Observer-Based Controller Design for Uncertain Large-Scale Systems with Time-Delays in Subsystem Interconnections*

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In this paper, the decentralized stabilization problem of uncertain large-scale systems with time-delays in subsystem interconnections is investigated. An observer-based controllers is developed using the Lyapunov method. For robust stability of the system, a sufficient condition is derived in terms of linear matrix inequalities (LMIs) which are solved by convex optimization algorithm. A numerical example is given to illustrate the proposed technique.

Key Words : Decentralized Control, Time-Delay, Lyapunov Method, Linear Matrix Inequality

1. Introduction

During the last decade, the problem of decentralized stabilization of large-scale systems with time-delays in subsystem interconnections has received considerable attention⁽¹⁾⁻⁽⁴⁾. Recently, Yan et al.⁽⁵⁾ presented new stabilization method using variable structure control technique, and the application is extended to the systems with parameter uncertainties. Furthermore, his method does not require any structural constraint on the target systems⁽¹⁾⁻⁽⁴⁾. However, parameter uncertainty is not included in input coefficient matrix and also it may yield conservative result since his stabilization condition is expressed in terms of matrix norms. Most of all, all the stabilization methods in the literature⁽¹⁾⁻⁽⁵⁾ including Yan's, are based on state feedback, so they can not be applicable to the systems if all the states are not available.

In this paper, we propose an observer-based controller for robust stabilization of uncertain large-

scale systems with time-delays in subsystem interconnections. The proposed technique does not require any restriction on system structure or any matching condition on system uncertainties. Using the Lyapunov method, the controller is designed based on the full state observer, and a delay-independent sufficient condition for robust stability is derived using the Lyapunov method. The condition is expressed in terms of two LMIs so that the controller parameters are found easily and effectively by various convex optimization algorithms⁽⁶⁾. Recently, the LMI approach becomes very powerful tool for various problems arising in system and control theory.

The organization of this paper is as follows. Section 2 provides formulation of the problem. The main result for designing observer-based controller is in section 3. In section 4, we provide an example to illustrate the application of proposed method. Finally, the conclusion is given in section 5.

Notation

R^n denotes n -dimensional Euclidean space, and I denotes identity matrix of appropriate dimension. The notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite).

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2. Problem Statement

Consider a large scale dynamic system which is composed of following N interconnected subsystems:

$$\begin{aligned}\dot{x}_i(t) &= [A_i + \Delta A_i(r_i(t))]x_i(t) + \sum_{j=1, j \neq i}^N [A_{ij} + \Delta A_{ij}(v_{ij}(t))]x_j(t - \tau_{ij}) + [B_i + \Delta B_i(s_i(t))]u_i(t) \\ y_i(t) &= [C_i + \Delta C_i(l_i(t))]x_i(t), \quad i=1, 2, \dots, N\end{aligned}\quad (1)$$

where $x_i(t) \in R^{n_i}$ is the state vector, $u_i(t) \in R^{m_i}$ is the control vector, and $y_i(t) \in R^{q_i}$ is the output vector. The system matrices A_i, B_i, C_i , and A_{ij} are of appropriate dimensions, the positive constant τ_{ij} denotes the time-delay in the interconnection, and $\Delta A_i(\cdot), \Delta B_i(\cdot), \Delta C_i(\cdot)$ and $\Delta A_{ij}(\cdot)$ are system parameter uncertainties. It is assumed that all pairs (A_i, B_i) and (A_i, C_i) are completely controllable and observable. The uncertainty enters the systems through the vectors $r_i(t) \in R^k, v_{ij}(t) \in R^l, s_i(t) \in R^p$, and $l_i(t) \in R^h$. Assume that $r_i(t), v_{ij}(t), s_i(t)$, and $l_i(t)$ are ranged in the following known compact sets $\mathcal{R}_i, \mathcal{V}_i, \mathcal{S}_i$, and $\mathcal{L}_i$, respectively^{(7),(8)}.

$$\begin{aligned}\mathcal{R}_i &\triangleq \{r_i : |r_{im}(t)| \leq \bar{r}_i, \text{ for } m=1, 2, \dots, k; \bar{r}_i \geq 0; \\ \mathcal{V}_i &\triangleq \{v_{ij} : |v_{ijm}(t)| \leq \bar{v}_i, \text{ for } m=1, 2, \dots, l; \bar{v}_i \geq 0; \\ \mathcal{S}_i &\triangleq \{s_i : |s_{im}(t)| \leq \bar{s}_i, \text{ for } m=1, 2, \dots, p; \bar{s}_i \geq 0; \\ \mathcal{L}_i &\triangleq \{l_i : |l_{im}(t)| \leq \bar{l}_i, \text{ for } m=1, 2, \dots, h; \bar{l}_i \geq 0.\end{aligned}\quad (2)$$

Furthermore, we assume that the uncertainties are of the 'rank-1' type^{(7),(8)}, i.e.,

$$\begin{aligned}\Delta A_i(r_i(t)) &= \sum_{m=1}^k A_{im} r_{im}(t), \\ \Delta A_{ij}(v_{ij}(t)) &= \sum_{m=1}^l A_{ijm} v_{ijm}(t), \\ \Delta B_i(s_i(t)) &= \sum_{m=1}^p B_{im} s_{im}(t), \quad \Delta C_i(l_i(t)) = \sum_{m=1}^h C_{im} l_{im}(t).\end{aligned}\quad (3)$$

