Pattern Formation Generated in a Winder System of Textile Machine*

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Elastic yarns in the winder system of a textile machine are deformed into a certain convex polygon in the process of winding at an operating speed of the bobbin holder rotating viscoelastically in contact with the drive roll. The polygonal deformation increases and develops into a serious vibration of the machine. This paper presents a model and a theoretical analysis to investigate the mechanism of this vibration phenomenon which is regarded as instability due to the time lag. The bobbin holder and the drive roll are both modeled by the one-mass one-shaft system. From the numerical computations using parameters obtained experimentally, the analytical results showed a good agreement with the phenomenon generated in the actual machines.

Key Words: Vibration of Rotating Body, Stability, Modeling, Contact Vibration, Rotary Machine, Viscoelastic Body, Time Delay System, Polygonal Deformation

1. Introduction

The authors have investigated pattern formation phenomena generated in the contact rotating system in the field of mechanical engineering. In the previous paper^{(1),(2)}, they analyzed the polygonal deformation of the roll-covering rubber occurring in the smoother roll and the gate roll size of paper making machines, and performed experiment on the polygonal deformation of the roll-covering rubber. As the results, they made clear the self-excited formation mechanism of a specific pattern, and confirmed a good agreement between analytical and experimental results. Moreover, they made clear the relationship between dependence of damping of viscoelastic rubber on strain

velocity and dependence of deformation recovery characteristic on time elapsed after unloading which are one of the most important elements to elucidate the pattern formation phenomena⁽³⁾.

Such pattern formation phenomena generated in the contact rotating systems are found in many fields such as regenerative chatter in cutting and grinding of machine tools⁽⁴⁾, polygonal hole in drilling and reaming⁽⁵⁾, hot and cold rolling in iron works⁽⁶⁾, chatter marks and brush marks occurring in tension leveler and brush roll respectively, barring generated in calendar rolls of paper making machine, partial worn VR mill, polygonal wear of automobile tire⁽⁷⁾, corrugation of rail⁽⁸⁾ and so on⁽⁹⁾. In relation to physics, flow, biology and entrainment⁽¹⁰⁾, the mathematical modeling for pattern formation phenomena is actively investigated.

In this report, regarding the polygonization of elastic yarn rolls as unstable vibration of a linear time delay system accompanied with the viscoelastic deformation of elastic yarn rolls, the phenomena generated in actual machine are elucidated. As the results, a very good agreement between numerical computational result and experimental result by an

* Received 1st May, 1998. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 60, No. 575, C (1994), p. 2285-2292 (Received 13th January, 1993)

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actual machine was confirmed.

2. Summary of Pattern Formation of Elastic Yarn Rolls

The winder system of textile machines treated in this paper is composed of two rotating machines with overhung weight as shown in Fig. 1, and they are a drive roll as driver (called DR shortly in what follows) and a bobbin holder as follower (called BH in what follows). The DR drives the BH through the elastic yarn wound around the paper roll set to the outer circumference of the BH. We call cylindrical part made of steel of DR in contact with elastic yarn rigid body part of DR (The shadowed portion in Fig. 4), and do a part winding elastic yarn (The shadowed portion in Fig. 4) and elastic yarn rolls rigid body of BH. The diameter of the rigid body part of DR is about 82 mm (constant value), and that of BH varies from 86 mm which is outer diameter of paper roll, to 146 mm as the elastic yarn is wound up. The axial length of DR and BH is about 500 mm. The rotating speed of DR is about 45 Hz which is held to be constant, and that of BH is decreased from about 45 Hz to 25 Hz because BH rotates at constant peripheral velocity. There-

Fig. 1 A winder system

Fig. 2 Vibration waveforms of BH and analytical result by FFT

fore, the center of gravity of the rigid body part of BH, the characteristic values of inertia (mass and moment of inertia around center of gravity), the characteristic values of the support stiffness and the contact stiffness between DR and BH vary all in the process of winding. Since it takes about one hour and 30 min. to wind up the elastic rolls up to final winding thickness of 30 mm, these changes are all very slow.

