Obtaining a Model and Control for a Rigid Body Motion System*

(Obtaining Inertia Parameters of a Rigid Body and Electromagnetic Characteristics for a Magnetic Levitation System)

Takashi ANDO** and Takuzo IWATSUBO***

This paper proposes a control method of a rigid body motion system which includes unknown nonlinearity by using motion learning. Defining the motion learning as achievement of objective motion control by obtaining the model of motion system, the proposed control structure consists of feedback controllers to stabilize the unstable motion system, a motion learning system to obtain nonlinear model and to linearize the motion system by using the obtained nonlinear model, and an inverse motion system to generate inputs to be applied to the linearized motion system. The obtained model is described as series of weighted basis functions, and the explicit learning rules of weights are derived by using Popov's hyperstable theory. The tuning rules of feedback gains are determined by using the obtained nonlinear models. Further, this paper shows the high performance of the proposed method by demonstrating an experiment of the tracking desired trajectory in the electromagnetic levitation and positioning system.

Key Words: Motion Control, Adaptive Control, Learning, Positioning, Electromagnetic Actuator, Nonlinear System, Inverse Dynamics

1. Introduction

Feedforward control is necessary for rapid and accurate tracking control with constituting inverse dynamics of a rigid body motion system. However, in the case of using electromagnetic force as driving force, the driving characteristics have nonlinearities depending on the positions or angles. Even more, because those characteristics vary according to heating, it becomes much more difficult to obtain the model of the rigid body motion system and constitute inverse dynamics.

Viewing such a background, in the field of adaptive control, compensating nonlinearity and obtaining an inverse dynamics have been tried, and

those trial have advanced to learning control of a motion system. Tsuji et al.⁽¹⁾ proposed an adaptive control method using neural networks for nonlinear system. Sanner et al.⁽²⁾ proposed an adaptive control method for nonlinear system describing nonlinearity as networks using Gaussian Radial Basis functions (GRBF). Hao et al.⁽³⁾ tried an adaptive nonlinear control for magnetic levitation using GRBF networks. Kemal et al.⁽⁴⁾ proposed an adaptive method of obtaining a model rapidly from multi-model for a robot manipulation.

However, researches in these fields, both obtaining a model of the objective control system and nonlinear compensation are executed by a single processors. This fact burden the processor with many tasks. In the future, more speed are demanded, and even more in the case that multi-variable system should be handled, more and more rapid processor is needed. Then, it is forecasted that the control performance strongly depend on expensive and high performance hardware. Therefore, such usual methods in which improvement of hardware is expected leave problems of cost and reliability.

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** Hyogo Prefectural Institute of Industrial Research, 3-1-12 Yukihiro-cho, Suma-ku, Kobe 654-0037, Japan. E-mail: ando@hyogo-kg.go.jp

*** Faculty of Engineering, Kobe University, 1-1 Rokko-dai-cho, Nada-ku, Kobe 657-0013, Japan

Then, aiming improvement of control performance for uncertain motion system, increase in speed of control system and the reduction in burden of processor, this paper propose a method of 1) Obtaining a nonlinear model present at drive system, 2) Linearization of the motion system using obtained nonlinear model, 3) Realization of high performed motion control by constitution of inverse dynamics of linearized motion system. In this study, motion learning is defined as an achievement of an objective motion control by obtaining a motion model and constitution of an inverse dynamics using the obtained model. Further, applying the proposed method to a magnetic levitation, tracking control of levitated table following objective trajectory is experimented. This experiment confirms the table follows the objective trajectory rapidly and shows the effectiveness of the proposed method.

2. Formulation of a Rigid Body Motion System

2.1 Formulation of a motion equation

In this paper, a problem of positioning a rigid body by magnetic force is considered. Figure 1 shows an illustration of the motion system consisting of a rigid body, electromagnets and noncontact displacement sensors. Using the symbols in Fig. 1, the motion equation is formulated as follows;

$$\left. \begin{aligned} m\ddot{x} &= f_1 + f_2 \\ I_z\ddot{\theta} &= -(l_M - y_G)f_1 + (l_M + y_G)f_2 - (l_M + x_G)f_3 \\ m\ddot{y} &= f_3 \end{aligned} \right\} \quad (1)$$

where, x , y and θ are the positions of the x -direction, y -direction, and the rotation around z -axis respectively; m is the mass of the rigid body; I_z is the inertia moment around the z -axis passing through the center of gravity; l_M is the distance between the centers of the magnets and coordinate axis; l_s is the distance between the centers of the sensors and the coordinate axis; x_G and y_G indicate coordinates of the center of gravity of the rigid body; f_i is the magnetic force of electromagnets given as the following equation;

