Nonlinear Vibration of the Coupled Textile/Rotor System by Finite Difference Method*

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The nonlinear vibration of the coupled textile/rotor system is investigated by using the finite difference method. Both the partial differential equation for the textile transverse vibration and the ordinary differential equation for the rotor whirling vibration are derived by Hamilton's principle. When the textile is wound either on or off the rotor, the physical model is non-conservative because the mass, inertia and unbalance magnitude of the rotor change with time, and also the length of the textile is time-dependent. A special technique of the finite difference method is developed for such a time-dependent length of the textile. The approach used here is with fixed-size grid and changing the grid number as the length varying. Finally, numerical examples are presented to demonstrate the effects of the rotating angular speed, rotating angular acceleration and nonlinear terms on the transient amplitudes of the coupled system.

Key Words: Nonlinear Vibration, Textile/Rotor System, Finite Difference Method

1. Introduction

Mechanical designers of the textile machine usually consider the textile and rotor independently in two distinct design processes. For example, much work has been done on the dynamic response of a textile under specified conditions, while a number of studies have been devoted to the steady-state response and stability of the rotor. However, the dynamic behavior of these two systems is interrelated when the textile is wound on or off by the rotor.

There is an important problem concerned with the string vibration with time-varying length. Kotera and Kawai⁽¹⁾ analyzed the free vibrations of a string with time-varying lengths by Laplace transformation. It should be noted that methods of solving the problems necessarily differ from the classical methods of

treating the fixed length problems. For instance, the concepts of the natural modes and frequencies become meaningless because as the length of the string varies the natural frequencies become time-dependent and the independence of the natural modes of oscillation is lost.

A related problem involving the oscillations and the influence of the reactive force on the motion of a textile machine rotor on which the textile is wound up was presented in a series of papers by Cventicanin⁽²⁾⁻⁽⁵⁾. The dynamics of a rotor with variable mass are given by Bessonov⁽⁶⁾. Usually, the rotor consists of a disk which is symmetrically mounted at the middle of the shaft, and the elastic force in the shaft is assumed to be nonlinear. The mass of the shaft is negligible in comparison to that of the disk. The mass of the rotor is varying due to winding on of the band. Cventicanin⁽⁷⁾ studied the textile machine rotor, in which the angular velocity is constant. The function of the rotor is to wind on a band of textile material. The free vibrations in the non-resonant case were analyzed and the solutions were found by use of the analytical method of multiple scaling. With a new procedure based on the Krylov-

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Bogoliubov method, Cventicanin⁽⁷⁾ observed the dynamic behavior of a rotor with variable parameters and small nonlinearity.

The purpose of this paper is to present the dynamic responses of a coupled textile/rotor system, which consists of an axially moving textile and a whirling rotor. The coupled dynamic equations for the textile/rotor system are derived by Hamilton's principle. We adopt the concept of Cventicanin⁽²⁾ and consider the Jeffcott rotor with variable mass, inertia and unbalance magnitude. As time proceeds, the number of grid changes in space. The finite difference approach of using fixed-size grid and changing its grid number is adopted. Finally, we present the numerical results including time-dependent length of textile and the radius of disks. The effects of the rotating angular speed, rotating angular acceleration, and nonlinear term on the transient amplitudes are shown for the coupled system.

2. Equations of Motion

2.1 Geometry description

The coupled textile/rotor system is shown in Fig. 1. Two fixed coordinate systems xoy and XOY are used to describe the dynamic configuration. It consists of a single, centrally located, unbalanced disk and an

Fig. 1.a Model of the coupled textile/rotor system

Fig. 1.b Deformed configuration of the coupled textile/rotor system

Fig. 1.c Grid arrangement of the textile

orthotropically elastic shaft, running in two rigid bearings. The shaft is assumed massless as compared with the large massive disk. The bearing supports are assumed to be rigid, with the shaft providing all the flexibility. The Jeffcott rotor model investigated here is similar to that used by Vance and Lee⁽⁸⁾. The generalized coordinate r is chosen to describe the whirling oscillation of the rotor system which has angular velocity ω . Since the rotor is whirling, the textile length is time-dependent. The transverse vibration of the textile is in the interval, $0 \leq x \leq l(t)$. The textile is subject to an initial tension T and has simple supports as the boundary conditions at $x=0$ and $l(t)$.

