Study on Vibratory Conveyance of Granular Materials*

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The flow of granular materials on the vibratory conveyor is studied both experimentally and analytically. The vibratory conveyor consists essentially of a near-horizontal conveying vessel, which is undergoing translational harmonic vibrations in a direction to the horizontal. The motion of individual particles contacting each other is solved by the discrete element method proposed by Cundall. The normal component of the contact force acting on a particle is firstly given by the sum of the forces due to the spring and the dashpot. Then, the tangential component of the contact force applies the Coulomb law of friction. It is shown that the mean velocity of transport of granular materials depends on the frequency and amplitude of the vibratory input as well as various physical parameters. Also, the numerically observed flow properties agree quantitatively with experimental results.

Key Words: Frictional Vibration, Numerical Analysis, Material Handling Equipment, Vibratory Conveyor, Granular Materials, Discrete Element Method, Collision

1. Introduction

When the material is on the vibrating surface, it may be conveyed by a combination of sliding and free flight. It is called a vibratory conveyance and has been widely used from feeding mechanical components to the transportation of raw materials. It has been quite extensively studied for many years. For example, Taniguchi, et al.⁽¹⁾, classified the many possible solutions. Okabe, et al.⁽²⁾, discussed the relations among the frictional and operating conditions and the motion forms. Since almost these investigations have been made on the efficient transport of such fragile objects as delicate mechanical components and electric parts, every previous work has been focused on a particle's motion without the interparticle collisions. For the conveying of granular materials such as raw materials, the flow pattern is surprisingly complex. Since the flow pattern depends on the interparticle

collisions, the analysis has been difficult compared to continuum model such as liquid. Recently, the discrete element method (It is termed 'DEM' as follows.) has been used to investigate the dynamics of the granular materials on soil mechanics or powder mechanics. In this method, individual particles are assumed as rigid body. Then, the interparticle collision is modeled by mechanical elements as springs and dashpots. Therefore, the dynamics of the entire system is determined by the analysis of the motion of the individual particles. In the present work, DEM is adapted for a dynamic analysis of vibratory conveyance of a shallow bed of granular materials. We discussed the dependence of any operating parameters such as the frequency and amplitude of the vibrating vessel on the transport condition of the granular materials. Also, the validity of this calculation method was investigated and the calculated results were compared with the experimental ones.

2. Vibratory Conveyance

Figure 1 shows a model of a vibratory conveyor. A vibrating vessel inclines to the horizontal at angle α as shown in this figure. When the vibrating vessel is undergoing translational harmonic vibrations (amplitude a and angular frequency ω) in the direction of angle β to the horizontal, granular materials on the

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Fig. 1 Model of vibratory conveyor

vessel may be conveyed by a combination of sliding and free flight. On the assumption that the motion of the vessel is independent of the flow pattern of the granular materials, the displacement of the vibrating vessel at time t is given by Eq. (1), using x - y coordinate system as shown in Fig. 1. In this figure, x_i and y_i denote the coordinates of any particle i .

$$\begin{cases} X = a \cdot \cos \beta \cdot \sin \omega t \\ Y = a \cdot \sin \beta \cdot \sin \omega t \end{cases} \quad (1)$$

3. Dynamic Analysis of Granular Materials

3.1 The equation of motion for particles

When the granular materials are conveyed, a particle is in contact with other particles or the vibrating vessel. Therefore, in order to investigate the behavior of the entire system, the equations of motion for all particles should be established and solved, considering the interactive effects due to friction and collision between two particles or between a particle and the vibrating vessel. Usually, granular materials consist of various shaped particles. However, in the present work, granular materials are first assumed as a shallow bed of cylinders of uniform size and density for simplicity of analysis. Then, the two dimensional treatment can be made of the motions of granular materials being conveyed. If the adhesive force between two particles can be neglected, the equation of motion for any particle i is given by

$$\begin{cases} m\ddot{x}_i = F_{xi} \\ m\ddot{y}_i = F_{yi} - mg \\ I\ddot{\theta}_i = T_i \end{cases} \quad (2)$$

where m is the mass of a particle, I is the moment of inertia of the particle and g is the acceleration of gravity. x_i and y_i are the coordinates of the particle, θ_i is the rotation angle of the particle. F_{xi} and F_{yi} are the x and y directional components of the total forces acting on particle i . T_i is the summation of the torque about an axis normal to the paper. The dot denotes a time derivative.

