N(0)$. In this paper, we consider a system with the components $N(0)$ and $N(1)$. In this case, we can put $\varepsilon_c^{(1)} = \varepsilon^{(1)}$ and $\varepsilon_s^{(1)} = 0$ by rotating the coordinate axis without losing generality, because the anisotropic nonlinear component of this system is only $N(1)$.

3. Theoretical Analysis

Generally, internal resonance phenomena may appear when natural frequencies satisfy a simple integral ratio in nonlinear systems^{(15),(16)}. In this section, we investigate the nonstationary oscillations of the Jeffcott rotor and other rotor systems with small discrepancies among the critical speeds. By comparing these results, we can clarify the influence of internal resonance.

3.1 The Jeffcott rotor

Equations of motion of the Jeffcott rotor are obtained by substituting $i_p = 0$ into Eq. (1). In the Jeffcott rotor, the internal resonance relation $p_f : p_b = 1 : (-1)$ holds and the major critical speed coincides precisely with the critical speed of backward harmonic resonance. By integrating Eq. (1) numerically, we get time histories and the FFT spectral diagrams of nonstationary oscillations in the system with only $N(0)$ and that with $N(0)$ and $N(1)$. The results are shown in Fig. 3. Although both systems have the same relation of internal resonance, a nonstationary oscillation in the system with $N(0)$ and

$N(1)$ has the backward harmonic component $-\omega$ in addition to the harmonic component $+\omega$, whereas the system with just $N(0)$ has only the harmonic component $+\omega$. We see that, with the angular acceleration $\lambda = 0.0015$, the rotor can pass the major critical speed in the system with $N(0)$ and $N(1)$ but cannot in the system with only $N(0)$.

Figure 4 shows the resonance curves and nonstationary response curves of harmonic component $+\omega$. The notations P_0 and $P(t)$ mean the amplitudes of the former and the latter, respectively. These amplitudes are obtained from $\sqrt{\theta_x^2 + \theta_y^2}$ of the harmonic component in each case. The values of the

Fig. 3 Time histories and FFT spectra in the Jeffcott rotor

Fig. 4 Amplitude variations through the critical speed of the Jeffcott rotor

parameters are the same as those in Fig. 3(b) except for the angular acceleration λ . The symbols $\blacktriangledown$ and $\triangledown$ show the critical speeds of the forward and the backward harmonic resonances, respectively. In resonance curves, solid lines represent stable solutions, and broken lines and chain lines represent unstable solutions. The difference between broken lines and chain lines show the difference in the unsatisfied condition in the Routh-Hurwitz criterion⁽¹⁷⁾. In the following figures for resonance curves, we use the same expressions. There is a critical angular acceleration λ_{cr} under which nonstationary oscillations become large along the stable branch. We call this a non-passable case. If λ is larger than λ_{cr} , nonstationary oscillations can exceed the critical speed and approach the low amplitude stable branch in the higher speed side asymptotically. We call this a passable case.

In phase space, stable and an unstable solutions are represented by a stable singular point and an unstable singular point, respectively. The former has a basin of attraction where a state point representing the rotor position is attracted to this singular point. When the rotational speed changes, these singular points move in phase space and the state point of the rotor follows a stable one. Suppose that the rotor is accelerated from $\omega=0.7$ in Fig. 4. In the case $\lambda=0.001$, the state point is attracted and follows the stable singular point corresponding to stable branch AB (non-

passable case). While in the case $\lambda=0.0015$, the state point trails the stable singular point and goes out from its basin of attraction. Then the state point enters into another basin of attraction and converges to stable branch FG (passable case). Therefore the possibility of passing the major critical speed is closely related to the relation between the basin of attraction and the state point of the rotor in phase space⁽⁵⁾⁻⁽⁷⁾.

3.1.1 Influence of the symmetric nonlinear component $N(0)$ To investigate the influence of the symmetric nonlinear component $N(0)$ on the nonstationary oscillation, amplitude variations in the nonstationary oscillations for two different values of parameter $\beta^{(0)}$ are shown in Fig. 5. Figure 5(a) shows that the resonance curves with dual peaks incline more strongly to the high speed side when $\beta^{(0)}$ becomes large. In this case, the rotor cannot pass the critical speed with acceleration $\lambda=0.0065$, which is much greater than the passable value in Fig. 4. The movement of the basin of attraction corresponding to branch AB is slower than the case of Fig. 4 for the same angular acceleration.

Figure 5(b) shows that the resonance curves become soft spring types when $\beta^{(0)}$ is small. As $\beta^{(0)}$ decreases from Fig. 4, the stable branch becomes upright and then turns left, and saddle-node bifurcation appears simultaneously at point H. When we accelerate from the lower speed side, the basin of attraction moves along stable branch AH and disappears at bifurcation point H. Therefore, the state point of the rotor enters another basin of attraction corresponding to stable branch ID. As a result, nonstationary oscillations can pass the critical speed with very small angular accelerations.