Since the connection point $x=l(t)$ is common to the disk and textile, this point on the textile has the same velocity and acceleration as that on the disk in the tangential direction. When the textile is wound on, the disk rotates in a clockwise direction and the shaft angular velocity ω is negative.

2.2 Vibration textile

The function of the rotor is to wind the textile on or off, so the rotor mass is variable. In Fig. 1.b, $R(t)$ is the radius of the disk, and ψ is the rotary angle. The textile is subject to an initial tension T , so this force acts along the tangent line of the disk in the undeformed configuration. The point $x=l(t)$ is one connection point between the textile and rotor, $R_A(t)$ and $R(t)$ are perpendicular each other at the point A. Since the disk has a whirling motion, the textile length is time-dependent. From the geometry of Fig. 1.b, the time-varying length of the textile is given by

$$l(t) = \sqrt{l_0^2 + r(t)^2 + 2l_0r(t)\cos\phi} - R^2(t). \quad (1)$$

2.3 Whirling rotor

The rotor is modeled as a rigid disk mounted on a massless shaft which is supported by two perfect rotating bearings. During winding of the textile on or off the disk, the effective mass and radius of the disk vary. The mass $m(t)$ and radius $R(t)$ are assumed to be given as the paper by Bessonov⁽⁶⁾ and Cventianin⁽²⁾ as

$$m(t) = m_0 - R_1 \rho \omega t \quad (2)$$

$$R(t) = \left(R_0^2 - \frac{R_1 h \omega t}{\pi} \right)^{\frac{1}{2}} \quad (3)$$

where m_0 and R_0 are, respectively, the mass and radius of the disk without textile, ω is the angular velocity of the rotor and has negative value for the textile to be wound up, t is time, $R_1 = R_0 + \frac{\bar{h}}{2}$, and $\bar{h}$ is the average thickness of the textile. The magnitude of the unbalance is given by the distance $e(t) = \overline{CM}$, where C is the geometric center of the disk and M is its mass center, as given by Bessonov⁽⁶⁾:

$$e(t) = -\frac{2}{m_0} R_1^2 \rho \sin \frac{\omega t}{2}. \quad (4)$$

The coordinate r gives the magnitude of shaft deflection, and the time derivative $\dot{\phi}$ gives the whirling speed. The instantaneous angular locations of the unbalance with respect to the plane of shaft bending is given by β . Considering the rotor in synchronous vibration, the angle β remains constant. Thus, the whirl speed equals the shaft speed, $\dot{\phi} = \omega$.

For the case that the whirl speed equals the shaft speed, $\dot{\phi} = \omega$, as given by Tsai⁽¹³⁾

$$w_{tt} + 2\dot{x}w_{xt} + \ddot{x}w_x - \dot{\phi}^2 w - \left(\frac{T}{\rho} - \dot{x}^2\right)w_{xx} - \frac{3}{2}\frac{EA}{\rho}w_x^2 w_{xx} = -3\dot{x}\dot{\phi} - x\ddot{\phi}, \quad 0 < x < l(t) \quad (5.a)$$

$$\begin{aligned} \dot{r} + \frac{\dot{m}(t)}{m(t)}\dot{r} + \left[\frac{k_x}{m(t)}\cos^2\beta + \frac{k_y}{m(t)}\sin^2\beta - \omega^2\right]r \\ = e\omega^2\cos\beta + 2\dot{e}\omega\sin\beta + e\frac{\dot{m}(t)}{m(t)}\dot{\phi}\sin\beta \\ - \dot{e}\frac{\dot{m}(t)}{m(t)}\cos\beta \\ - \ddot{e}\cos\beta - g\sin\phi - \frac{T}{m(t)}[\cos\phi\cos\phi \\ - \sin\phi\sin\phi] \\ - \frac{(r+l_0\cos\phi)}{2m(t)l(t)}[\rho\dot{x}^2 + \rho l^2(t)\dot{\phi}^2] \\ - Tw_x^2(l(t), t) - \frac{1}{4}EAwx_x^4(l(t), t) \end{aligned} \quad (5.b)$$