Let us consider how to derive the contact force acting on a particle and how to obtain F_{xi} , F_{yi} and T_i from it in the following sections.

Fig. 2 Contact force

3.2 Normal and tangential components of contact force

Figure 2 shows the contact or collision between two particles i and j (It's termed 'collision' as follows). In this figure, $P_i(x_i, y_i)$ and $P_j(x_j, y_j)$ mean the centers of two particles i and j . Consider the normal component f_N and tangential component f_T of the contact force. The normal component of the contact force is modeled by the sum of the spring force P_N and viscous damping force and expressed as in the following Eq. (3). The tangential component of the contact force is given in Eq. (4) considering Coulomb law of friction.

$$f_N = P_N + c_N \dot{\delta}_N \quad (3)$$

$$f_T = -\mu f_N \dot{\delta}_T / |\dot{\delta}_T| \quad (4)$$

$$\delta_N = (u_i - u_j) \cos \phi_{ij} + (v_i - v_j) \sin \phi_{ij} \quad (5)$$

$$\begin{aligned} \dot{\delta}_T = & -(u_i - u_j) \sin \phi_{ij} + (v_i - v_j) \cos \phi_{ij} \\ & + r(\dot{\theta}_i + \dot{\theta}_j) \end{aligned} \quad (6)$$

$$\delta_N = 2r - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (7)$$

$$\sin \phi_{ij} = (y_j - y_i) / \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad (8)$$

However, we let $f_N = f_T = 0$, if $\delta_N < 0$. In these expressions, subscript N and T refer to the normal and tangential direction, respectively. u_i and u_j are the velocities in the x direction of particles i and j , respectively. v_i and v_j are velocities in the y direction of particles i and j , respectively. δ is the displacement, c_N is the damping coefficient, μ is the coefficient of the friction between two particles or the friction between a particle and the vibrating vessel.

3.3 Spring constant and damping coefficient

According to the Hertzian theory of elastic contact for the case of cylinders, it is pointed out that the relation between the repulsive force P_N in Eq. (3) and the displacement δ_N is given by

$$P_N = k_N \delta_N \quad (9)$$

where k_N is the normal spring constant⁽⁴⁾. In our case of the collision between two cylinders, we let P_N and δ_N be P and δ stated later in Eq. (11), respectively. In our case of the collision between a cylinder and the vibrating vessel, we similarly let P_N and δ_N be P_i and δ_i stated later in Eq. (13), respectively. In the case of the collision between two cylinders of uniform size

Fig. 3 The relation between the repulsive force and the normal displacement

and density, if we regard dynamic behavior as the same as static one, the width b of contact area and the displacement δ due to the approach of central axis of a cylinder to the other are given by

$$b = \sqrt{2DP(1-\nu^2)/(\pi LE)} \quad (10)$$

$$\delta = \frac{2P}{\pi L} \frac{1-\nu^2}{E} \left(1.8864 + \ln \frac{L}{2b} \right), \quad (11)$$

respectively, where E is the modulus of elasticity of the cylinder and ν is the Poisson's ratio. D and L are the diameter and the length of the cylinder, respectively. P is the compressive applied force.