Fig. 1 A rigid body motion system

$$f_i = k_{Pi}(y_i)i_{Pi}^2 - k_{Ni}(y_i)i_{Ni}^2 \quad (2)$$

where y_i is the output of each noncontact displacement sensor; $k_{Pi}(y_i)$ and $k_{Ni}(y_i)$ are functions that describe nonlinearity of magnets depending on gaps. I_{Pi} and I_{Ni} are the electric current that drive electromagnets EM_{Pi} and EM_{Ni} respectively. Since the time constants are much smaller than those of motion system, the influence of coil inductance is neglectable. Suppose the resistance of electromagnets EM_{Pi} and EM_{Ni} are indicated by R_{Pi} and R_{Ni} respectively, the relation between driving electric current and electric potentials of electromagnets follows Ohm's law; $I_{Pi} = v_{Pi}/R_{Pi}$, $I_{Ni} = v_{Ni}/R_{Ni}$. Then, v_{Pi} and v_{Ni} are given as the following equations;

$$v_{Pi} = p_i(v_{Bi} + v_i)^{1/2} \quad (3)$$

$$v_{Ni} = p_i(v_{Bi} - v_i)^{1/2} \quad (4)$$

where v_{Pi} and v_{Ni} are the outputs of the controller to control electromagnets; v_{Bi} is constant bias electric potential; p_i is rate of electric potential. From Ohm's law and Eqs. (2)–(4), the relation between v_i and f_i is given as the following equations;

$$f_i = k_i(y_i)v_i + f_{Di}(y_i) \quad (5)$$

$$k_i(y_i) = p_i^2 \left(\frac{k_{Pi}(y_i)}{R_{Pi}^2} + \frac{k_{Ni}(y_i)}{R_{Ni}^2} \right) \quad (6)$$

$$f_{Di}(y_i) = p_i^2 v_B \left(\frac{k_{Pi}(y_i)}{R_{Pi}^2} - \frac{k_{Ni}(y_i)}{R_{Ni}^2} \right) \quad (7)$$

By these equations, it is obvious that f_i includes nonlinearity $k_i(y_i)$ and $f_{Di}(y_i)$ that depend on the position y_i of the rigid body. In order to introduce the motion equation for y_i , the coordinates are transformed as the following equation;

$$\begin{bmatrix} x \\ l_s\theta \\ y \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ -0.5 & 0.5 & 0 \\ -0.5 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (8)$$

where θ in Eq.(8) is multiplied by l_M so that the dimensions of elements are the same. Using Eq.(8), The motion equation (1) is transformed as the following equation;

$$M\ddot{\mathbf{y}} = \mathbf{L}(\mathbf{y})\mathbf{K}(\mathbf{y})\mathbf{v} + \mathbf{L}(\mathbf{y})\mathbf{f}_D(\mathbf{y}) \quad (9)$$

where $\mathbf{y} = [y_1 \ y_2 \ y_3]^T$; $\mathbf{K}(\mathbf{y}) = \text{diag}\{k_1(y_1), k_2(y_2), k_3(y_3)\}$; $\mathbf{v} = [v_1 \ v_2 \ v_3]^T$; $\mathbf{f}_D = [f_{D1}(y_1) \ f_{D2}(y_2) \ f_{D3}(y_3)]^T$. $\mathbf{M}$ is an inertia matrix and $\mathbf{L}$ is a matrix determined by the location of electromagnets described as the following equations;

$$\mathbf{M} = m \begin{bmatrix} 0.5 & 0.5 & 0 \\ -0.5I_z/ml_s l_M & 0.5I_z/ml_s l_M & 0 \\ -0.5 & 0.5 & 1 \end{bmatrix} \quad (10)$$

$$\mathbf{L}(\mathbf{y}) = \begin{bmatrix} 1 & 1 & 0 \\ -1 + y_G/l_M & 1 + y_G/l_M & -1 - x_G/l_M \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

By the way, it is easy to measure m precisely, however, I_z , x_G and y_G are difficult to measure. Therefore, $\mathbf{M}$ and $\mathbf{L}$ are unknown or uncertain.

