Analysis of Chatter Vibration in an Automotive Wiper Assembly*

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The objective of this paper is to analyze chatter vibrations in an automotive wiper assembly which consists of two blades driven by a direct current motor via one link. For this purpose, first, the characteristics of the chatter vibrations in a currently used wiper are examined experimentally. Then, by assuming that the wiper is modeled by a two-degree-of freedom system, the equations of motion are derived. They are solved for practical wiping speeds. By comparing the results with the experimental results, it is found that the derived equations well predict the experimental results. Based on the equations it is concluded that the chatter vibration is mode-locked motion. Finally, the equations of motion are solved for lower wiping speeds. It is predicted that for these speeds, chaotic motion as well as quasi-periodic and periodic motion can occur.

Key Words : Frictional Vibration, Self-Excited Vibration, Stick-Slip Motion, Mode-Locked Motion, Chaos

1. Introduction

Chatter vibrations are often observed in an automotive wiper. Their occurrence spoils comfortable driving. To find an effective way to control chatter vibrations, we attempt to clear the mechanism of their occurrence.

A front wiper system has two blades. They are attached to the cowl at the driver's side and the passenger's side. Each blade is supported by an arm which moves to and fro around the pivot. This motion is given by the rotating motion of a DC motor via a pantographic link.

On the motion of this system, some studies⁽¹⁾⁻⁽³⁾ are conducted. In these studies, it is concluded theoretically and experimentally that chatter vibration is self-excited vibration caused by a friction with negative slope in its characteristic curve. Also it is concluded experimentally that the steady state motion is a kind of stick motion. But, we found that the results

of these studies are insufficient for predicting some characteristics of chatter vibrations we observed in a currently used wiper. So we first attempt to derive the equations of motion which enable to explain these characteristics.

Because of its diversity, the behavior of a system induced by a dry-friction force attracts interest of many researchers⁽⁴⁾⁻⁽⁹⁾. The wiper is just a system subject to a dry-friction force. Its behavior may also be interest. So, based on the equations derived above, we investigate the behavior of the wiper system not only for practically used wiping speeds but also for lower speeds.

2. Characteristics of Chatter Vibrations

The motion of a wiper when chatter vibrations are occurring is measured by accelerometers which are put on both arms of the wiper assembly. An example of the time history and the spectrum of the motion obtained in this way are shown in Fig. 1. The figure shows that there are two types of motion: one in which the two blades deflect in the same direction (in-phase type) and the other in which the two blades deflect in opposite direction (out-of-phase type). It is found from the motion pictures taken by a high speed video system that the above two types of motion accompany stick-slip motion. It is confirmed that

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(a) First-mode-locked vibration (In plane type)

(b) Second-mode-locked vibration (Out of plane type)

Fig. 1 Chatter vibration in an automotive wiper

Fig. 2 Mathematical model of wiper

which type of motion occurs depends on the initial conditions, and that the motion of out-of-phase type is more frequently observed than that of in-phase type. Also it is confirmed by a hammer test, that the frequencies of the motion of in-phase type and out-of-phase type coincide with the first and second natural frequencies. In the following, the motion of in-phase type will be referred to as the first-mode-locked vibration and that of out-of-phase type as the second-mode-locked vibration.

3. Equations of Motion

We attempt to derive the equations of motion. For this purpose, we consider a two-degree-of-freedom system shown in Fig. 2. In this figure, the quantities with subscripts D and P are related to driver's and passenger's side, respectively. The lines with notations L_i ($i=D, P$) indicate the positions which the wiper arms take when no deflections occur. The quantities θ_i ($i=D, P$) are the angular deflections with respect to the position L_i . The notations $\dot{\phi}_i$ are the angular velocity of the arms. The notations l_i are the length of the arm between the pivot center and the top. Then

$$\dot{z}_i = (\dot{\theta}_i + \dot{\phi}_i)l_i \quad (i=D, P) \quad (1)$$

represent relative velocities of the blades with respect to L_i at the position of the top of the wiper arms. The quantities I_i ($i=D, P$) are the moments of inertia, and K_D, K_P, K_M are the coefficients of stiffness. The quantities M_i ($i=D, P$) are the moments induced by the friction force between the wiper blades and the windshield.