In the case of the collision between a cylinder and the vibrating vessel, we can use the values obtained when a cylinder is statically pressed on an elastic plate under the compressive applied force P_1 . Therefore, the width b_1 of contact area and the displacement δ_1 are given by

$$b_1 = \sqrt{\frac{2DP_1}{\pi L} \left(\frac{1-\nu^2}{E} + \frac{1-\nu_1^2}{E_1} \right)} \quad (12)$$

$$\delta_1 = \frac{P_1}{\pi L} \left(\frac{1-\nu^2}{E} + \frac{1-\nu_1^2}{E_1} \right) \left(1.8864 + \ln \frac{L}{2b_1} \right) \quad (13)$$

where E_1 and ν_1 are the modulus of elasticity and the Poisson's ratio of the vibrating vessel, respectively. P_1 is the compressive applied force. Supposing that the static relations of Eq. (10)-(13) hold for the interparticle collision and the collision with the vibrating vessel and setting δ_N equal to δ or δ_1 , the relation between the normal displacement δ_N and P_N can be obtained as shown in Fig. 3. The conditions used in the calculation are the same as the experimental conditions as shown later. The particles are made of acrylic resin (The modulus of elasticity E is 300 MPa, the Poisson's ratio ν is 0.32, diameter is 10×10^{-3} m and the length of the cylinder is 8×10^{-2} m). The vibrating vessel is made of steel (The modulus of elasticity E is 206 GPa, the Poisson's ratio ν is 0.3). In this figure, solid lines are the approximated lines from the relation between δ_N and P_N . P_N is exactly proportional to δ_N as shown in Fig. 3, then the relation can be approximated by Eq. (9). The spring constant k_N is

Fig. 4 Relative motion of a particle

determined from the slope.

Next, consider the damping force that is referred to as collision energy dissipation. In the case of the interparticle collision and collision with the vibrating vessel, supposing that the duration of the collision equals to the half period of the damped harmonic oscillation and using the coefficient e of restitution, the damping coefficient c_N is expressed by

$$c_N = \frac{2\sqrt{m \cdot k_N}}{\sqrt{1 + (-\pi/\ln e)^2}} \quad (14)$$

Each particle conveyed may be in contact with other particles. However, it is supposed that k_N and c_N can be defined by Eqs. (9) and (14), respectively.

3.4 Contact force during the sliding motion

Granular materials may be conveyed on the vibratory vessel, doing and repeating not only the interparticle collisions but also relative sliding motions. Let us consider the contact force acting on the particle during the sliding motion on the vibrating vessel. The contact force can be divided into two components, i.e. the normal force N and the frictional force F , as shown in Fig. 4.

Using the coefficient μ_w of friction between a particle and the vessel, the relation between F and N is expressed as the following Eq. (15).

$$\left. \begin{aligned} P > \mu_w N &\cdots F = \mu_w N \\ -\mu_w N \leq P \leq \mu_w N &\cdots F = P \\ P < -\mu_w N &\cdots F = -\mu_w N \end{aligned} \right\} \quad (15)$$

where P and N are the maximum static friction force and the normal force, respectively. When the particle remains on the vessel, P and N are given in Eq. (16). P and N can be derived as follows. The equations of motion for the particle are first expressed as $m\ddot{x} = F \cos \alpha - N \sin \alpha$ and $m\ddot{y} = F \sin \alpha + N \cos \alpha - mg$ in x - y coordinates. Then we can have Eq. (16) by transforming x - y coordinate system into x_0 - y_0 coordinate system and setting $\ddot{x}_0 = 0$ and $\ddot{y}_0 = 0$.

$$\left. \begin{aligned} P &= m\{g \sin \alpha - a\omega^2 \cos(\beta - \alpha) \sin \omega t\} \\ N &= m\{g \cos \alpha - a\omega^2 \sin(\beta - \alpha) \sin \omega t\} \end{aligned} \right\} \quad (16)$$

To substitute the contact force F and N into the Eq. (2) of motion for the particle, F and N may be expressed in components F_x and F_y along x and y directions, respectively.