Here also we assume that the constant matrices A_{im}, A_{ijm}, B_{im} and C_{im} are of the form

$$\begin{aligned}A_{im} &= d_{im} e_{im}^T, \quad B_{im} = f_{im} g_{im}^T, \quad A_{ijm} = h_{ijm} w_{ijm}^T, \\ C_{im} &= \phi_{im} \psi_{im}^T\end{aligned}\quad (4)$$

where $d_{im}, e_{im}, f_{im}, w_{ijm}, h_{ijm}, \phi_{im} \in R^{n_i}, g_{im} \in R^{m_i}, \psi_{im} \in R^{q_i}$.

Now, we consider a full state observer for nominal part of the system (1)

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_i \tilde{x}_i(t) + \sum_{j=1, j \neq i}^N A_{ij} \tilde{x}_j(t - \tau_{ij}) + B_i u_i(t) \\ &\quad + L_i(y_i(t) - C_i \tilde{x}_i(t)),\end{aligned}\quad (5)$$

and the control law for the system (1)

$$u_i(t) = -K_i \tilde{x}_i(t)\quad (6)$$

in which $\tilde{x}_i(t) \in R^{n_i}$ is the observer state vector, and K_i and L_i are the constant feedback matrix and observer gain matrix with appropriate dimensions, respectively.

Define error vector $e_i(t)$ as

$$e_i(t) = x_i(t) - \tilde{x}_i(t).\quad (7)$$

With the control law (6), the resulting closed-loop

subsystem becomes

$$\begin{aligned}\dot{x}_i(t) &= (A_i - B_i K_i)x_i(t) + \sum_{j=1}^N [A_{ij} + \Delta A_{ij}]x_j(t - \tau_{ij}) \\ &\quad + B_i K_i e_i(t) \\ &\quad + \Delta A_i x_i(t) - \Delta B_i K_i (x_i(t) - e_i(t)).\end{aligned}\quad (8)$$

Then, the problem is to find the feedback gain matrix K_i of the control law (6) and the observer gain matrix L_i of the full state observer (5) so that the closed-loop system (8) is asymptotically stabilized.

3. Decentralized Stabilization

We synthesize the gain matrices K_i and L_i as follows:

$$K_i = \gamma_{ci} B_i^T P_{ci}, \quad L_i = \gamma_{oi} P_{oi} C_i^T, \quad i=1, 2, \dots, N\quad (9)$$

where γ_{ci} and γ_{oi} are positive scalars, and P_{ci} and P_{oi} are positive-definite matrices.

Then, using Lyapunov method, we derive a theorem for robust stability of the the systems.

The following lemma and definitions are necessary for derivation of the theorem.

Lemma 1⁽⁶⁾: The LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0,$$

is equivalent to

$$R(x) > 0, \quad Q(x) - S(x)R(x)^{-1}S(x)^T > 0,$$

where $Q(x) = Q(x)^T, R(x) = R(x)^T$ and $S(x)$ depend affinely on x .

Define

$$\begin{aligned}D_i &= \bar{r}_i \sum_{m=1}^k d_{im} d_{im}^T, \quad E_i = \bar{r}_i \sum_{m=1}^k e_{im} e_{im}^T, \quad F_i = \bar{s}_i \sum_{m=1}^p f_{im} f_{im}^T \\ G_i &= \bar{s}_i \sum_{m=1}^p g_{im} g_{im}^T, \quad H_i = \sum_{j=1, j \neq i}^N \bar{v}_i \sum_{m=1}^l h_{ijm} h_{ijm}^T, \\ W_{ij} &= \bar{v}_i \sum_{m=1}^l w_{ijm} w_{ijm}^T \\ \Phi_i &= \bar{l}_i \sum_{m=1}^h \phi_{im} \phi_{im}^T, \quad \Psi_i = \bar{l}_i \sum_{m=1}^h \psi_{im} \psi_{im}^T,\end{aligned}\quad (10)$$

and

$$\begin{aligned}M_1(X_{ci}, \gamma_{ci}) &\equiv X_{ci} A_i^T + A_i X_{ci} + X_{ci} (2E_i + \Psi_i \\ &\quad + (N-1)I + 2W_i) X_{ci} \\ &\quad - \{\gamma_{ci} B_i B_i^T - 2F_i - 2\gamma_{ci}^2 B_i G_i B_i^T - D_i - H_i - A_{di}\} \\ M_2(X_{oi}, \gamma_{oi}) &\equiv A_i^T X_{oi} + X_{oi} A_i + X_{oi} (D_i + 2F_i \\ &\quad + H_i + A_{di}) X_{oi} \\ &\quad - \{2\gamma_{oi} C_i^T C_i - \gamma_{oi}^2 C_i^T \Phi_i C_i - (N-1)I \\ &\quad - P_{ci} B_i (\gamma_{ci} + 2\gamma_{ci}^2 G_i) B_i^T P_{ci}\}\end{aligned}\quad (11)$$

where

$$\begin{aligned}\mathcal{A}_{di} &= \sum_{j=1, j \neq i}^N A_{ij} A_{ij}^T, \quad \mathcal{W}_i = \sum_{j=1, i \neq j}^N W_{ij}, \\ X_{ci} &= P_{ci}^{-1}, \quad X_{oi} = P_{oi}^{-1}.\end{aligned}\quad (12)$$