The pattern formation phenomenon of the elastic yarn rolls generated in an actual machine is summarized as follows:

(1) The coupled vibration between DR and BH begins to grow when the roll thickness of the elastic yarn becomes about 6 mm (i.e. in 15 min. from the beginning of winding), and the unstable vibration continues intensively up to about 10 mm in thickness. And after that the unstable vibration does not occur and the rotation is smooth. The amplitudes of the unstable vibration at the tops of the overhung parts of BH and DR attain 2–3 mm and 1–2 mm in peak-to-peak respectively and the vibration accompanied with collisions is often developed.

(2) Figure 2 illustrates the typical vibration waveforms and the power spectrum during unstable vibration generated when the rotational speed of DR is 44 Hz (peripheral speed is 675 m/min), the thickness of elastic yarn rolls is 12 mm (the rotational speed of BH is 33 Hz) and the contact force between DR and BH is 147 N. From the figure, the coupled vibration with frequency of 98 Hz between DR and BH occurs intensively, and this frequency is about three times the rotational speed of BH, and it coincides with a natural frequency of the winder system. Then, the elastic yarn rolls are deformed into a triangle pattern. It was found out from the observation by stroboscope that the coupled vibration mode was out-of-phase between DR and BH, and the polygonal number of the deformed elastic yarn was three. If the winder is stopped quickly after the occurrence of an unstable vibration, the triangle pattern of elastic yarn rolls return back to a circle. Therefore, the deformation of the elastic yarn is viscoelastic.

(3) The higher the contact force between DR and BH is, the easier the unstable vibration takes place.

(4) If an O-ring made of rubber is attached to the outer ring of the bearing located at the outside of DR as a countermeasure, the unstable vibration tends to be hard to occur.

(5) The pattern formation phenomenon occurs only in a limited range of rotational speed of DR and the thickness of the elastic yarn.

3. Modeling and Theoretical Analysis

In this section, the generation condition of pattern

formation phenomenon of elastic yarn rolls mentioned in the previous section is theoretically taken into account.

The rigid body parts of DR and BH have both cylinder-like shape which is slender in the axial direction. Hence, the gyration effect is very small, since the moment of inertia around rotating shaft is small in comparison with the moment of inertia around a diameter through the center of gravity of rigid body part. In addition, the mass of rigid body part is clearly large, comparing with that of rotating shaft. Moreover, taking into account the fact that unstable vibration occurs in the vertical direction to the contact plane, DR and BH are both modeled as one rigid body and one shaft system to simplify the analysis. This unstable bending vibration generated in the vertical plane is formulated, neglecting the gyroscopic effect and the precession of rotary machines. The contact area between DR and BH (called nip part in what follows) is assumed to be in straight line and the slip at the nip part is neglected.

We set the lateral and the angular displacements at the center of gravity G of the rigid body part to x and θ respectively, as shown in Fig. 3. The physical quantities with subscripts "d" and "b" denote the ones with respect to DR and BH respectively. The nip part is modeled by the simplest three parameter model representing the viscoelastic characteristic which is composed of an instantaneous elastic deformation part having distributed stiffness coefficient k_e , and a retarded elastic deformation part having distributed stiffness coefficient k_v , and distributed viscous damping coefficient c_v . Such elastic yarn rolls are mounted on the cylindrical rigid body part of BH at equal spaces in the axial direction. The number of the

elastic yarn rolls is n , and the width of every elastic yarn roll is commonly l . x_{di} and x_{bi} are lateral displacements of DR and BH at the center of the i -th yarn roll respectively, and x_{ri} is the lateral displacement at the middle point in the three-parameter model illustrating the elastic yarn. It is assumed that the viscoelastic characteristics of every elastic yarn roll are same to each other, and the mass effects at the nip part are neglected. The lateral and the angular displacements at the middle points of the elastic yarns are denoted by x_r and θ_r defined at the point G_d respectively because those middle points are distributed linearly. z_{di} and z_{bi} are the distances in the axial direction from the points G_d and G_b to the center of the i -th elastic yarn respectively at which the origin of local coordinate ξ_i is defined, where $i=1, \dots, n_r$.

Then, we obtain the following relation (cf. Fig. 3):

$$\left. \begin{aligned} x_{di} &= x_d + z_{di}\theta_d, & x_{bi} &= x_b + z_{bi}\theta_b \\ x_{ri} &= x_r + z_{ri}\theta_r, & z_{di} &= z_{bi} + e \end{aligned} \right\} \quad (1)$$

where e is the axial distance between points G_d and G_b .