2.2 Nonlinear feedback system

In this section the feedback system to stabilize the motion system is introduced. A feedback rule of the control output $\mathbf{v}$ is given as the following equation for the motion equation;

$$\mathbf{v} = -\mathbf{K}_D \dot{\mathbf{y}} - \mathbf{K}_P(\mathbf{y} - \mathbf{u} + \mathbf{u}_B) \quad (12)$$

where $\mathbf{K}_D = \text{diag}\{k_{D1}, k_{D2}, k_{D3}\}$ is differential gain; $\mathbf{K}_P = \text{diag}\{k_{P1}, k_{P2}, k_{P3}\}$ is proportional gain; $\mathbf{u}_B = [u_{B1}, u_{B2}, u_{B3}]^T$ is offset term of drift force $\mathbf{f}_D$. By substituting Eq.(12) into Eq.(9), nonlinear feedback system is derived as the following equation;

$$\begin{aligned} \mathbf{J}(\mathbf{y}) \ddot{\mathbf{y}} + \mathbf{K}_D \mathbf{K}(\mathbf{y}) \dot{\mathbf{y}} + \mathbf{K}_P \mathbf{K}(\mathbf{y})(\mathbf{y} - \mathbf{u}) \\ = \mathbf{f}_D(\mathbf{y}) - \mathbf{K}_P \mathbf{K}(\mathbf{y}) \mathbf{u}_B \end{aligned} \quad (13)$$

where

$$\begin{aligned} \mathbf{J}(\mathbf{y}) &= \mathbf{L}(\mathbf{y})^{-1} \mathbf{M} \\ &= 0.5m \begin{bmatrix} 1 + 0.5\nu_1(\mathbf{y}) & -0.5\nu_2(\mathbf{y}) & -\nu_3(\mathbf{y}) \\ -0.5\nu_1(\mathbf{y}) & 1 + 0.5\nu_2(\mathbf{y}) & \nu_3(\mathbf{y}) \\ -1 & 1 & 2 \end{bmatrix} \end{aligned} \quad (14)$$

$$\begin{bmatrix} \nu_1(\mathbf{y}) \\ \nu_2(\mathbf{y}) \\ \nu_3(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} I_z/m l_s l_M + y_G/l_M - x_G/l_M \\ I_z/m l_s l_M - y_G/l_M - x_G/l_M \\ 1 - x_G/l_M \end{bmatrix} \quad (15)$$

3. Modeling of Nonlinearity and Tuning Gains

3.1 Linearization by gain tuning

At first, linearization of the motion system by gain tuning is introduced. Suppose it is possible to tune the gains as the following equations using constant matrices $\mathbf{A} = \text{diag}\{a_1, a_2, a_3\}$, $\mathbf{B} = \text{diag}\{b_1, b_2, b_3\}$;

$$\mathbf{K}_D = \mathbf{A} \mathbf{K}^{-1}(\mathbf{y}) \quad (16)$$

$$\mathbf{K}_P = \mathbf{B} \mathbf{K}^{-1}(\mathbf{y}) \quad (17)$$

$$\mathbf{u}_B = \mathbf{B}^{-1} \mathbf{f}_D(\mathbf{y}) \quad (18)$$

the nonlinearities $\mathbf{K}(\mathbf{y})$, $\mathbf{f}_D(\mathbf{y})$ in Eq.(13) are linearized and the nonlinear feedback system becomes the following equation;

$$\mathbf{J}(\mathbf{y}) \ddot{\mathbf{y}} + \mathbf{A} \dot{\mathbf{y}} + \mathbf{B}(\mathbf{y} - \mathbf{u}) = 0 \quad (19)$$

By constituting the inverse dynamics for this control system, it is possible to generate the input $\mathbf{u}$ from objective trajectory $\mathbf{y}_M = [y_{M1}, y_{M2}, y_{M3}]^T$;

$$\mathbf{u} = \mathbf{B}^{-1}(\mathbf{J}(\mathbf{y}) \ddot{\mathbf{y}}_M + \mathbf{A} \dot{\mathbf{y}}_M + \mathbf{B} \mathbf{y}_M) \quad (20)$$

Substituting Eq.(20) into (19), error system is given as the following equation;

$$\mathbf{J}(\mathbf{y}) \ddot{\mathbf{e}} + \mathbf{A} \dot{\mathbf{e}} + \mathbf{B} \mathbf{e} = 0 \quad (21)$$

where $\mathbf{e} = \mathbf{y}_M - \mathbf{y}$. By determining $\mathbf{A}$ and $\mathbf{B}$ such that Eq.(21) is stable, it is possible to obtain that $\mathbf{e} \rightarrow 0$. Therefore, the trajectory of motion system coincide with the objective trajectory. However, since $\mathbf{K}(\mathbf{y})$, $\mathbf{f}_D(\mathbf{y})$, $\mathbf{M}$ and $\mathbf{L}(\mathbf{y})$ are unknown or uncertain, it is impossible to realize Eqs. (16)–(18). Then, a method for obtaining the parameters and the nonlinearities as models will be mentioned below.