and the boundary conditions are

$$w(0, t) = 0, \quad w(l(t), t) = 0 \quad (6.a, b)$$

The mass, inertia and unbalance magnitude of the rotor are time-varying when the textile is wound on or off, therefore the nonlinear governing equations (5.a, b) include the time-dependent mass $m(t)$, inertia $I(t)$ and eccentricity $e(t)$. The nonhomogeneous terms, $-3\dot{x}\dot{\phi} - x\ddot{\phi}$, in equation (5.a) are the whirling effects of the rotor on the textile vibration. The terms including $w_x(l(t), t)$ in equation (5.b) are the end effect at $x=l(t)$ of the textile vibration on the rotor whirling.

The terms containing EA in equations (5.a, b) are due to the geometric nonlinearity of the textile. If they are neglected for small amplitude transverse vibration of the textile, the governing equation (5.a) becomes linear. However, equation (5.b) is still nonlinear, due to the slope terms $w_x(l(t), t)$ at boundary $x=l(t)$. The emphasis is placed on the moving boundary condition of the coupled textile/rotor system. The connection point $x=l(t)$ is not specified and its position moves with time. The boundary position $x=l(t)$ will be solved simultaneously with equations (5.a, b).

In this paper, the longitudinal elastic deformation of the textile is neglected, so every point along the textile has the same axial traveling velocity $\dot{x}$ and

acceleration $\ddot{x}$, which are given by

$$\dot{x} = -R(t)\dot{\phi}, \quad 0 < x < l(t) \quad (7.a)$$

$$\ddot{x} = -\dot{R}(t)\dot{\phi} - R(t)\ddot{\phi}, \quad 0 < x < l(t) \quad (7.b)$$

The axial traveling velocity $\dot{x}$ is a positive value and the textile moves along the positive x axis direction. In the case of constant angular velocity, the radius $R(t)$ of the disk and the axial velocity $\dot{x}$ of the textile are nonlinear functions of time, $\dot{R}(t)$ is not equal to zero, and the axial traveling acceleration $\ddot{x}$ also exists.

3. Finite Difference Technique

For the problems with nonconservative mass, mass is generally not automatically conserved, that is, new mass elements enter domain and, depending on boundary conditions, some mass elements may leave the domain. Literature on an axially moving beam develops a more elegant finite element method with varying domain is given by Stylianou and Tabarrok^{(9),(10)}. In this paper, the finite difference technique is developed to solve the coupled textile/rotor problem with time-dependent length.

The explicit forward scheme is adopted in finite difference method. Since the textile length is time-dependent, we let the virtual string length l_T , which is constant and larger than the true length $l(0)$ at $t=0$, be divided into $n+1$ fixed number of grid point (see Fig. 1.c). Choosing the suitable step sizes h and k such that $x_i = (i-1)h$, $i=1, \dots, n+1$ and $t_j = (j-1)k$, $j=1, 2, \dots$, and denoting by $w(x_i, t_j)$ the approximated values of $w(x, t)$ at the points (ih, jk) for $i=1, 2, \dots, n+1$; $j=1, 2, \dots$, we discretize the term $\frac{\partial^2 w}{\partial t \partial x}(x_i, t_j)$ by the backward-difference quotient in time and central differences in space. The other terms are obtained by using the centered-difference quotients for the partial derivatives in the space and time.