Though the great part of the elastic yarn deformation immediately after passing through the nip part recovers in one revolution period T of BH, the residual deformation is fed back when reentering into the nip part next time, and a specific polygonization is self-excitedly formed on the elastic yarn rolls. Namely, the generation mechanism of this polygonal deformation is concerned with retardation. The amount of retarded elastic deformation of the elastic yarn immediately after passing through the nip part remains by a factor $\exp(-\alpha T)$ times that of its deformation after one revolution period T , and this residual deformation is fed back to the nip part again, where α is the deformation recovery coefficient of the elastic yarn, and the elastic yarn is modeled by the three parameter model. Since the rotational speed of DR is in the high range of 40 to 50 Hz, the deformation recovery coefficient α depends on the revolution period of BH^{(1),(3)}. Hence, we use here a standard value corresponding to the average rotational speed of BH for simplicity and dependence of time elapsed is neglected.

In winder system, the vibration characteristic values of DR and its rotational speed are both constant. On the other hand, the center of gravity of rigid body part of BH, characteristic values of inertia, support stiffness at the center of gravity and rotating period T of BH vary with time as the elastic yarn is wound up. However, since the system characteristics vary very slowly, we obtain the equations of motion by using the characteristic values at a certain time t here.

Fig. 3 Modeling of winder system

Under the assumption mentioned above, the equations of motion with respect to the center of gravity of DR and BH are represented as

$$\left. \begin{aligned} M_d \ddot{x}_d + c_{d1} \dot{x}_d + c_{d2} \dot{\theta}_d + k_{d1} x_d + k_{d2} \theta_d \\ + \sum_{i=1}^{n_r} \{F_{di}(t) - \exp(-\alpha T) F_{di}(t-T)\} = 0 \\ I_d \ddot{\theta}_d + c_{d2} \dot{x}_d + c_{d3} \dot{\theta}_d + k_{d2} x_d + k_{d3} \theta_d \\ + \sum_{i=1}^{n_r} \{N_{di}(t) - \exp(-\alpha T) N_{di}(t-T)\} = 0 \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} M_b \ddot{x}_b + c_{b1} \dot{x}_b + c_{b2} \dot{\theta}_b + k_{b1} x_b + k_{b2} \theta_b \\ - \sum_{i=1}^{n_r} F_{bi}(t) = 0 \\ I_b \ddot{\theta}_b + c_{b2} \dot{x}_b + c_{b3} \dot{\theta}_b + k_{b2} x_b + k_{b3} \theta_b \\ - \sum_{i=1}^{n_r} N_{bi}(t) = 0 \end{aligned} \right\} \quad (3)$$

where t is time, M and I are mass and moment of inertia around the diameter at time t respectively, and $k_{1,2,3}$ and $c_{1,2,3}$ are support stiffness and damping coefficients at the center of gravity respectively. $F_i(t)$ is a shearing force exerting on the i -th elastic yarn, and $N_i(t)$ is a moment around the center of gravity caused by the distributed shearing force f_i of the nip part. These are obtained from the following expressions for $i=1, \dots, n_r$.

$$\left. \begin{aligned} F_{di}(t) &= \int_{-l/2}^{l/2} f_{di} d\xi_i, F_{bi}(t) = \int_{-l/2}^{l/2} f_{bi} d\xi_i \\ N_{di}(t) &= \int_{-l/2}^{l/2} f_{di} (z_{di} + \xi_i) d\xi_i \\ N_{bi}(t) &= \int_{-l/2}^{l/2} f_{bi} (z_{bi} + \xi_i) d\xi_i \\ f_{di} &= c_v \dot{u}_i(t) + k_v u_i(t), f_{bi} = k_e v_i(t) \\ u_i(t) &= x_{di}(t) - x_{ri}(t) + \{\theta_d(t) - \theta_r(t)\} \xi_i \\ v_i(t) &= x_{ri}(t) - x_{bi}(t) + \{\theta_r(t) - \theta_b(t)\} \xi_i \end{aligned} \right\} \quad (4)$$