Theorem: The closed-loop system (8) is asymptotically stable if there exist positive definite matrices X_{ci}, X_{oi} , and positive scalars γ_{ci}, γ_{oi} which satisfy the following LMIs:

$$\begin{aligned}
\mathcal{Q}_{i1}(X_{ci}, \gamma_{ci}) = & \begin{bmatrix} Q_{i1}(X_{ci}, \gamma_{ci}) & X_{ci}E_i^{1/2} & X_{ci}W_i^{1/2} \\ E_i^{1/2}X_{ci} & -\frac{1}{2}I & 0 \\ W_i^{1/2}X_{ci} & 0 & -\frac{1}{2}I \\ X_{ci} & 0 & 0 \\ \Psi_i^{1/2}X_{ci} & 0 & 0 \\ \gamma_{ci}G_i^{1/2}B_i^T & 0 & 0 \end{bmatrix} \\
& \begin{bmatrix} X_{ci} & X_{ci}\Psi_i^{1/2} & \gamma_{ci}B_iG_i^{1/2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{1}{(N-1)}I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -\frac{1}{2}I \end{bmatrix} < 0 \\
\mathcal{Q}_{i2}(X_{oi}, \gamma_{oi}) = & \begin{bmatrix} Q_{i2}(X_{oi}, \gamma_{oi}) & X_{oi}F_i^{1/2} & X_{oi}\mathcal{A}_{di}^{1/2} \\ F_i^{1/2}X_{oi} & -\frac{1}{2}I & 0 \\ \mathcal{A}_{di}^{1/2}X_{oi} & 0 & -I \\ D_i^{1/2}X_{oi} & 0 & 0 \\ H_i^{1/2}X_{oi} & 0 & 0 \\ \gamma_{oi}\Phi_i^{1/2}C_i & 0 & 0 \end{bmatrix} \\
& \begin{bmatrix} X_{oi}D_i^{1/2} & X_{oi}H_i^{1/2} & \gamma_{oi}C_i^T\Phi_i^{1/2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix} < 0 \quad (13)
\end{aligned}$$

where

$$\begin{aligned}
Q_{i1}(X_{ci}, \gamma_{ci}) &= X_{ci}A_i^T + A_iX_{ci} - \gamma_{ci}B_iB_i^T + 2F_i \\
&+ \mathcal{A}_{di} + D_i + H_i \\
Q_{i2}(X_{oi}, \gamma_{oi}) &= A_i^T X_{oi} + X_{oi}A_i - 2\gamma_{oi}C_i^T C_i \\
&+ (N-1)I + P_{ci}B_i(\gamma_{ci} + 2\gamma_{ci}G_i\gamma_{ci})B_i^T P_{ci}.
\end{aligned}$$

Proof: Consider a Lyapunov functional candidate

$$\begin{aligned}
V &= \sum_{i=1}^N V_i \\
&= \sum_{i=1}^N \left\{ x_i^T P_{ci} x_i + e_i^T P_{oi}^{-1} e_i \right. \\
&+ \sum_{j=1, i \neq j}^N \int_{t-\tau_{ij}}^t [x_j^T(s)x_j(s) + 2x_j^T(s)W_{ij}x_j(s)] ds \\
&+ \left. \sum_{j=1, i \neq j}^N \int_{t-\tau_{ij}}^t e_j^T(s)e_j(s) ds \right\}. \quad (14)
\end{aligned}$$

The time derivative of V is given by

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^N \left\{ \dot{x}_i^T P_{ci} x_i + x_i^T P_{ci} \dot{x}_i + \dot{e}_i^T P_{oi}^{-1} e_i + e_i^T P_{oi}^{-1} \dot{e}_i \right. \\
&+ \sum_{j=1, i \neq j}^N [x_j^T x_j - x_j^T(t-\tau_{ij})x_j(t-\tau_{ij})] \\
&+ 2 \sum_{j=1, i \neq j}^N [x_j^T W_{ij} x_j - x_j^T(t-\tau_{ij})W_{ij}x_j(t-\tau_{ij})] \\
&+ \left. \sum_{j=1, i \neq j}^N [e_j^T e_j - e_j^T(t-\tau_{ij})e_j(t-\tau_{ij})] \right\}. \quad (15)
\end{aligned}$$

By definition (7), we can get the error dynamic equation as

$$\begin{aligned}
\dot{e}(t) &= (A_i - L_i C_i)e_i(t) + \sum_{j \neq i}^N A_{ij}e_j(t - \tau_{ij}) \\
&+ \Delta A_i x_i(t) \\
&+ \sum_{j \neq i}^N \Delta A_{ij}x_j(t - \tau_{ij}) - L_i \Delta C_i x_i(t) + \Delta B_i u_i(t). \quad (16)
\end{aligned}$$