Figure 6 shows the maximum amplitude P_{max} of the harmonic oscillation component $+\omega$ (see Fig. 4) as a function of the angular acceleration λ for various values of $\beta^{(0)}$. When we decrease the acceleration λ

Fig. 5 Influence of nonlinear component $N(0)$ (The Jeffcott rotor)

Fig. 6 Variations of the maximum amplitude for various $N(0)$ (The Jeffcott rotor)

from a large value, this characteristic curve becomes vertical at a certain value of λ . This value means the critical angular acceleration λ_{cr} that designates the boundary between passable and a non-passable cases. The smaller the amount of $\beta^{(0)}$, the smaller the critical angular acceleration λ_{cr} becomes. However, if the resonance curves have a stable branch of a soft spring type for small $\beta^{(0)}$ (as shown in Fig. 5(b)), we must pay attention to the maximum amplitude obtained in a deceleration case.

3.1.2 Influence of the asymmetric nonlinear component $N(1)$ To clarify the influence of the asymmetric nonlinear component $N(1)$, amplitude variations of the nonstationary oscillations for various values of the parameter $\epsilon^{(1)}$ are investigated. The values of other parameters are the same as those in Fig. 4.

Figure 7(a) shows a case in which $\epsilon^{(1)}$ becomes large, the influence of internal resonance becomes stronger and the shape of the resonance curves becomes more complex. When the rotational speed increases from the lower speed side, the basin of attraction of the stable singular point corresponding to branch AH disappears at point H where the saddle-node bifurcation occurs. Then the state point representing the rotor position moves into the basin of

(a) System with $N(0)$ and $N(1)$ (effected by internal resonance)

(b) System with $N(0)$ (no effects of internal resonance)

Fig. 7 Influence of nonlinear component $N(1)$ (The Jeffcott rotor)

another stable branch. If the angular acceleration is not so large (when $\lambda=0.001$), the nonstationary solution follows stable branch DE which expands to the large amplitude region (a non-passable case). However, if the angular acceleration is large enough (when $\lambda=0.0015$), the nonstationary solution is captured by the basin of attraction corresponding to stable branch FG and the amplitude decreases (a passable case).

When $\epsilon^{(1)}$ becomes small, the influence of internal resonance decreases. When $\epsilon^{(1)}=0$, we get the result shown in Fig. 7(b), in which the shape of the resonance curves is the same as that of solitary resonance. The basin of attraction of the stable singular point corresponding to stable branch AB moves smoothly and continuously as the rotational speed increases. As a result, a large angular acceleration is necessary to escape this basin of attraction.

Figure 8 shows the influence of $\epsilon^{(1)}$. When the value of $\epsilon^{(1)}$ decreases, the critical angular acceleration λ_{cr} increases and it becomes more difficult to pass the major critical speed.

3.2 Transition from a system with internal resonance to a system with solitary resonance

Nonlinear resonance is also influenced by internal resonance if the discrepancies among critical speeds are small⁽¹¹⁾. In the 2 DOF inclination model, parameter i_p influences on the degree of discrepancy among critical speeds. When i_p increases from zero, the shape of resonance curves changes from that of the Jeffcott rotor. As i_p increases further, the shape converges asymptotically to that of solitary resonance.

We investigated the influence of internal resonance on the nonstationary oscillation while paying attention to the transition process from internal resonance to solitary resonance.

3.2.1 Systems with slight discrepancies among critical speeds In this 2 DOF system, the action of gyroscopic motion and the discrepancies among the

Fig. 8 Variations of the maximum amplitude for various $N(1)$ (The Jeffcott rotor)

critical speeds are determined by the value of i_p . Figure 9(a) for $i_p=0.03$ shows the resonance curves and nonstationary oscillations in a system with slight discrepancies among critical speeds. As i_p increases from that in Fig. 4, branch AB combines with branch CDE and then separates into branch AJDE and branch BKC. In Fig. 9(a), when we sweep slowly from the lower speed side, amplitude P changes up and down with steep slope following branch AJDE.

The nonstationary oscillations for various angular accelerations λ are shown in this diagram. We see that it is possible to pass the major critical speed with an angular acceleration much smaller than that of Fig. 4. Because the stable resonance curve AJDE is complex, the basin of attraction of the singular point corresponding to this stable curve moves quickly when rotational speed changes slowly. As a result, the state point of the rotor easily enters another basin of attraction corresponding to stable branch FG.