Substituting these into (5.a, b), the most advanced time-step approximation $w(x_i, t_{j+1})$ and $r(t_{j+1})$, are obtained, respectively, as

$$\begin{aligned} w(x_i, t_{j+1}) = k^2 \left\{ \left[\frac{(T/\rho - \dot{x}^2)}{h^2} - \frac{\ddot{x}}{2h} \right] w(x_{i+1}, t_j) \right. \\ + \left[\frac{2\dot{x}}{hk} - \frac{1}{k^2} \right] w(x_i, t_{j-1}) \\ + \left[\frac{2}{k^2} - \frac{2\dot{x}}{hk} + \dot{\phi}^2 - \frac{2(T/\rho - \dot{x}^2)}{h^2} \right] w(x_i, t_j) \\ + \left[\frac{2(T/\rho - \dot{x}^2)}{h^2} + \frac{\ddot{x}}{2h} + \frac{2\dot{x}}{hk} \right] w(x_{i-1}, t_j) \\ + [(w^2(x_{i+1}, t_j) - 2w(x_{i+1}, t_j)w(x_{i-1}, t_j) \\ + w^2(x_{i-1}, t_j))(w(x_{i+1}, t_j) - 2w(x_i, t_j) \\ + w(x_{i-1}, t_j))] \frac{3EA}{8\rho h^3} - \frac{2\dot{x}}{hk} w(x_{i-1}, t_{j-1}) - 3\dot{x}\dot{\phi} \\ \left. - x_i\ddot{\phi} \right\}, \quad 2 \leq i \leq n, \quad 2 \leq j \end{aligned} \quad (8.a)$$

$$\begin{aligned}
r(t_{j+1}) = & \left(\frac{2mk^2}{2m + \dot{m}k} \right) \\
& \times \left\{ \left[\frac{2}{k^2} - \frac{k_x}{m} \cos^2 \beta - \frac{k_y}{m} \sin^2 \beta + \omega^2 \right] r(t_j) \right. \\
& + \left[\frac{\dot{m}}{2m} - \frac{1}{k^2} \right] r(t_{j-1}) + e\omega^2 \cos \beta + 2\dot{e}\omega \sin \beta \\
& - g \sin \phi - \frac{T}{m} [\cos \phi \cos \psi - \sin \phi \sin \psi] \\
& - \frac{(r + l_0 \cos \psi)}{2ml} [\rho \dot{x}^2 + \rho l^2(t_j) \dot{\phi}^2 \\
& \left. - Tw_x^2(l(t_j), t_j) - \frac{1}{4} EAw_x^4(l(t_j), t_j) \right] \Big\}, \quad 2 \leq j
\end{aligned} \quad (8.b)$$

where the nonlinear term is shown in (8.a) and the terms $m = m(t_j)$, $e = e(t_j)$ and the slope at $x = l(t_j)$ is $w_x(l(t_j), t_j)$.

It should be noted that the grid number ir is needed in equation (8.a). However, we provided the total grid number is $n+1$. In Fig. 1.c, the textile length is $l(t_j)$ which locate in the interval (x_{ir}, x_{ir+1}) , the difference equation of the slope at $x = l(t_j)$ is

$$w_x(l(t), t) = \frac{w_{ir} - w(l(t_j), t_j)}{x_{ir} - l(t_j)} = \frac{w_{ir}}{x_{ir} - l(t_j)} \quad (9)$$

For the special case, the derivative $w_x(l(t), t)$ may be infinity when the time-dependent length $l(t_j)$ locates at the grid position x_{ir} . Therefore, we modify the above equation as

$$w_x(l(t), t) = \frac{w_{ir-1} - w(l(t_j), t_j)}{x_{ir-1} - l(t_j)} = \frac{w_{ir-1}}{x_{ir-1} - l(t_j)} \quad (10)$$

The boundary conditions give $w(x_1, t_j) = w(l(t), t_j) = 0$, $j = 1, 2, \dots$. The initial conditions give $w(x_i, t_1) = f(x_i)$, $w_t(x_i, t_1) = g(x_i)$ and $r(t_1), \dot{r}(t_1)$. Equations (8.a) and (8.b) imply that the most advanced time-step $(j+1)$ st requires values from the j th and $(j-1)$ th time steps. This produces a minor starting problem (Burden and Faires⁽¹¹⁾) since values for $j=1$ are given by the initial conditions, but values for $j=2$, which are needed in equations (8.a, b) to compute $w(x_i, t_3)$, $i = 1, 2, \dots, n+1$ and $r(t_3)$, must be obtained from the initial-velocity conditions $w_t(x_i, t_1) = g(x_i)$, $i = 1, 2, \dots, n+1$ and $\dot{r}(t_1)$. Following Burden and Faires⁽¹¹⁾, we obtain