3.2 Modeling of nonlinearity

Here, the method of obtaining nonlinearity $\mathbf{K}(\mathbf{y})$, $\mathbf{f}_D(\mathbf{y})$, $\mathbf{M}$ and $\mathbf{L}(\mathbf{y})$ as models is mentioned. Suppose it

Fig. 2 The shapes and the locations of the basis functions

is possible to describe $\mathbf{K}(\mathbf{y})$ and $\mathbf{f}_D(\mathbf{y})$ as the following equations, using continuous linear independent basis functions $\phi_j(y_i)$ ($j=1, \dots, n$);

$$k_i(y_i) = \sum_{j=1}^n w_{ij}^K \phi_j(y_i) \quad (22)$$

$$f_{Di}(y_i) = \sum_{j=1}^n w_{ij}^D \phi_j(y_i) \quad (23)$$

where w_{ij}^D and w_{ij}^K are weight coefficients. For such description using basis functions, the conditions below are necessary. (a) The nonlinearity is bounded. (b) The nonlinearity is continuous and smooth. In general an actual motion system can not include infinite parameters. Therefore, condition (a) is satisfied naturally. In the magnetic levitation system, there are no frictions. Therefore, condition (b) is also satisfied. Selecting basis functions that have their maximum value at their center θ_i and decrease smoothly to 0 as departing from their center shown in Fig. 2 and locating $\phi_j(y_i)$ properly, it is possible to describe nonlinearity as Eqs. (22) and (23). Here, the basis function is given as the following equation;

$$\phi_j(y_i) = \left\{ 1 - \frac{1}{1 + e^{\frac{1}{\sigma}(y_i - \theta_j)}} \right\} \frac{1}{1 + e^{\frac{1}{\sigma}(y_i - \theta_j)}} \quad (24)$$

where σ is a constant that determines the steepness of $\phi_j(y_i)$. The reason why the basis functions are described as above will be mentioned in Section 4.2. Describing nonlinearity as Eqs. (22) and (23), the problem of obtaining the models is replaced by estimating the weight coefficients w_{ij}^D and w_{ij}^K . The estimates of w_{ij}^D and w_{ij}^K are indicated by $\hat{w}_{ij}^D$ and $\hat{w}_{ij}^K$, respectively. $\hat{k}_i(y_i)$ and $\hat{f}_{Di}(y_i)$ are obtained as

$$\hat{k}_i(y_i) = \sum_{j=1}^n \hat{w}_{ij}^K \phi_j(y_i) \quad (25)$$

$$\hat{f}_{Di}(y_i) = \sum_{j=1}^n \hat{w}_{ij}^D \phi_j(y_i) \quad (26)$$

3.3 Gain tuning rule

Here, gain tuning rule so that $\mathbf{y}$ coincides with $\mathbf{y}_M$ will be mentioned. Using nonlinear model Eqs. (25) and (26), tuning rule of $\mathbf{K}_D(\mathbf{y})$, $\mathbf{K}_P(\mathbf{y})$ and $\mathbf{u}_B$ are given as the following equations;

$$k_{Di} = \frac{a_i}{\hat{k}_i(y_i)} \quad (27)$$

$$k_{Pi} = \frac{b_i}{\hat{k}_i(y_i)} \quad (28)$$

$$u_{Bi} = \frac{\hat{f}_{Di}(y_i)}{b_i} \quad (29)$$

or, $K_D = A\tilde{K}(y)^{-1}$, $K_P = B\tilde{K}(y)^{-1}$, $u_B = B^{-1}\hat{f}_D(y)$.

In order to constitute inverse dynamics, the inertia moment is estimated by the following equation;

$$\hat{I}_Z = \hat{w}_0^l m l_M l_s \quad (30)$$

Since the center of gravity of the rigid body moves according to its position, it is estimated by the following equations;

$$\hat{x}_G = x + \hat{w}_0^x l_s \quad (31)$$

$$\hat{y}_G = y + \hat{w}_0^y l_s \quad (32)$$

4. Obtaining a Model

4.1 Introduction of error system and its asymptotical hyperstable condition

Here, the error system between the objective trajectory and the motion trajectory is introduced.