3.2.2. System with some discrepancies among critical speeds Figure 9(b) shows resonance curves and nonstationary oscillations of the system with some discrepancies among critical speeds. In this case, the shape of the resonance curves is similar except in stability to that of the rotor system where resonances occur solitarily. In the vicinity of the

chain unstable branch, almost periodic motions may occur^{(11),(17)}. In this system, the influence of internal resonance on nonstationary oscillations decreases, and the critical angular acceleration becomes larger than that in Fig. 9(a).

If the value of i_p increases further, for example $i_p=0.6$, resonance occurs solely and the influence of internal resonance disappears. The resonance curves and nonstationary oscillations in this case are shown in the previous paper⁽⁴⁾.

3.2.3 Influence of angular accelerations Figure 10 shows the influence of i_p , which determines the magnitudes of the differences among critical speeds. From the comparison between the case with internal resonance ($i_p=0$) and the case with no internal resonance ($i_p=0.6$), we see that the critical angular acceleration of the former case (designated by λ_a) is a little smaller than that of the latter case (designated by λ_b).

When there is a slight discrepancy ($i_p=0.03$), the critical angular acceleration λ_e is much smaller than the other two cases, if the narrow, non-passable ranges $\lambda_c-\lambda_d$ and $\lambda_e-\lambda_f$ are neglected.

By comparing these three cases, we see that the system can pass the major critical speed most easily when the critical speeds are slightly different from each other. Namely, if we utilize the relation for internal resonance in designing rotor systems, the rotor can pass the major critical speed very easily with a small amplitude even if the acceleration is small.

4. Experiment

4.1 Experimental setup

The experimental setup is shown in Fig. 11(a). An elastic shaft with circular cross section with a length $l=701$ mm and a diameter $d=12$ mm was simply supported at the upper side by a self-aligning double-ball bearing (#1200) and fixed at the lower side by a single-row deep groove ball bearing (#

Fig. 9 Influence of discrepancies among critical speeds

Fig. 10 Variations in the maximum amplitude for various discrepancies among critical speeds

6204). A disk mounted on the shaft can be moved to various positions. The diameter and the thickness of the disk are 300 mm and 14 mm, respectively. The notation a is the distance of the rotor from the lower bearing. The position of rotor a is related to the magnitude of gyroscopic moment.

In this system, the possible angular acceleration λ that can be realized is about 15–90 (rpm/sec). Maximum amplitudes measured in the x and y directions are also limited to about 4 mm to prevent accidents.

When we consider this system as a 4 DOF concentrated mass system in which the deflection and inclination of the rotor are coupled, the natural frequency diagram is obtained as shown in Fig. 11(b). The natural frequencies p_f and p_b of a 2 DOF inclination model correspond to p_2 and p_3 , respectively.

When the rotor is located at the center of the shaft, there is no coupling between a deflection motion and an inclination motion. We can then divide the motion in this 4 DOF system into the 2 DOF deflection motion and the 2 DOF inclination motion. In this case, the lower two natural frequencies p_2 and p_3 correspond to those of deflection motion, and the higher two natural frequencies p_1 and p_4 are those of inclination motion. Because this deflection motion is not influenced by the gyroscopic moment, the forward natural frequency p_2 and the backward natural frequency p_3 are independent of the rotational speed ω and are constant. Therefore the critical speeds of some nonlinear resonances coincide at the major critical speed.

When the rotor is not at the center of the shaft,

Fig. 11 Experimental system

the system is a 4 DOF system with coupling between a deflection motion and an inclination motion. A natural frequency diagram of such a system is shown in Fig. 11(b). The natural frequencies p_2 and p_3 of the lower mode in which the deflection motion is dominant are influenced by the gyroscopic moment, and they change as a function of rotational speed ω . The rate of change of the natural frequencies p_2 and p_3 with ω depend on the rotor position. If we analyze the motion of this lower mode by a 2 DOF system, we should use an inclination model shown in Fig. 1(b). In this case, the parameter i_p in the theoretical analysis reflects the distance of the rotor from the center of the shaft.

This experimental setup has nonlinear spring characteristics due to the angular clearance of the single-row deep groove ball bearing. The amount of this angular clearance is $0.2^\circ - 0.5^\circ$, and this corresponds to a length of 3 mm–6 mm at the upper bearing position. We use the notation δ to represent the distance between the centerlines of the upper and lower bearings.

When the setup is well-assembled ($\delta \approx 0$), the system has an isotropic nonlinear spring characteristic. In this case, the system corresponds to the system with only $N(0)$ in the theoretical analysis.

When the assembly condition is not so good, the system has the characteristics of an asymmetric nonlinear spring. In this case, the system corresponds to a system with $N(0)$ and $N(1)$ in the theoretical analysis.

In this study, we investigated the following three cases.