Substituting (8) and (16) into (15), we have

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^N \left\{ x_i^T (A_i^T P_{ci} + P_{ci}A_i - 2\gamma_{ci}P_{ci}B_iB_i^T P_{ci})x_i \right. \\
&+ 2\gamma_{ci}x_i^T P_{ci}B_iB_i^T P_{ci}e_i \\
&+ 2x_i^T P_{ci} \sum_{j=1, i \neq j}^N (A_{ij} + \Delta A_{ij})x_j(t - \tau_{ij}) \\
&+ 2x_i^T P_{ci} \Delta A_i x_i - 2\gamma_{ci}x_i^T P_{ci} \Delta B_i B_i^T P_{ci} x_i \\
&+ 2x_i^T P_{ci} \Delta B_i \gamma_{ci} B_i^T P_{ci} e_i \\
&+ e_i^T (A_i^T P_{oi}^{-1} + P_{oi}^{-1}A_i - 2\gamma_{oi}C_i^T C_i)e_i \\
&+ 2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N A_{ij}e_j(t - \tau_{ij}) + 2e_i^T P_{oi}^{-1} \Delta A_i x_i \\
&+ 2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N \Delta A_{ij}x_j(t - \tau_{ij}) \\
&- 2\gamma_{ci}e_i^T P_{oi}^{-1} \Delta B_i B_i^T P_{ci} x_i + 2\gamma_{ci}e_i^T P_{oi}^{-1} \Delta B_i B_i^T P_{ci} e_i \\
&- 2\gamma_{oi}e_i^T C_i^T \Delta C_i x_i \\
&+ \sum_{j=1, i \neq j}^N [x_j^T x_j - x_j^T(t - \tau_{ij})x_j(t - \tau_{ij})] \\
&+ 2 \sum_{j=1, i \neq j}^N [x_j^T W_{ij} x_j - x_j^T(t - \tau_{ij})W_{ij}x_j(t - \tau_{ij})] \\
&+ \left. \sum_{j=1, i \neq j}^N [e_j^T e_j - e_j^T(t - \tau_{ij})e_j(t - \tau_{ij})] \right\}. \quad (17)
\end{aligned}$$

Here, in order to obtain the required upper bound for $\dot{V}$, we utilize the following well-known inequality

$$2|x^T y| \leq \beta x^T x + \frac{1}{\beta} y^T y$$

for any real number $\beta > 0$, and any vectors with appropriate dimensions x and y . Now, we apply this inequality to various terms in (17). For example, the three terms, $2x_i^T P_{ci} \Delta A_i x_i$, $2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N \Delta A_{ij}x_j(t - \tau_{ij})$, and $2x_i^T P_{ci} \sum_{j=1}^N A_{ij}x_j(t - \tau_{ij})$ of (17), are expanded as follows:

$$\begin{aligned}
2x_i^T P_{ci} \Delta A_i x_i &= 2x_i^T P_{ci} \sum_{m=1}^k d_{im} e_{im}^T r_{im}(t) x_i \\
&\leq x_i^T K_{ci} \bar{r}_i \sum_{m=1}^k d_{im} d_{im}^T P_{ci} x_i + x_i^T \bar{r}_i \sum_{m=1}^k e_{im} e_{im}^T x_i \\
&= x_i^T P_{ci} D_i P_{ci} x_i + x_i^T E_i x_i, \quad (18) \\
2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N \Delta A_{ij}x_j(t - \tau_{ij}) &= 2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N \sum_{m=1}^l h_{ijm} w_{ijm}^T v_{ijm}(t) x_j(t - \tau_{ij}) \\
&\leq e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N \bar{v}_i \sum_{m=1}^l h_{ijm} h_{ijm}^T P_{oi}^{-1} e_i \\
&+ \sum_{j=1, i \neq j}^N \{x_j^T(t - \tau_{ij}) \bar{v}_i \sum_{m=1}^l w_{ijm} w_{ijm}^T x_j(t - \tau_{ij})\} \\
&= e_i^T P_{oi}^{-1} H_i P_{oi}^{-1} e_i + \sum_{j=1}^N x_j^T(t - \tau_{ij}) W_{ij} x_j(t - \tau_{ij}) \quad (19)
\end{aligned}$$

and

$$2x_i^T P_{ci} \sum_{j=1}^N A_{ij}x_j(t - \tau_{ij}) \leq x_i^T P_{ci} \sum_{j=1}^N A_{ij} A_{ij}^T P_{ci} x_i$$

$$\begin{aligned}
& + \sum_{j=1}^N x_j(t - \tau_{ij})^T x_j(t - \tau_{ij}) \\
& = x_i^T P_{ci} \mathcal{A}_{di} P_{ci} x_i + \sum_{j=1}^N x_j(t - \tau_{ij})^T x_j(t - \tau_{ij}). \quad (20)
\end{aligned}$$