The equilibrium conditions of shearing forces $[F_{bi}, F_{di}]$ and moments $[N_{bi}, N_{di}]$ acting on the contact part of the i -th elastic yarn roll are represented as

$$\left. \begin{aligned} F_{bi}(t) - \{F_{di}(t) - \exp(-\alpha T) F_{di}(t-T)\} &= 0 \\ N_{rbi}(t) - \{N_{rdi}(t) - \exp(-\alpha T) N_{rdi}(t-T)\} &= 0 \\ N_{rdi}(t) &= \int_{-l/2}^{l/2} f_{di} \xi_i d\xi_i, N_{rbi}(t) = \int_{-l/2}^{l/2} f_{bi} \xi_i d\xi_i \end{aligned} \right\} \quad (5)$$

For the stability analysis of the system treated, we perform the following variable transform to Eqs. (1) to (5) so that the revolution period of BH becomes 2π .

$$\tau = \omega_b t, \omega_b = 2\pi/T \quad (6)$$

where ω_b is the rotational angular velocity of BH.

Next, performing Laplace transform to Eqs. (1) to (5), setting all the initial values to zero, and some rearrangement of the resulting equations yields the following equation:

$$\mathbf{A}\mathbf{X} = \mathbf{0} \quad (7)$$

where

$$\left. \begin{aligned} \mathbf{A} &= {}^t\mathbf{A} = [a_{pq}] : (p, q = 1, \dots, 4) \\ \mathbf{X} &= (X_d, \Theta_d, X_b, \Theta_b) \\ a_{11} &= (M_d \omega_b^2 s^2 + c_{d1} \omega_b s + k_{d1}) y(s) - a_{31} \\ a_{21} &= (c_{d2} \omega_b s + k_{d2}) y(s) - a_{32} \\ a_{31} &= -n_r w(s), a_{41} = -\sum_i z_{bi} w(s) \\ a_{22} &= (I_d \omega_b^2 s^2 + c_{d3} \omega_b s + k_{d3}) y(s) \\ &\quad + (\sum_i z_{di}^2 + n_r l^2 / 12) w(s) \\ a_{32} &= -\sum_i z_{di} w(s) \\ a_{42} &= -(\sum_i z_{bi} z_{di} + n_r l^2 / 12) w(s) \\ a_{33} &= (M_b \omega_b^2 s^2 + c_{b1} \omega_b s + k_{b1}) y(s) - a_{31} \\ a_{43} &= (c_{b2} \omega_b s + k_{b2}) y(s) - a_{41} \\ a_{44} &= (I_b \omega_b^2 s^2 + c_{b3} \omega_b s + k_{b3}) y(s) \\ &\quad + (\sum_i z_{bi}^2 + n_r l^2 / 12) w(s) \\ w(s) &= K_e f(s), y(s) = K_e + f(s) \\ f(s) &= (K_v + C_v \omega_b s) \{1 - \exp(-2\pi s - \alpha T)\} \\ K_e &= k_e l, K_v = k_v l, C_v = c_v l \end{aligned} \right\} \quad (8)$$

Symbol " t " denotes the transpose, X_d, Θ_d, X_b and Θ_b are Laplace transform of functions x_d, θ_d, x_b and θ_b respectively, s is a variable of Laplace transform and $\sum_i$ denotes the total summation from $i=1$ to n_r . The time delay element included in the term of $f(s)$ described above is a cause of unstable vibration, that is, of pattern formation phenomenon of elastic yarn.

From Eq. (7), the characteristic equation is given by

$$\det \mathbf{A} = 0 \quad (9)$$

Since there exists a time lag, there are an infinite number of characteristic roots s of Eq. (9). The system is stable if all the real parts of these roots are negative. On the other hand, it is unstable if more than one real part are positive.

Applying the argument principle⁽¹¹⁾, Eq. (9) has, in general, a finite number of unstable characteristic roots (an even number when the conjugate roots are counted) with positive real parts for a given set of parameters when the system is unstable^{(1),(2)}. We apply the Newton-Raphson iteration method to the simultaneous algebraic equations which are obtained by setting the real and the imaginary parts of equations resulting from the substitution of $s = \sigma + \sqrt{-1}N$ (σ and N are real, $N > 0$) into Eq. (9) to zero, and we calculate σ and N directly. Then, if there exist a solution of $\sigma > 0$, this root corresponds to an unstable characteristic root at an angular velocity ω_b of BH. As the system vibrates at a circular frequency $\omega_b N$, that is, N times rotational angular velocity, when converted into the real time t using Eq. (6), N gives important information related to the polygonal number of the elastic yarn rolls.