- Setup 1: $a = (1/2)l$: Perfect coincidence among critical speeds (corresponds to $i_p = 0$)
- Setup 2: $a = (1/4)l$: Solitary resonance
- Setup 3: $a = (3/8)l$: Slight discrepancies among critical speeds (corresponds to $i_p = 0.03$)

4.2 Experimental results

In the experiment, we measured deflections in the x and y directions during acceleration through the major critical speed for various values of distance a of the rotor position and various values of distance δ .

Figure 12 shows the maximum amplitude $P_{\max}$ of the harmonic component $+\omega$ during acceleration through the major critical speed in Setup 1, Setup 2 and Setup 3. The symbol $\circ$ shows the maximum value of the amplitude $\sqrt{x^2 + y^2}$ of the harmonic component $+\omega$ obtained by spectral analysis of the time histories in the x and y directions. Some examples of these time histories are also shown in the figure. By comparing these results, we can determine the effect of rotor position, namely, the effect of the internal resonance.

In Setup 1 where the rotor is located at the center

of the shaft, the gyroscopic moment does not work in its deflection motion, and critical speeds of harmonic resonance $[+p_2]$ and backward harmonic resonance $[-p_3]$ coincide. This system is under the influence of internal resonance and corresponds to the case $i_p=0$ shown in Fig. 4 in the theoretical analysis. In Setup 2 the gyroscopic moment works, and critical speeds of harmonic resonance $[+p_2]$ and backward harmonic resonance $[-p_3]$ are separate enough from each other. As resonance occurs solely in this system, non-stationary oscillation is not influenced by internal resonance.

(a) Setup 1 ($a=(1/2)l$ $\delta \approx 2$ mm)

(b) Setup 2 ($a=(1/4)l$ $\delta \approx 2$ mm)

(c) Setup 3 ($a=(3/8)l$ $\delta \approx 1$ mm)

Fig. 12 Variations in the maximum amplitude for various rotor positions (Experimental results)

From the comparison of the maximum amplitude curves in Fig. 12(a) and Fig. 12(b), we see that the critical angular acceleration of Setup 1 is smaller than that of Setup 2. In Setup 2, we could pass the major critical speed with the angular acceleration $\lambda \approx 92$ (rad/sec), but could not pass with an acceleration smaller than that. As a result, we confirm that the system influenced by internal resonance can pass the major critical speed with smaller angular acceleration than the system with no effects of internal resonance.

In Fig. 12(c), we show the result in Setup 3, which has slight discrepancies among critical speeds. The influence of the internal resonance on the non-stationary oscillation in this system is remarkable. We could pass the major critical speed with the slowest acceleration within the possible acceleration range in experiments, and the maximum amplitudes for every acceleration much decrease from those in Fig. 12(a) and Fig. 12(b). For example, the case $i_p=0.03$ shown in Fig. 9(a) in the theoretical analysis corresponds to this case.

Next we investigate the influence of the asymmetric nonlinear component $N(1)$ caused by the distance δ . In Fig. 12(c) and Fig. 13, the setup (the rotor position) is the same but the distance δ is different. Figure 13 shows the experimental result of maximum amplitude in the case with a distance $\delta \approx 3$ mm. In this case, as the case Fig. 12(c), we could pass the major critical speed with the slowest acceleration. In addition, all the maximum amplitudes for every acceleration are smaller than those in Fig. 12(c).

These experimental results qualitatively coincide with the theoretical results. As a result, we confirm the validity of the theoretical analysis.

Summarizing these results, we see that, to pass the major critical speed with small acceleration and small maximum amplitude, the system with slight differences between critical speeds and asymmetric

Fig. 13 Influence of the distance δ on the variation in the maximum amplitude (Experimental results)

nonlinearity is most preferable.

5. Conclusions

From the theoretical and experimental analyses concerning the nonstationary oscillations during acceleration through the major critical speed, we achieved the following results:

(1) In the Jeffcott rotor and other rotor systems with slight discrepancies among the critical speeds, the nonstationary oscillations during acceleration through the major critical speed are influenced by the internal resonance when it has characteristics of an asymmetric spring.

(2) In nonlinear rotor systems influenced by internal resonance, there are cases in which three stable branches exist at certain rotational speeds. In such cases, the angular acceleration determines to which of these stable branches the nonstationary oscillation is attracted.

(3) In nonlinear rotor systems influenced by internal resonance, we can pass the major critical speed easier than in the systems that are not effected by internal resonance.

(4) In nonlinear rotor systems influenced by internal resonance, it becomes easier to pass the major critical speed as the isotropic nonlinear component $N(0)$ decreases.

(5) In nonlinear rotor systems influenced by internal resonance, it becomes easier to pass the major critical speed as the asymmetric nonlinear component $N(1)$ increases.

(6) To pass the major critical speed with small acceleration and small maximum amplitude, the system with slight differences among critical speeds and with asymmetric nonlinearity is most preferable.

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