In a similar fashion, bounds of other terms in (17) are derived as

$$\begin{aligned}
& 2x_i^T P_{ci} B_i \gamma_{ci} B_i^T P_{ci} e_i \leq \gamma_{ci} x_i^T P_{ci} B_i B_i^T P_{ci} x_i \\
& \quad + \gamma_{ci} e_i^T P_{ci} B_i B_i^T P_{ci} e_i, \\
& 2x_i^T P_{ci} \sum_{j=1, i \neq j}^N \Delta A_{ij} x_j(t - \tau_{ij}) \leq x_i^T P_{ci} H_i P_{ci} x_i \\
& \quad + \sum_{j=1, i \neq j}^N x_j^T(t - \tau_{ij}) W_{ij} x_j(t - \tau_{ij}), \\
& -2x_i^T P_{ci} \Delta B_i \gamma_{ci} B_i^T P_{ci} x_i \leq x_i^T P_{ci} F_i P_{ci} x_i \\
& \quad + \gamma_{ci}^2 x_i^T P_{ci} B_i G_i B_i^T P_{ci} x_i, \\
& 2x_i^T P_{ci} \Delta B_i \gamma_{ci} B_i^T P_{ci} e_i \leq x_i^T P_{ci} F_i P_{ci} x_i \\
& \quad + \gamma_{ci}^2 e_i^T P_{ci} B_i G_i B_i^T P_{ci} e_i, \\
& 2e_i^T P_{oi}^{-1} \sum_{j=1, i \neq j}^N A_{ij} e_j(t - \tau_{ij}) \leq e_i^T P_{oi}^{-1} \mathcal{A}_{di} P_{oi}^{-1} e_i \\
& \quad + \sum_{j=1, i \neq j}^N e_j(t - \tau_{ij})^T e_j(t - \tau_{ij}), \\
& 2e_i^T P_{oi}^{-1} \Delta A_i x_i \leq e_i^T P_{oi}^{-1} D_i P_{oi}^{-1} e_i + x_i^T E_i x_i, \\
& -2e_i^T P_{oi}^{-1} \Delta B_i \gamma_{ci} B_i^T P_{ci} x_i \leq e_i^T P_{oi}^{-1} F_i P_{oi}^{-1} e_i \\
& \quad + \gamma_{ci}^2 x_i^T P_{ci} B_i G_i B_i^T P_{ci} x_i, \\
& 2e_i^T P_{oi}^{-1} \Delta B_i \gamma_{ci} B_i^T P_{ci} e_i(t) \leq e_i^T P_{oi}^{-1} F_i P_{oi}^{-1} e_i \\
& \quad + \gamma_{ci}^2 e_i^T P_{ci} B_i G_i B_i^T P_{ci} e_i, \\
& -2e_i^T \gamma_{oi} C_i^T \Delta C_i x_i \leq \gamma_{oi}^2 e_i^T C_i^T \Phi_i C_i e_i + x_i^T \Psi_i x_i. \quad (21)
\end{aligned}$$

Then, using (18), (19), (20) and (21), we have a bound of $\dot{V}$ as

$$\begin{aligned}
\dot{V} \leq & \sum_{i=1}^N \left\{ x_i^T [A_i^T P_{ci} + P_{ci} A_i - P_{ci} (\gamma_{ci} B_i B_i^T - 2F_i \right. \\
& - 2\gamma_{ci}^2 B_i G_i B_i^T - D_i - H_i - \mathcal{A}_{di}) P_{ci} \\
& + 2E_i + \Psi_i] x_i + 2 \sum_{j=1, i \neq j}^N x_j^T(t - \tau_{ij}) W_{ij} x_j(t - \tau_{ij}) \\
& + \sum_{j=1, i \neq j}^N x_j^T(t - \tau_{ij}) x_j(t - \tau_{ij}) \Big\} \\
& + \sum_{i=1}^N \left\{ e_i^T [A_i^T P_{oi}^{-1} + P_{oi}^{-1} A_i + P_{oi}^{-1} (D_i + 2F_i + H_i \right. \\
& + \mathcal{A}_{di}) P_{oi}^{-1} - 2\gamma_{oi} C_i^T C_i \\
& + \gamma_{oi}^2 C_i^T \Phi_i C_i + P_{ci} B_i (\gamma_{ci} + 2\gamma_{ci}^2 G_i) B_i^T P_{ci}] e_i \\
& + \sum_{j=1, i \neq j}^N e_j^T(t - \tau_{ij}) e_j(t - \tau_{ij}) \Big\} \\
& + \sum_{i=1}^N \left\{ \sum_{j=1, i \neq j}^N [x_j^T x_j - x_j^T(t - \tau_{ij}) x_j(t - \tau_{ij})] \right. \\
& + \sum_{j=1, i \neq j}^N [e_j^T e_j - e_j^T(t - \tau_{ij}) e_j(t - \tau_{ij})] \\
& + 2 \left[\sum_{j=1, i \neq j}^N x_j^T W_{ij} x_j \right. \\
& \left. \left. - \sum_{j=1, i \neq j}^N x_j^T(t - \tau_{ij}) W_{ij} x_j(t - \tau_{ij}) \right] \right\}. \quad (22)
\end{aligned}$$

Note that

$$\sum_{i=1}^N \sum_{j=1, i \neq j}^N x_j^T(t) x_j(t) = (N-1) \sum_{i=1}^N x_i^T(t) x_i(t), \quad (23)$$

$$\sum_{i=1}^N \sum_{j=1, i \neq j}^N x_j^T(t) W_{ij} x_j(t) = \sum_{i=1}^N x_i^T(t) \left(\sum_{j=1, i \neq j}^N W_{ji} \right) x_i(t), \quad (24)$$

$$\sum_{i=1}^N \sum_{j=1, i \neq j}^N e_j^T(t) e_j(t) = (N-1) \sum_{i=1}^N e_i^T(t) e_i(t). \quad (25)$$