Furthermore, in order to indicate the locus of the characteristic root the real part of which is σ , the Newton-Raphson method is also applied to the simultaneous equations described above, regarding them as

Fig. 4 Outline of shape and dimension of shaft system (unit: mm)

the simultaneous equations with respect to ω and N . In order to obtain the boundary between stable and unstable regions, we need only to compute the simultaneous equations by setting σ to zero.

4. Numerical Computational Results and Discussions

Figure 4 shows the outline dimensions of the shaft systems of DR and BH for the actual machine. Both rotating machines are supported by ball bearings, and the shadowed parts correspond to the rigid body parts except the elastic yarn rolls of BH. The number of the elastic yarn rolls n_r is 8, and the width l is 44 mm.

Table 1 shows the inertia characteristic values, the viscoelastic characteristic values of elastic yarn, deformation recovery coefficient α_1 , the distance e between the center of gravity of DR and BH, and support stiffness and damping coefficient at the centers of gravity when the elastic yarn's thickness h is 10 mm. Though the deformation recovery coefficient α_1 of the elastic yarn depends upon the revolution period of BH, we use here a standard value indicated in Table 1 corresponding to the average rotational speed of BH by referring to the experimental results of rubber in the previous reports^{(1),(3)}. The support stiffness coefficients at the center of gravity of the rigid body parts are calculated by means of the transfer influence coefficient⁽¹²⁾, where all the linear spring constants of bearings are set to 49 MN/m. The i -th position mounting the elastic yarn rolls on DR from the dimensions of an actual machine is expressed as

$$z_{di} = 57.5(i - 0.5) - 187, \quad (i = 1, \dots, 8) \quad (10)$$

where unit is mm.

(1) First, the feature of the generation of the unstable vibration is investigated for the case that roll thickness is fixed at $h = 10$ mm, and the rotational speed of BH is changed.

Table 2 shows the computational values of the first to the fourth undamped natural frequencies denoted by f_{N1} to f_{N4} of the winder system respective-

Table 1 Standard parameter values at $h = 10$ mm

Parameter	Value	Parameter	Value
M_d	8.81 kg	k_{d1}	33.9 MN/m
M_b	9.61 kg	k_{d2}	-7.14 MN
I_d	0.188 kgm ²	k_{d3}	1.66 MNm
I_b	0.237 kgm ²	k_{b1}	102 N/m
K_e	100 kN/m	k_{b2}	-29.6 MN
K_v	702 kN/m	k_{b3}	8.90 Nm
C_v	0.10 kNs/m	c_{d1}, c_{b1}	9.2 Ns/m
α_1	6.0 1/s	c_{d2}, c_{b2}	0.0 Ns/rad
e	36.8 mm	c_{d3}, c_{b3}	10 ms/rad

Table 2 Theoretical natural frequencies of a winder system (Hz)

f_{N1}	f_{N2}	f_{N3}	f_{N4}
78.68	103.0	562.1	1101

Fig. 5 Relationship between f_b and N at $h = 10$ mm

ly, where the parameters at $h = 10$ mm were used. Since the frequency of the unstable vibration generated in the winder system is about 100 Hz, the unstable vibration with very high frequencies like the third and the fourth natural frequencies is not dealt with in this report.

Figure 5 shows the relationship between imaginary part N of the characteristic roots of Eq. (9) and the rotational speed of BH $f_b = \omega_b/2\pi$ at $h = 10$ mm. Dotted and solid thick lines show the stable ($\sigma < 0$) and the unstable ($\sigma > 0$) solutions respectively. Curves Ω_1 and Ω_2 represent the ratios of the first and the second natural frequencies to the rotational speed of BH respectively which are illustrated by solid fine line in the figure. However, curve Ω_1 is omitted here because it lies on top of the dotted line. It is clear from the figure that the unstable characteristic roots exist in the vicinity of the second natural frequency, in addition, only in the region in which N is near an integer, and the unstable regions become narrow as N is increased. Furthermore, the unstable vibration may occur at the frequency lower than the second natural frequency as there exist the unstable characteristic roots on the left side of curve Ω_2 . The polygonization phenomenon of the elastic yarn rolls occurs in the second mode in which the vibration phase of DR and

BH are opposite, and that of the first mode in which the vibrations of DR and BH are in phase can never occur, because DR and BH of the winder system are designed almost symmetrically.