$$\begin{aligned}
w(x_i, t_2) = & f(x_i) + kw_t(x_i, t_1) + \frac{k^2}{2} w_{tt}(x_i, t_1) \\
= & f(x_i) + kg(x_i) + \frac{k^2}{2} \left\{ \left(\frac{T}{r(t_1)h^2} - \frac{\dot{x}^2}{h^2} \right) \right. \\
& \times [f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))] + \dot{\phi}^2 w(x_i, t_1) \\
& - \frac{2\dot{x}}{2h} [g(x_{i+1}) - g(x_{i-1})] + \frac{\ddot{x}}{2h} [f(x_{i+1}) - f(x_{i-1})] \\
& \left. - 3\dot{x}\dot{\phi} - x_i\ddot{\phi} \right\}, \quad 2 \leq i \leq ir, \quad 2 \leq j
\end{aligned} \quad (11.a)$$

and

$$\begin{aligned}
r(t_2) = & r(t_1) + k\dot{r}(t_1) + \frac{k^2}{2} \ddot{r}(t_1) \\
= & r(t_1) + k\dot{r}(t_1) + \frac{k^2}{2} \left\{ \frac{-\dot{m}}{m} \dot{r}(t_1) \right.
\end{aligned}$$

$$\begin{aligned}
& - \left(\frac{k_x}{m} \cos^2 \beta + \frac{k_y}{m} \sin^2 \beta - \omega^2 \right) r(t_1) \\
& + e\omega^2 \cos \beta + 2\dot{e}\omega \sin \beta - g \sin \phi \\
& - \frac{T}{m} [\cos \phi \cos \psi - \sin \phi \sin \psi] \\
& - \frac{(r(t_1) + l_0 \cos \psi)}{2ml} [\rho \dot{x}^2 - \rho l^2(t_1) \dot{\phi}^2 \\
& \left. - Tw_x^2(l(t_1), t_1) - \frac{1}{4} EAw_x^4(l(t_1), t_1) \right] \Big\}, \quad 2 \leq j
\end{aligned} \quad (11.b)$$

The governing differential equations (5.a, b) is transformed into a system of nonlinear difference equations (8.a, b; 11.a, b). Since the length $l(t)$ of the string is time-dependent, special programming considerations are necessary.

To solve the finite difference equations (5.a, b), the special techniques are made as following:

(i) It is seen that the step size h is held constant. The textile length $l(t)$ is time-dependent when the textile wound either on or off the rotor. Thus the grid number of the textile changes with time.

(ii) The obvious approach of using fixed-size grid and changing its grid number requires a large number of small grid. It is seen from Fig. 1.c that actual grid number ir is needed for the time-dependent length while we provide the total grid number $n+1$. For saving the computational time, we choose $n+1$ close to ir .

(iii) To solve the differential equation (5.a) of the textile, the length $l(t)$ must be solved first. However, the length $l(t)$ is coupled with the rotor whirling amplitude $r(t)$ and shown in Eq. (1). Therefore, the Eq. (5.b) must be solved first to obtain $r(t)$. Then, we obtain $l(t)$ from Eq. (1) and $w(x, t)$ from Eq. (5.a).

(iv) Similarly, we obtain $r(t_{j+1})$ from difference Eq. (8.b) and $l(t_j)$ from Eq. (1). In each time step, it is necessary to obtain the grid number ir by comparing the time-dependent length $l(t_j)$ with the grid position x_{ir} . Then, we solve $w(x_i, t_{j+1})$ from the difference equation (8.a).