Therefore,

$$\begin{aligned}
\dot{V} \leq & \sum_{i=1}^N \left\{ x_i^T [A_i^T P_{ci} + P_{ci} A_i - P_{ci} (\gamma_{ci} B_i B_i^T - 2F_i \right. \\
& - 2\gamma_{ci}^2 B_i G_i B_i^T - D_i - H_i - \mathcal{A}_{di}) P_{ci} \\
& + 2E_i + \Psi_i + (N-1)I + 2W_i] x_i \Big\} \\
& + \sum_{i=1}^N \left\{ e_i^T [A_i^T P_{oi}^{-1} \right. \\
& + P_{oi}^{-1} A_i + P_{oi}^{-1} (D_i + 2F_i + H_i + \mathcal{A}_{di}) P_{oi}^{-1} \\
& - 2\gamma_{oi} C_i^T C_i + \gamma_{oi}^2 C_i^T \Phi_i C_i + (N-1)I \\
& \left. + P_{ci} B_i (\gamma_{ci} + 2\gamma_{ci}^2 G_i) B_i^T P_{ci}] e_i \right\}. \quad (26)
\end{aligned}$$

Using the definitions of M_i , M_2 , X_{ci} , and X_{oi} in (11) and (12), (26) is rewritten as

$$\dot{V} \leq \sum_{i=1}^N \{ x_i^T X_{ci}^{-1} M_1 X_{ci}^{-1} x_i + e_i^T M_2 e_i \}.$$

Therefore, the sufficient condition for robust stability of the system (1) is

$$M_1 < 0 \text{ and } M_2 < 0. \quad (27)$$

By Lemma 1, two inequalities of (27) are equivalent to two LMIs of (13). This completes the proof.

4. Numerical Example

Consider a large-scale system which is composed of three subsystems given below. Each subsystem is coupled through delayed states:

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ 3 & 3 \end{bmatrix} x_1(t) + \begin{bmatrix} 0.2 & -0.1 \\ -0.1 & 0.2 \end{bmatrix} x_2(t-0.4)$$

$$\begin{aligned}
& + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.15 \end{bmatrix} x_3(t-1) + \Delta A_1(r_1) x_1(t) \\
& + \Delta A_{12}(v_{12}) x_2(t-0.4) + \Delta A_{13}(v_{13}) x_3(t-1) \\
& + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) + \Delta B_1(s_1) u_1(t),
\end{aligned}$$

$$y_1(t) = [1.5 \ 1.5] x_1(t) + \Delta C_1(l_1) x_1(t),$$

$$\dot{x}_2(t) = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} x_1(t-0.5)$$

$$\begin{aligned}
& + \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} x_3(t-1) + \Delta A_2(r_2) x_2(t) \\
& + \Delta A_{21}(v_{21}) x_1(t-0.5) + \Delta A_{23}(v_{23}) x_3(t-1) \\
& + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_2(t) + \Delta B_2(s_2) u_2(t),
\end{aligned}$$

$$y_2(t) = [1 \ 0] x_2(t) + \Delta C_2(l_2) x_2(t),$$

$$\dot{x}_3(t) = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} x_3(t) + \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{bmatrix} x_1(t-0.5)$$

$$\begin{aligned}
& + \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0.2 \end{bmatrix} x_2(t-0.4) + \Delta A_3(r_3) x_3(t) \\
& + \Delta A_{31}(v_{31}) x_1(t-0.5) + \Delta A_{32}(v_{32}) x_2(t-0.4) \\
& + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_3(t) + \Delta B_3(s_3) u_3(t),
\end{aligned}$$

$$y_3(t) = [1 \ 0] x_3(t) + \Delta C_3(l_3) x_3(t),$$

where

$$\Delta A_1(\cdot) = \begin{bmatrix} 0 & 0 \\ 0.05 \sin(t) & 0.1 \sin(2t) \end{bmatrix},$$

$$\Delta A_{12}(\cdot) = \begin{bmatrix} 0 & 0 \\ 0.05 \sin(t) & 0.03 \sin(2t) \end{bmatrix},$$

$$\Delta A_{13}(\cdot) = \begin{bmatrix} 0.05 \sin(t) & 0 \\ 0 & 0.02 \sin(t) \end{bmatrix},$$