Figures 6 and 7 show the loci of the solution for the standard parameters indicated in Table 1, where the variables are only the damping coefficients c_{d1} , c_{b1} and c_{d3} , c_{b3} respectively and for the cases where $\sigma=0$, 0.01 and 0.02 rad^{-1} . The abscissa represents the rotational speed f_b of BH, and the ordinate represents c_{d1} ($=c_{b1}$) and c_{d3} ($=c_{b3}$). The curves for $\sigma=0$ indicate the boundaries between stable and unstable regions where the regions under the curves corresponds to the latter. The number n in the figure expresses the nearest integer to N ($n \geq N$), and we call this the polygonal number of the elastic yarn rolls. In both figures, only the unstable regions for $n=2$ to 6 are illustrated, but the larger the viscous damping coefficients are, and the larger the polygonal number are, the narrower the unstable regions are.

For example, if the rotational speed f_b of BH is 33 Hz (i.e. the rotational speed of DR is about 42 Hz), then it is possible that the unstable vibration of $n=3$ occurs for the small damping coefficients. This corresponds to the fact that in the actual machine, the elastic yarn rolls were deformed into triangle at about $f_d=44$ Hz, and the heavy vibration occurred at the frequency three times rotational speed of BH.

Fig. 6 Effect of damping upon unstable regions ($h=10$ mm)

Fig. 7 Effect of damping upon unstable regions ($h=10$ mm)

(2) It was already mentioned that the inertia characteristic values of BH and the support stiffness coefficients vary very slowly in the winding process of the elastic yarn rolls in the winder system. Neglecting the effect of the variation of these characteristics with time here, and regarding the winding process as a quasi-static process, the unstable regions in the winding process are calculated.

First, we regard the inertial characteristic values, the center of gravity of BH and the viscoelastic coefficients of elastic yarn as the function of the thickness h of elastic yarn rolls, and the former two values are calculated from the geometrical view point.

Assuming no slipping at the contact point, the relationship between angular velocities ω_d of DR and ω_b of BH is given by

$$\omega_b = (D_d/D_b)\omega_d, D_b = D_p + 2h \quad (11)$$

where D_d , D_p and D_b are the outer diameters of DR, the paper roll in BH and the elastic yarn rolls respectively. The paper roll of BH on which the elastic yarn is wound up, is fixed on the rigid body part.

When modeling the viscoelastic characteristic of the elastic yarn roll at a high speed revolution as in winder system, in general, the retarded stiffness coefficient K_v is estimated much higher than the instantaneous stiffness coefficient K_e . In this report, we set $K_v(h) = 7K_e(h)$ according to the previous report⁽¹⁾ to analyze the stability. Furthermore, the relationship between the instantaneous stiffness coefficients K_e (kN/m) of the elastic yarn and the thickness h of the elastic yarn roll was obtained from the static experiment. The result is shown in Appendix. The relation is approximated by the following function of power under the condition that $K_e(h)$ is infinite at $h=0$, and it is naught at $h=\infty$:

$$K_e(h) = 334h^{-0.522} \quad (12)$$

There are only paper roll and no elastic yarn rolls at $h=0$, so that the viscoelastic deformation does not occur. Hence, the viscoelastic characteristic of the elastic yarn roll may not be remarkable when h is extremely small. In order to incorporate this effect into the stability analysis, the dependence of deformation recovery coefficient α on the thickness h is simply expressed as

$$\alpha(h) = \begin{cases} \alpha_1 & (h \geq 10 \text{ mm}) \\ \alpha_1 \left[1 + \frac{5(h-10)^2}{h^2} \right] & (h < 10 \text{ mm}) \end{cases} \quad (13)$$

where α_1 is a standard value of α for the case of $h=10$ mm, and the function $\alpha(h)$ satisfies the continuity at $h=10$ mm and $\alpha(0)=\infty$.