4. Illustrative Examples

The examples given here are chosen to discuss the dynamic responses of the textile/rotor system when the rotary angular velocity ω is accelerated, decelerated and taken as a periodic perturbation. Finally, we also make a comparison between the linear and nonlinear effects on the transient amplitudes of the textile, whirling deflections of rotor and the time-dependent textile length and disk radius. It is seen that the amplitudes of the textile/rotor system are governed by two nonlinearly coupled differential equations (5.a, b). Equation (5.a) is a wave equation and the explicit finite-difference method for such a

Fig. 2 Comparison the numerical results for various length step sizes. $h=0.0025$ m (—), $h=0.005$ m (---), $h=0.01$ m (-.-). (a) Middle deflection of the textile (b) Deflections of the shaft (c) Time-dependent length $l(t)$ of the textile (d) Time-dependent radius $R(t)$ of the disk (e) Time history of angular velocity $\dot{\phi}(t)$ (f) Time history of angular acceleration $\ddot{\phi}(t)$

equation has stability problems. In fact, it is necessary that $\lambda = \sqrt{\frac{T}{\rho}} \frac{k}{h} \leq 1$ for the method to be stable (Issaacson and Keller⁽¹²⁾).

The parameter values are $T=100$ N, $\rho=1$ kg/m, $l_0=1$ m, $m_0=4.95$ kg, $R_0=0.1$ m, $m=200$ kg, $h=0.02$ m, $e(t) = -(2/m_0)R_0^2 \rho \sin(\omega t/2)$ and the values of rotor stiffness are chosen as $k_x=k_y=493\,430$ N/m. We consider the textile and rotor with the following initial conditions: $f(x_i)=g(x_i)=0$, $i=1, 2, \dots, ir$, and $r(t_1)=\dot{r}(t_1)=0$. The zero initial conditions are given

Fig. 3 Influence of angular speed of the rotor on the transient amplitudes for the cases, $\omega=-\pi$ (---); $\omega=-2\pi$ (—). (a) Middle deflection of the textile (b) Deflections of the shaft (c) Time-dependent length $l(t)$ of the textile (d) Time-dependent radius $R(t)$ of the disk (e) Time history of angular velocity $\dot{\phi}(t)$ (f) Time history of angular acceleration $\ddot{\phi}(t)$

for us to obtain the motion-induced vibration.

Figure 2 shows the compared results for the length step size $h=0.01$ m, 0.005 m and 0.0025 m, respectively, and the time step size $k=0.0001$ sec. It is seen that the length step size affects the middle deflection of the textile (Fig. 2.a) but has no effect on the deflections of the shaft (Fig. 2.b), the time-dependent length $l(t)$ of the textile (Fig. 2.c), the radius $R(t)$ of the disk (Fig. 2.d), the time histories of angular velocity $\dot{\phi}(t)$ (Fig. 2.e), and angular acceleration $\ddot{\phi}(t)$ (Fig. 2.f). For saving time and analyzing the

qualitative features of whole system, we take $h=0.005$ m in our following investigations in spite of the fact that much less h will obtain better accuracy. For the case with the step sizes $k=0.0001$ sec and $h=0.005$ m, we have $\lambda=0.005<1$, and the Eqs.(8.a, b) are stable.

The results with the angular velocities $\omega=-\pi$ and $\omega=-2\pi$ are compared in Fig. 3. The transient curves of the textile at center are shown in Fig. 3.a. It is noted that the amplitude builds up and then diminishes in a regular pattern. A beating phenomenon may occur, since the forcing frequency is close to, but not exactly equal to, the nature frequency of the string system. It is found that the amplitudes at $\omega=-2\pi$ are larger than those at $\omega=-\pi$. The transient vibrations of the shaft are shown in Fig. 3.b. Figure 3.c shows that time-dependent textile length which decreases with time. It seems that the decreasing is not linear. Figure 3.d shows the rotor radius $R(t)$ increases with time. Figures 3.e-f show that angular velocity $\dot{\phi}$ and angular acceleration $\ddot{\phi}$ of the string, and both have beating phenomenon. These results give a reasonably good picture of the qualitative features of the coupled system.