$$\begin{aligned}
\Delta A_2(\cdot) &= \begin{bmatrix} 0 & 0 \\ 0.3 \cos(t) & 0.1 \cos(2t) \end{bmatrix}, \\
\Delta A_{21}(\cdot) &= \begin{bmatrix} 0 & 0.07 \sin(t) \\ 0 & 0.03 \sin(2t) \end{bmatrix}, \\
\Delta A_{23}(\cdot) &= \begin{bmatrix} 0.05 \sin(t) & 0 \\ 0 & 0.02 \sin(2t) \end{bmatrix}, \\
\Delta A_3(\cdot) &= \begin{bmatrix} 0 & 0 \\ 0.15 \sin(t) & 0 \end{bmatrix}, \Delta A_{31}(\cdot) = \begin{bmatrix} 0 & 0 \\ 0 & 0.03 \sin(t) \end{bmatrix}, \\
\Delta A_{32}(\cdot) &= \begin{bmatrix} 0 & 0 \\ 0 & 0.01 \sin(2t) \end{bmatrix}, \\
\Delta B_1(\cdot) &= \begin{bmatrix} 0 \\ 0.02 \sin(t) \end{bmatrix}, \Delta B_2(\cdot) = \begin{bmatrix} 0.02 \sin(t) \\ 0.02 \sin(2t) \end{bmatrix}, \\
\Delta B_3(\cdot) &= \begin{bmatrix} 0 \\ 0.02 \cos(t) \end{bmatrix}, \\
\Delta C_1(\cdot) &= [0.02 \cos(t) \ 0.01 \cos(2t)], \\
\Delta C_2(\cdot) &= [0.03 \sin(t) \ 0], \Delta C_3(\cdot) = [0.05 \sin(t) \ 0].
\end{aligned}$$

Then, we can express the above uncertainty matrices as

$$\begin{aligned}
\Delta A_1(\cdot) &= A_{11}r_{11} + A_{12}r_{12}, \Delta A_2(\cdot) = A_{21}r_{21} + A_{22}r_{22}, \\
\Delta A_3(\cdot) &= A_{31}r_{31}, \\
\Delta A_{12}(\cdot) &= A_{121}v_{121} + A_{122}v_{122}, \\
\Delta A_{13}(\cdot) &= A_{131}v_{131} + A_{132}v_{132}, \\
\Delta A_{21}(\cdot) &= A_{211}v_{211} + A_{212}v_{212}, \\
\Delta A_{23}(\cdot) &= A_{231}v_{231} + A_{232}v_{232}, \\
\Delta A_{31}(\cdot) &= A_{311}v_{311}, \Delta A_{32}(\cdot) = A_{321}v_{321}, \\
\Delta B_1(\cdot) &= B_{11}s_{11}, \Delta B_2(\cdot) = B_{21}s_{21}, \Delta B_3(\cdot) = B_{31}s_{31}, \\
\Delta C_1(\cdot) &= C_{11}l_{11} + C_{12}l_{12}, \Delta C_2(\cdot) = C_{21}l_{21}, \\
\Delta C_3(\cdot) &= C_{31}l_{31},
\end{aligned}$$

where

$$\begin{aligned}
A_{11} &= d_{11}e_{11}^T, A_{12} = d_{12}e_{12}^T, A_{21} = d_{21}e_{21}^T, A_{22} = d_{22}e_{22}^T, \\
A_{31} &= d_{31}e_{31}^T, \\
A_{121} &= h_{121}w_{121}^T, A_{122} = h_{122}w_{122}^T, A_{131} = h_{131}w_{131}^T, \\
A_{132} &= h_{132}w_{132}^T, \\
A_{211} &= h_{211}w_{211}^T, A_{212} = h_{212}w_{212}^T, A_{231} = h_{231}w_{231}^T, \\
A_{232} &= h_{232}w_{232}^T, \\
A_{311} &= h_{311}w_{311}^T, A_{321} = h_{321}w_{321}^T, \\
B_{11} &= f_{11}g_{11}^T, B_{21} = f_{21}g_{21}^T, B_{31} = f_{31}g_{31}^T, \\
C_{11} &= \phi_{11}\psi_{11}^T, C_{12} = \phi_{12}\psi_{12}^T, C_{21} = \phi_{21}\psi_{21}^T, C_{31} = \phi_{31}\psi_{31}^T.
\end{aligned}$$

From above, we choose

$$\begin{aligned}
d_{11} &= d_{12} = d_{21} = d_{22} = d_{31} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
h_{121} &= h_{122} = h_{132} = h_{212} = h_{232} = h_{311} = h_{321} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
e_{12} &= e_{22} = f_{11} = f_{31} = \phi_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
w_{122} &= w_{132} = w_{211} = w_{212} = w_{232} = w_{311} = w_{321} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
e_{11} &= e_{21} = e_{31} = w_{121} = w_{131} = w_{231} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
h_{131} &= h_{211} = h_{231} = \phi_{11} = \phi_{21} = \phi_{31} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
f_{21} &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
g_{11} &= g_{21} = g_{31} = \phi_{11} = \phi_{12} = \phi_{21} = \phi_{31} = 1
\end{aligned}$$

Fig. 1 Closed-loop state trajectories of the subsystem 1

and further,

$$\begin{aligned}
\bar{r}_1 &= 0.1, \bar{r}_2 = 0.3, \bar{r}_3 = 0.15, \bar{v}_1 = 0.05, \bar{v}_2 = 0.07, \\
\bar{v}_3 &= 0.03, \bar{s}_1 = \bar{s}_2 = \bar{s}_3 = 0.02, \bar{l}_1 = 0.02, \bar{l}_2 = 0.03, \bar{l}_3 \\
&= 0.05.
\end{aligned}$$

First, we find the positive solutions of the first LMI of (13) for the subsystem 1 as

$$P_{c1} = \begin{bmatrix} 6.0040 & 3.4479 \\ 3.4479 & 3.9920 \end{bmatrix}, \gamma_{c1} = 2.9638.$$