From the geometrical view point, the variation of the inertia characteristic values of the rigid body part of BH and the position of the center of gravity for the case of the thickness h is calculated by

$$\left. \begin{aligned}
M_b(h) &= M_{b0} + n_r m_r(h) \\
I_b(h) &= I_{b0} + [e(h) - e_0]^2 M_{b0} \\
&\quad + m_r(h) \sum_i z_{bi}^2 \\
&\quad + \frac{\rho l n_r}{48} [D_b^2(3D_b^2/4 + l^2) \\
&\quad - D_p^2(3D_p^2/4 + l^2)] \\
m_r(h) &= \rho \pi l [(D_p + 2h)^2 - D_p^2]/4 \\
e(h) &= \frac{e_0 M_{b0} + e_r n_r m_r(h)}{M_b(h)}
\end{aligned} \right\} \quad (14)$$

where ρ is density of elastic yarn, m_r is mass of a elastic yarn roll, e_r is a distance between the center of gravity of rigid body part of DR and the center of gravity of only elastic yarn rolls on BH, and it is defined as positive when the latter is relatively located at the top of the overhung part. The physical quantities with subscript 0 are the ones for the case of $h=0$.

Using the expressions described above and the values of parameters as shown in Table 3, we calculate the values of M_b , I_b , e , K_e , K_v , α and ω_b for the case of the thickness h , use the damping coefficients as shown in Table 1, and compute the generation regions of unstable vibration of the winder system as the thickness h is changed.

Figure 8 shows the unstable regions occurring

Table 3 Parameter values of a winder system

Parameter	Value
M_{b0}	8.68 kg
I_{b0}	0.220 kgm ²
D_p	85.7 mm
D_d	82.0 mm
l	44 mm
e_0	36.1 mm
e_r	43.2 mm
ρ	0.88×10^3 kg/m ³

Fig. 8 Unstable regions in the plane of thickness h and rotational speed of BH

through over the winding process of the elastic yarn rolls. The abscissa represents the wound thickness h ($0 \leq h \leq 30$ mm), and the ordinate represents the rotational speed f_b of BH. The number $n=2-6$ indicates the polygonal number of the elastic yarn rolls with polygonization pattern. The inside of the wing-like shapes corresponds to the unstable region in which the polygonization phenomena with the corresponding polygonal number n may occur. In Fig. 8, three kinds of the rotational speeds of DR [50 Hz (dashed line), 45 Hz (solid line) and 40 Hz (dot-dashed line)] are simultaneously illustrated in the figure. With increasing the thickness h , the rotational speed of BH is gradually decreased along these lines.

In the case of $f_d=45$ Hz, the winder system comes into an unstable region a short while from the start of winding, and the polygonal deformation of $n=3$ is gradually increased. The pattern formation process grows very slowly, because the values of positive real part of the unstable characteristic roots are remarkably small as shown in Figs. 6 and 7 comparing with, for example, the unstable vibration of the rotating body due to journal bearings. Then, the unstable vibration accompanied with the elastic yarn rolls deformed into triangle begins to occur gradually, and grows to a heavy vibration in the self-excited pattern formation process. As the rotational speed is decreased with increasing the diameter of the elastic yarn rolls, the winder system escapes from the unstable region and the unstable vibration disappears because of variation of the inertia characteristic values, support stiffness and so on. After that, the unstable vibration does not occur until the winding is finished ($h=30$ mm).

In the case of $f_d=50$ Hz, the winder system comes into the unstable region of $n=3$ at the end of winding. On the other hand, in the case of $f_d=40$ Hz, the unstable vibration never occurs through over the winding.

From Fig. 8, we can see the unstable vibrations of $n=2$ (elliptic) and of $n=4, 5$ may occur at the higher and the lower rotational speeds of DR respectively. Especially, the unstable region of $n=2$ obstructs the increase of the operation speed. The smaller the polygonal number is, the wider the unstable regions are, and the unstable regions shift to the side on which the rotational speed of BH is relatively higher. In Fig. 8, the relationship between the thickness of the elastic yarn rolls h and (the second natural frequency/polygonal number) $= f_{N2}/n$ is indicated with dot-dashed line. There exist the unstable regions in only the limited rotational speed of BH where f_{N2} is in the vicinity of n times the rotational speed of BH. Therefore, it is necessary not to bring this dot-dashed line

Fig. 9 Influence of α upon unstable regions

Fig. 10 Time historical response of pattern formation of elastic yarn roll

close to the curve of rotational speed of BH essentially, in order to avoid the unstable regions.