Using the notations given above, the equations of motion for a wiper on the i 's ($i=D, P$) side are

when $\dot{z}_i \neq 0$

$$I_i \ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i)$$

when $\dot{z}_i = 0$, $|R_i| \geq N_i l_i \mu_0$

$$I_i \ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i)$$

when $\dot{z}_i = 0$, $|R_i| \leq N_i l_i \mu_0$

$$I_i \ddot{\theta}_i = 0 \quad (\dot{\theta}_i = \dot{\phi}_i) \quad (2)$$

Here R_i and D_i are the moments produced by the restoring force and the damping force, given as follows:

$$R_D = k_D \theta_D - k_{DP} \theta_P, \quad R_P = k_P \theta_P - k_{PD} \theta_D$$

$$D_D = c_D \dot{\theta}_D - c_{DP} \dot{\theta}_P, \quad D_P = c_P \dot{\theta}_P - c_{PD} \dot{\theta}_D$$

$$k_D = K_D(K_P + K_M)/(K_D + K_P + K_M) \quad (3)$$

$$k_{DP} = k_{PD} = K_D K_P / (K_D + K_P + K_M)$$

$$k_P = K_P(K_D + K_M)/(K_D + K_P + K_M)$$

$$c_D = c_{DP}, \quad c_P = c_{DP} + c_P, \quad c_{DP} = c_{PD} = c_{DP}$$

Also, moments M_i ($i=D, P$) are given as follows:

$$M_i(\dot{z}_i) = N_i l_i \mu(\dot{z}_i) \quad (4)$$

where N_i is the force pressed down on the arm by the spring. Finally the coefficient of friction μ is given as

$$\mu(\dot{z}_i) = \mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 \dot{z}_i + \mu_2 \dot{z}_i^3 \quad (i=D, P) \quad (5)$$

which approximates the experimental results.

To check the validity of the equations of motion, we solve them numerically. We take the values of the parameters as follows:

$$\left. \begin{aligned} I_D &= 4.07 \times 10^{-2} (\text{kg} \cdot \text{m}^2), \\ I_P &= 3.67 \times 10^{-2} (\text{kg} \cdot \text{m}^2), \\ K_D &= 7.20 \times 10^2 (\text{N} \cdot \text{m/rad}), \\ K_P &= 7.51 \times 10^2 (\text{N} \cdot \text{m/rad}), \\ K_M &= 3.53 \times 10^2 (\text{N} \cdot \text{m/rad}), \\ C_{DP} &= 1.00 \times 10^{-2} (\text{N} \cdot \text{m} \cdot \text{s/rad}), \\ C_P &= 1.00 \times 10^{-2} (\text{N} \cdot \text{m} \cdot \text{s/rad}), \quad l_D = 4.70 \times 10^{-1} (\text{m}), \\ l_P &= 4.50 \times 10^{-1} (\text{m}), \quad \dot{\phi}_P = 1.16 \dot{\phi}_D (\text{rad/s}), \\ N_D &= 7.35 (\text{N}), \quad N_P = 5.98 (\text{N}), \\ \mu_0 &= 1.18, \quad \mu_1 = -9.84 \times 10^{-1}, \quad \mu_2 = 4.74 \times 10^{-1} \end{aligned} \right\} \quad (6)$$

In Fig. 3 we show an example of the time histories obtained in this way. These time-histories are similar to those in Fig. 1. The phase planes in Fig. 3 show that the mode-locked vibrations accompany the stick-slip motion. In Fig. 4 we show the amplitudes of displacements against the wiping speeds $V (= \dot{\phi}_D l_D)$. For comparison, we add in Fig. 4 those obtained experimentally. The figure shows that the equations well explain the experimental results described in the

Fig. 3 Time-histories of numerical solutions

Fig. 4 Comparison between experimental results and numerical solutions

previous section.

4. Dynamic Behavior of the Wiper System

Based on the equations of motion obtained above, we discuss the dynamic behavior of the wiper system not only for practically used wiping speeds but also for lower wiping speeds. We solve the equations of motion numerically for enough long time till a steady state solution is obtained. Then using the later part of the solution, we determine the values of θ_D , θ_P and $\dot{\theta}_P$ at 200 instants when $\dot{\theta}_D=0$ is valid. In this way we obtain the Poincaré map (θ_D , θ_P , $\dot{\theta}_P$). Here, we show the values of θ_D with respect to the wiper speed V to obtain the bifurcation diagram as shown in Fig. 5(a). For comparison, approximate solutions using the principle of harmonic balance (HB)⁽¹⁰⁾ are also shown by the lines (theoretical: HB) in the figure. We also obtain Lyapunov spectra as shown in Fig. 5(b). In the following, using Fig. 5, we discuss dynamic behaviors of a wiper system.

(a) Bifurcation diagram

(b) Lyapunov spectra

Fig. 5 Solutions in the low wiping speeds

4.1 Two kind of mode-locked vibrations for practically used wiping speeds

While wiping speed is enough high, equilibrium point of Eq.(2) is stable, as shown in the previous chapter. This means that no chatter vibrations occur. This situation lasts until the wiper speed decreases down to the point $V=0.8$ m/s indicated by C1 in Fig. 5(a). When the wiping speed decreases beyond C1, the stable equilibrium point becomes unstable, and a periodic solution representing the first-mode-locked vibration with slip motion appears. This means that the chatter vibration occurs. When the wiping speed decreases further across the point C2, a periodic solution representing the second-mode-locked vibration with slip motion appears in addition to the first-mode-locked vibration. The wiping speed decreases further across the point P1 and P2, the first-mode-locked and then the second-mode-locked periodic solution begins to accompany stick motion. Thus the chatter vibrations can be regarded as the mode-locked periodic solutions with stick motion.