Then, using P_{c1} and γ_{c1} , the solutions of the second LMI of (13) are found as

$$P_{o1} = \begin{bmatrix} 0.1071 & 0.2402 \\ 0.2402 & 9.7414 \end{bmatrix}, \gamma_{o1} = 27.1627.$$

Similarly, the other solutions of the LMIs (13) for the subsystem 2 and 3 are as follows:

$$P_{c2} = \begin{bmatrix} 2.0823 & 0.2267 \\ 0.2267 & 1.2593 \end{bmatrix}, \gamma_{c2} = 1.4660$$

$$P_{o2} = \begin{bmatrix} 0.4897 & 0.4221 \\ 0.4221 & 0.5521 \end{bmatrix}, \gamma_{o2} = 34.3040$$

$$P_{c3} = \begin{bmatrix} 1.3261 & -0.1277 \\ -0.1277 & 1.3141 \end{bmatrix}, \gamma_{c3} = 0.7600$$

$$P_{o3} = \begin{bmatrix} 0.2078 & 0.1727 \\ 0.1727 & 0.3029 \end{bmatrix}, \gamma_{o3} = 16.2556.$$

Therefore, the stabilizing control laws $u_i(t) = -K_i \tilde{x}(t)$, $i=1, 2, 3$ of three subsystems are found as follows:

$$\begin{aligned}
u_1(t) &= -\gamma_{c1} B_1^T P_{c1} \tilde{x}_1(t) = -[10.2187 \ 11.8314] \tilde{x}_1(t), \\
u_2(t) &= -\gamma_{c2} B_2^T P_{c2} \tilde{x}_2(t) = -[3.3851 \ 2.1784] \tilde{x}_2(t), \\
u_3(t) &= -\gamma_{c3} B_3^T P_{c3} \tilde{x}_3(t) = -[0.9107 \ 0.9016] \tilde{x}_3(t).
\end{aligned}$$

The system responses of the example is simulated by digital computer. The initial condition functions for the three subsystems are arbitrary chosen as follows

$$\begin{aligned}
x_1(t) &= [1 \ -0.5]^T, \text{ for } -0.5 \leq t \leq 0, \\
x_2(t) &= [1.5 \ -1.5]^T, \text{ for } -0.4 \leq t \leq 0, \\
x_3(t) &= [-1 \ 0.5]^T, \text{ for } -1 \leq t \leq 0.
\end{aligned}$$

Figures 1, 2, and 3 show state responses of the three subsystems, and also Fig. 4 shows the stabilizing observer-based control law, $u_1(t)$, $u_2(t)$, and $u_3(t)$, for

Fig. 2 Closed-loop state trajectories of the subsystem 2

Fig. 4 Control laws of the subsystems

Fig. 3 Closed-loop state trajectories of the subsystem 3

the subsystems. In the figure, one can see the three subsystems are robustly stabilized irrespective of parameter uncertainties.

5. Conclusion

In this paper, the robust stabilization of uncertain large-scale systems with time-delays in the subsystem interconnections is investigated. The presented observer-type controller does not require all the state variables for the feedback, and any constraint on system structure or matching condition on system uncertainty are not imposed. Using the Lyapunov method, we derive a delay-independent sufficient condition for robust stabilization of the systems. Since the stability condition is expressed in terms of LMI, it is less conservative than Yan's⁽⁵⁾. Finally, a numerical example is given for illustration of controller design, and simulation result shows that the system is well

stabilized by the observer-based controller developed in this paper.

References

- (1) Lee, T.N. and Radovic, U.L., Decentralized Stabilization of Linear Continuous and Discrete-Time Systems with Delays in Interconnections, *IEEE Trans. Autom. Control*, Vol. 33 (1988), p. 757.
- (2) Geromel, J.C. and Yamakami, A., Stabilization of Continuous and Discrete Linear Systems Subjected to Control Structure Constraint, *Int. J. Control*, Vol. 3 (1982), p. 429.
- (3) Hu, Z., Decentralized Stabilization of Large Scale Interconnected Systems with Delays, *IEEE Trans. Autom. Control*, Vol. 39 (1994), p. 180.
- (4) Trinh, H. and Aldeen, M., A Comment on Decentralized Stabilization of Large Scale Interconnected Systems with Delays, *IEEE Trans. Autom. Control*, Vol. 40 (1995), p. 914.
- (5) Yan, J. -J., Tsai, J. S. -H. and Kung, J. -C., Robust Decentralized Stabilization of Large-Scale Delay Systems via Sliding Mode Control, *Trans. ASME, J. Dyn. Syst. Meas. Control*, Vol. 119 (1997), p. 307.
- (6) Boyd, S., Ghaoui, L.E., Feron, E. and Balakrishnan, V., *Linear Matrix Inequalities in System and Control Theory*, (1994), SIAM books, Philadelphia.
- (7) Wang, W. and Cheng, C., Stabilizing Controller and Observer Synthesis for Uncertain Large-Scale Systems by the Riccati Equation Approach, *IEE Proceedings-D*, Vol. 139 (1992), p. 72.
- (8) Petersen, I.R., A Riccati Equation Approach to the Design of Stabilizing Controllers and Observers for a Class of Uncertain Linear Systems, *IEEE Trans. Autom. Control*, Vol. 30 (1985), p. 904.