Figure 9 shows the influence of the deformation recovery coefficient upon the unstable vibration. For example, the value of the deformation recovery coefficient α of the elastic yarn is expected to change by changing the kinds of elastic yarn, or by alteration of the circumferential temperature. Figure 9 shows the same result as shown in Fig. 8 for $\alpha_1=4, 6$ and 8 ($1/s$). As the deformation recovery coefficient α_1 becomes small, the unstable region corresponding to each polygonal number becomes wide. Therefore, if the deformation recovery coefficient becomes smaller than the standard value 6 , not only the unstable vibration deformed into triangle but also the one with polygonal number of 4 are possible to occur within the ordinary rotational speed of BH.

Notations $\times$, $\triangle$ and $\circ$ in Fig. 9 represent the experimental results of heavy, light and almost no vibrations respectively in the winding process for the several rotational speeds of DR. The heavy vibrations means the occurrence of the pattern formed into

triangle of the elastic yarn rolls. The experimental results show a good agreement with the analytical ones for the deformation recovery coefficient $\alpha_1=6$.

(3) The time historical response of pattern formed on the elastic yarn roll located in the outside by numerical simulation⁽¹⁾ is shown in Fig. 10 for the case of $f_d=45$ Hz and $h=10$ mm in Fig. 8 operation condition of which belongs to the unstable region of deformed into triangle. Figure 10 shows the growth process of pattern formation when the deformation has proceeded after a long revolutions, and the deformation of the elastic yarn roll during 10 revolutions of BH from dotted to solid lines is illustrated exaggeratedly. It is clear from Fig. 10 that this pattern is deformed into triangle, and the pattern formation on the elastic yarn roll is not fixed on the elastic yarn roll but grows in size and moves in the antirotational direction of BH as the number of revolution is increased.

5. Conclusions

The authors made modeling of the pattern formation phenomena (polygonization) of the elastic yarn rolls generated in the winder system of textile machine, analyzed it theoretically and compared the numerical computational results with the experimental ones from an actual machine. The results obtained are summarized as follows:

(1) The viscoelastic characteristics of the elastic yarn roll were modeled as the three parameter model, and the generation mechanism of pattern formation of the elastic yarn rolls was made clear, regarding the phenomena as the stability problem of the linear retarded system. And it was confirmed that the analytical results agreed well with those from actual machines.

(2) For the elastic yarn rolls the diameter of which was fixed temporarily, the relation between the rotational speed of BH and the pattern formation regions generating the polygonization was dealt with. The occurrence regions of pattern formation are comparatively easily estimated from an approximate relationship of (rotational speed of BH) $\times$ (polygonal number) = (natural frequency of the winder system).

(3) The unstable vibration occurring in the winding process of the elastic yarn rolls was analyzed taking the changes of the inertia characteristic values of the rigid body part due to the shift of the center of gravity of BH, support stiffness coefficients, contact stiffness of elastic yarn roll between DR and BH and the rotational speed of BH into account quasi-statically. As the results, it was made clear that the pattern formation is generated in the limited regions specified from the rotational speed of BH and the

Fig. 11 Relationship between thickness h and spring constant K_e

thickness of the elastic yarn rolls, the smaller the polygonal number is, the wider the unstable regions are, and the unstable regions shift to the side on which the rotational speed of BH is relatively higher and the deformation recovery coefficient of the elastic yarn takes part in the width of the unstable regions strongly.

(4) The patterns formed on the elastic yarn roll are not fixed on the elastic yarn roll but grow in size and move in the antirotational direction of BH as the number of revolution is increased.

Appendix Experimental Results for Stiffness Coefficient of Elastic Yarn Roll

The stiffness coefficient K_e of an elastic yarn roll with the thickness h subjected to a press loading 20 N was measured statically as h is changed, and obtained from the displacement of the elastic yarn roll. The result is shown in Fig. 11. Symbol $\bigcirc$ is experimental value and solid line is a curve of an experimental equation which is approximated with a function of power with respect to h . From Fig. 11, the stiffness coefficient K_e is almost in inverse proportion to the square root of the thickness of the elastic yarn roll, and is decreased with increasing h nonlinearly. Therefore, the stiffness coefficient of the nip part between DR and BH is decreased with increasing the diameter of the elastic yarn roll of BH, which is one of the causes varying the natural frequencies of the winder system.

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