$$\left. \begin{aligned} F_{rx} &= F \cos \alpha - N \sin \alpha \\ F_{ry} &= F \sin \alpha + N \cos \alpha \end{aligned} \right\} \quad (17)$$

3.5 External force acting on a particle

Next, consider the components F_{xi} and F_{yi} of the contact force along x and y directions and the moment T_i of the force about an axis normal to the paper. When the particle i is in contact with the particle j ($j=1\sim n$) as shown in Fig. 2, the total forces F_{xi} and F_{yi} acting on the particle i and the summation of the moment T_i are expressed by Eq.(18). When the particle isn't in contact with the vessel, we let F_{xi} , F_{yi} and F equal to zero.

$$\left. \begin{aligned} F_{xi} &= \sum_j (-f_n \cdot \cos \phi_{ij} - f_T \cdot \sin \phi_{ij}) + F_{rx} \\ F_{yi} &= \sum_j (-f_n \cdot \sin \phi_{ij} + f_T \cdot \cos \phi_{ij}) + F_{ry} \\ T_i &= r \cdot (\sum_j f_T + F) \end{aligned} \right\} \quad (18)$$

3.6 Procedure of the simulation

We give here a short description of the procedure of the simulation method for the vibratory conveyance used in this paper. We first consider the motion of a particle and determine the contact force acting on the particle derived from Eqs. (3), (4) and (17). By using this force, the components F_{xi} and F_{yi} of the contact force along x and y directions and the moment T_i about the center axis of the particle are obtained by Eq.(18). Then, the motion for the particle is analyzed by using Eq.(2). Same procedure is carried out for all particles. The Eq.(2) of motion is analyzed by means of Runge-Kutta method. We should be careful in determining the time step Δt in numerical integration. The bigger the time step Δt , the more unstable calculation becomes. The smaller the time step, the more stable the calculation, but the calculation time is longer. Therefore, we determined the time step Δt by considering both the stability of calculation and its convergence.

4. Analytical Results

The computational conditions used for the simulation are shown in Table 1. These computational conditions were also carried out under identical conditions in the experiment as mentioned later. The spring constant is derived from the slope of the line in Fig. 3. The coefficient of friction means the coefficient of static friction and it was measured with acrylic resin cylinder and steel plate. In the free fall experiment, the coefficient of restitution is measured with the video camera and the image processing system. In these measurements, the temperature and humidity were kept constant. Usually, the vibrating vessel may be inclined to the horizontal with an angle α less than the angle of friction. The cylindrical particles are so easy to roll that we let α be zero in the present work.

Figure 5 shows a plot of the conveying velocity

Table 1 Condition of calculation

Particle	
Total number	23, 36, 109
Diameter	16 mm
Length	80 mm
Spring constant	
Particle-Particle	5.8×10^6 N/m
Particle-Vessel	11.5×10^6 N/m
Coefficient of friction	
Particle-Particle	0.52
Particle-Vessel	0.25
Coefficient of restitution	
Particle-Particle	0.62
Particle-Vessel	0.76

Fig. 5 Effect of time step Δt

V_m versus the amplitude of vibrating vessel by using 23 cylindrical particles. This figure shows the effect of time step Δt on the analytical results. We define the conveying velocity V_m to be the mean conveying displacement along the horizontal direction per number of particle divided by the setting time. We decided the setting time to be 1.155 sec. This was based on the reason why during the time, 23 particles can move to the full size of vibrating vessel as shown later in the experiment. Therefore, we determined that time step $\Delta t = 10^{-5}$ sec should be used in all of the simulation in the present work, so that the calculation becomes stable and the motion for particles converges.

Figure 6 shows the effect of the amplitude a and the period T of the vibrating vessel on the conveyance of 23 particles in the same plane as Fig. 5. It is clear that the conveying velocity increases as the amplitude of the vibrating vessel increases or the period of the vibrating vessel decreases.