In Fig. 4, we make a comparison of the rotary angular velocity $\omega=-10+\cos(\Omega_f t)$ by setting $\Omega_f=\pi c_2/l_0\sqrt{1-v_0/c_2}$ (principal resonance) and $2\pi c_2/l_0\sqrt{1-v_0/c_2}$ (parametric resonance), respectively. The other parameters are the same in Fig. 3. It is observed that the amplitudes (Figs. 4.a, b) of both the textile and rotor, the time dependent $l(t)$ (Fig. 4.c) and $R(t)$ (Fig. 4.d), and the angular velocity (Fig. 4.e) and acceleration (Fig. 4.f) are larger and are fluctuated more excitedly when the rotary angular velocity is $\Omega_f=2\pi c_2/l_0\sqrt{1-v_0/c_2}$. It is seen that the system is unstable.

In Fig. 5, we show the comparison of the rotary angular velocity $\omega=-10+\omega_1\cos(\Omega_f t)$ by setting $\Omega_f=\pi c_2/l_0\sqrt{1-v_0/c_2}$, and $\omega_1=0.5$, $\omega_1=1$ and $\omega_1=1.5$, respectively. The other parameters are the same as in Fig. 3. Obviously, as the perturbation amplitude ω_1 increases, the fluctuated amplitudes increase. We note that not only the transient amplitudes of textile become larger but also the transient deflections of that shaft have beating phenomena. It is due to the fact that the textile and rotor are coupled.

In Fig. 6, we consider the effect of the nonlinear term of the textile and rotor. The curves are obtained by making the rotary angular speed of the shaft equal to -2π and the nonlinear term $EA=900$ T. The other parameters are the same in Fig. 3. It is observed that the amplitudes of the textile (Fig. 6.a) are larger and the period of oscillation is shorter when the nonlinearity is included. Figure 6.b shows the transient magnitudes of shaft deflection, which are the same for both

Fig. 4 Comparison the dynamic responses by taking the rotary angular velocity $\omega=-10+\cos(\Omega_f t)$, $\Omega_f=\pi c_2/l_0\sqrt{1-v_0/c_2}$ (—) and $2\pi c_2/l_0\sqrt{1-v_0/c_2}$ (-----). (a) Middle deflection of the textile (b) Deflections of the shaft (c) Time-dependent length $l(t)$ of the textile (d) Time-dependent radius $R(t)$ of the disk (e) Time history of angular velocity $\dot{\phi}(t)$ (f) Time history of angular acceleration $\ddot{\phi}(t)$

the linear and nonlinear cases. It is the fact that the nonlinear coupling effect between the textile and rotor is small for the motion-induced vibration case. Figure 6.c shows the time-dependent textile length. Figure 6.d shows the time-dependent radius of disk. These curves are also the same for both the linear and nonlinear cases.

5. Conclusion

A model of the coupled textile/rotor system that includes the transverse textile vibration and rotor

Fig. 5 Comparison the dynamic responses by taking the rotary angular velocity $\omega = -10 + \omega_1 \cos(\Omega_f t)$, $\Omega_f = \pi c_2 / l_0 \sqrt{1 - v_0/c_2}$, $\omega_1 = 0.5$ (---), $\omega_1 = 1$ (-----) and $\omega_1 = 1.5$ (—). (a) Middle deflection of the textile (b) Deflections of the shaft (c) Time-dependent length $l(t)$ of the textile (d) Time-dependent radius $R(t)$ of the disk (e) Time history of angular velocity $\dot{\phi}(t)$ (f) Time history of angular acceleration $\ddot{\phi}(t)$

whirling is successfully simulated by the finite difference method. Especially, we develop a specific technique that the grid number changes while its step size remains fixed. The non-constant traveling speed and the nonlinear terms are considered in the textile. The rotary angular velocities with acceleration, deceleration and periodic perturbation are also considered in the rotor. The finite difference numerical results give reasonable demonstration for the nonlinear coupled system. The time-dependent length of the textile and the radius of the rotor, the angular velocity and

Fig. 6 Nonlinear effects on the transient amplitudes and whirling deflection. $EA = 900$ T (—), $EA = 0$ (-----). (a) Middle deflection of the textile (b) Deflections of the shaft (c) Time-dependent length $l(t)$ of the textile (d) Time-dependent radius $R(t)$ of the disk

acceleration are also included in the numerical results.

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