4.2 Bifurcation from the first-mode-locked to chaotic vibration for low wiping speeds

We discuss here the case in which the wiping speed is further lowered. First the bifurcation from the first-mode-locked to chaotic vibration is examined. Fig. 6(a) illustrates Poincaré maps projected on (θ_P ,

Fig. 6 Bifurcation of first-mode-locked solution

$\dot{\theta}_p$) planes, for three values of V in the region R1 in Fig. 5(a). In this figure, $\circ$ and $\bullet$ show points for transient state and for post transient state, respectively. For the same values of V , trajectories on the phase plane ($\theta_d, \dot{\theta}_d$) and the Fourier spectra are shown in Fig. 6(b)–(c). In Fig. 6(c), Ω_1 and Ω_2 are predominant frequencies which are nearly equal to the natural frequencies of the linearized equation of Eq. (2). It can be seen that the mode-locked vibrations are accompanied by a motion with its frequency synchronized with one of the natural frequencies. It can also be seen from Fig. 5(a), Fig. 6(a), (b) that the first-mode-locked periodic solution becomes unstable, and the quasi-periodic vibration takes place, that the quasi-periodic vibration becomes unstable, and chaotic vibration takes place. These results correspond to those predicted by the Lyapunov spectra shown in Fig. 5(b). Thus the point C3 can be considered as the transition boundary at which periodic motion becomes unstable, and C4 as the boundary between quasi-periodic and chaotic motion. When V decreases across the point C3, complex-conjugate pair of characteristic multipliers traverses the unit circle of complex plane, so the bifurcation can be considered as a kind of Neimark's type.

4.3 Bifurcation from the second-mode-locked to chaotic vibration for low wiping speeds

Next, the transition from the second-mode-locked to chaotic vibration is studied. The second-

Fig. 7 Bifurcation of 2nd-mode-locked solution

mode-locked periodic solution bifurcates at the point C6. To see this behavior in detail, Poincaré maps for five wiping speeds are shown in Fig. 7(a). From this figure it can be seen that the stable second-mode-locked vibration changes its stability to quasi-periodic vibration, and then the quasi-periodic vibration is absorbed into the chaotic attractor which had bifurcated from the first-mode-locked vibration. As seen in the phase plane in Fig. 7(a) and the time history in Fig. 7(b), the transition from the quasi-periodic vibration to chaotic vibration accompanies the jump in the amplitudes.

4.4 Behavior of the wiper system after the second-mode-locked vibration is merged with chaotic vibration

After the second-mode-locked vibration is merged with chaotic vibration bifurcated from the first-mode-locked vibration, periodic vibrations appear in the region R2 in Fig. 5(a), the characters of the periodic vibrations being different from those observed before. These vibrations have periods three or six times as large as those of the first-mode-locked vibrations. Here, the three periodic vibrations are considered. Their time history, trajectory on the phase plane, and the Fourier spectrum are shown in Fig. 8. The Fourier spectrum shows that the vibration is synchronized by the relation $7\Omega_1 = 3\Omega_2$, where Ω_1, Ω_2 are the frequency of the first-mode-locked solution and that of the second-mode-locked solution, respectively. When appropriate initial conditions are given, transient phenomenon from chaotic vibration to the three periodic vibration is observed.

Chaotic vibrations observed in narrow regions outside of region R2 have intermittency. The time

(a) Time history and trajectories on the phase plane

(b) Fourier spectrum

Fig. 8 Periodic vibration in the region R2

(a) Time history and trajectory on the phase plane

(b) Fourier spectrum

Fig. 9 Chaotic vibration in narrow region outside of region R2

history and the Fourier spectrum are shown in Fig. 9.

5. Conclusions

First, we conducted experiments, and found that the chatter vibrations in an automotive wiper are either the first-mode-locked vibration or the second-mode-locked vibration. We also found that the mode-locked vibrations accompany the stick-slip motion.

Next, assuming that the wiper is modeled by a two-degree-of-freedom system, we derived the equations of motion. By comparing their solutions with experimental results, we confirmed the validity of the equations.

Finally, using the equations of motion, we discussed the dynamic behavior of the wiper for low wiping speeds. The behavior can be summarized as follows. As the wiping speed decreases, the first-mode-locked vibration is changed into a quasi-periodic vibration. As the wiping speed decreases further, the motion

becomes unstable, and the quasi-periodic vibration is changed into chaotic vibration. The second-mode-locked vibration is suddenly changed into a chaotic vibration, accompanying the jump in the amplitudes. In the range of very low wiping speeds, periodic window with period three and six times as large as that of the first-mode-locked periodic vibration appears. Transient chaotic motion is also observed in the periodic window.

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