Figure 7 shows the examples of the flow pattern of 109 cylindrical particles obtained from the numerical simulation. Initially, particles are piled up evenly

Fig. 6 Effect of period T (a) $T=0.01$ s, $a=0.8$ mm, $\alpha=0^\circ$, $\beta=20^\circ$ (b) $T=0.01$ s, $a=0.8$ mm, $\alpha=0^\circ$, $\beta=60^\circ$ (c) $T=0.02$ s, $a=0.8$ mm, $\alpha=0^\circ$, $\beta=20^\circ$

Fig. 7 Flow pattern (calculation)

in the vibrating vessel. Then, the vessel is subjected to vibration under the same condition as shown in Fig. 7. In the case of (b), the direction β of vibration to the horizontal is larger than that in the case of (a). In the case of (c), the period T of the vibrating vessel is longer than that in the case of (a). It is clearly

Fig. 8 Experimental apparatus

observed that, as time passes, the particles piled up evenly changes to crumble down and the flow occurs from left to right. As β increases, the particles are easy to jump up on the vibrating vessel and it looks like the performance of vibratory conveyance decreases. The reason is that vertical component of vibrations increases when β increases. It is easy to expect this phenomenon. The flow pattern seems to be wave-like. When period T decreases, the wave forms in the direction of conveying and the conveying velocity also seems to increase. DEM has the advantage to show this flow pattern of granular materials. Much of the previous theoretical analysis has been concerned with the motion of a particle and has not been able to predict the flow pattern like this.

5. Experiment

To verify the validity of the numerical method, the experiment was conducted by the experimental apparatus as shown in Fig. 8. In this apparatus, the vibrating vessel on the sliding guide is subjected to vibrations by using the AC servo motor through the ball screw. The angle of inclination of the vibration mechanism as well as the vibrating vessel can be changed. To secure sinusoidal vibrations, AC servo motor is controlled by the personal computer via the controller. The displacement of the vibrating vessel was measured with noncontact displacement sensor and we confirmed that the vessel was undergoing vibrations according to demand. The contamination of all acrylic resin cylinders was removed by clean cloth and we paid attention to keep the vibration vessel clean. In the experiment, cylindrical particles with marks at the cross sections were initially piled up evenly. From their marks, the displacements of the particles were measured by the image processing system. In the experiment, temperature and humidity of the room were kept constant for the purpose that

Fig. 9 Comparison of experimental and calculated results

these were independent of the experimental result.

Figure 9 shows the comparison of experimental results to calculated ones for the conveyance of 23 particles. Both this figure and Fig. 5 have the same ordinate and abscissa. From these two figures, it is clear that the calculated result shows the experimental trend with reasonable accuracy, in which the conveying velocity increases with the increase of the amplitude of the vibrating vessel. Figure 10 shows the typical examples of the comparison of flow pattern between experimental and calculated results. (a) is the image data recorded VTR and (b) is the calculated result. It is found from the marks at the cross sections of the particles that the particles may be conveyed by rotating action. For each layer of four layers bed, the calculated rotational motion of the particles is found to agree with the experimental result. Therefore, the approach in the present work is effective to estimate the vibratory conveyance of granular materials.

6. Conclusions

The vibratory conveyance of granular materials was investigated by means of DEM, in which granular materials are treated as many cylinders and we analytically studied the foundation of estimating the flow pattern of granular materials. The contact forces between two particles and between a particle and vibrating vessel were divided into the normal and tangential components. The normal component of contact force was modeled by the sum of the forces due to the spring and the viscous damping. The tangential component of contact force was given by the Coulomb laws of friction. The validity of the theory was confirmed by the experimental and calculated results. This numerical method is very practical one, compared to previous theoretical analysis in which it has been focussed on a particle's motion.

(a) Experimental

(b) Calculated

Fig. 10 Comparison of flow pattern between Experimental and calculated results ($T=0.1$ s, $a=0.9$ mm, $\alpha=0^\circ$, $\beta=20^\circ$)

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