First, input u of the motion system is given by the inverse dynamics as the following equation;

$$u = B^{-1}(\tilde{J}(y)\dot{y}_M + A\dot{y}_M + B y_M) \quad (33)$$

where $\tilde{J}(y)$ is estimation of $J(y)$. Substituting Eq. (33) into Eq. (13), the error system is gotten as the following equations;

$$J(y)\ddot{e} + A\dot{e} + Be = -r \quad (34)$$

$$r = \tilde{J}(y)\dot{y}_M - \tilde{K}(y)v_{FB} - \tilde{f}_D(y) \quad (35)$$

Error output is given as the following equation;

$$\varepsilon = e + C\dot{e} \quad (36)$$

where, $v_{FB} = \text{diag}\{v_{FB1}, v_{FB2}, v_{FB3}\}$,

$$v_{FBi} = -\frac{1}{\hat{k}_i(y_i)}\{a_i\dot{y}_i + b_i(y_i - u_i) + \hat{f}_{Di}(y_i)\} \quad (37)$$

$$\tilde{K}(y) = \hat{K}(y) - K(y) \quad (38)$$

$$\tilde{f}_D(y) = \hat{f}_D(y) - f_D(y) \quad (39)$$

$$\begin{aligned} \tilde{J}(y) &= \hat{J}(y) - J(y) \\ &= \frac{m}{4} \begin{bmatrix} \tilde{v}_1(y) & -\tilde{v}_2(y) & -2\tilde{v}_3(y) \\ -\tilde{v}_1(y) & \tilde{v}_2(y) & 2\tilde{v}_3(y) \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \quad (40)$$

where $\tilde{*} = * - \hat{*}$.

According to Popov's hyperstable theory⁽⁷⁾, the error system converge to 0 in the case that the following integral inequality is satisfied, and that C in Eq. (36) is determined so that the transfer function of Eq. (34) is strictly positive real.

$$\int_0^t \varepsilon^T r dt \geq -\delta^2 \quad (41)$$

where ε^T is transposed vector of ε , δ^2 is a positive real constant independent of t .

4.2 Parameter learning rule for obtaining nonlinear model

Here, parameter learning rule necessary for obtaining the model will be mentioned. The parameter learning rules to model nonlinearities are given as the following equations;

$$\dot{\hat{w}}_{ij}^K = -\alpha_K \varepsilon_i \phi_j(y_i) v_{FBi} \quad (42)$$

$$\dot{\hat{w}}_{ij}^D = -\alpha_D \varepsilon_i \phi_j(y_i) \quad (43)$$

where α_K and α_D are positive real constants. Parameter learning rule to estimate the inertia moment and the center of gravity is given as the following equations;

$$\dot{\hat{w}}_0^I = \alpha_I \ddot{\theta}_M (\varepsilon_1 - \varepsilon_2) \quad (44)$$

$$\dot{\hat{w}}_0^X = \alpha_X \dot{y}_M (\varepsilon_1 - \varepsilon_2) \quad (45)$$

$$\dot{\hat{w}}_0^Y = \alpha_Y \ddot{x}_M (\varepsilon_1 - \varepsilon_2) \quad (46)$$

where α_I , α_X and α_Y are positive real constants.

Lemma The integral inequality (41) is satisfied by the learning rules (42)–(46).

Proof Substituting (35) into Eq. (41),

$$\int_0^t \varepsilon^T r dt = \int_0^t \varepsilon^T \{\tilde{J}(y)\dot{y}_M - \tilde{K}(y)v_{FB} - \tilde{f}_D(y)\} dt \quad (47)$$

For the first term;

$$\begin{aligned} \int_0^t \varepsilon^T \tilde{J}(y)\dot{y}_M dt &= \int_0^t \{\underbrace{\tilde{w}_0^I \ddot{\theta}_M (\varepsilon_1 - \varepsilon_2)}_{\dot{\tilde{w}}_0^I / \alpha_I} \\ &\quad + \underbrace{\tilde{w}_0^X \dot{y}_M (\varepsilon_1 - \varepsilon_2)}_{\dot{\tilde{w}}_0^X / \alpha_X} + \underbrace{\tilde{w}_0^Y \ddot{x}_M (\varepsilon_1 - \varepsilon_2)}_{\dot{\tilde{w}}_0^Y / \alpha_Y}\} dt \\ &= \int_0^t \left\{ \frac{1}{\alpha_I} \tilde{w}_0^I \dot{\tilde{w}}_0^I + \frac{1}{\alpha_X} \tilde{w}_0^X \dot{\tilde{w}}_0^X + \frac{1}{\alpha_Y} \tilde{w}_0^Y \dot{\tilde{w}}_0^Y \right\} dt \\ &= \frac{1}{2} \int_0^t \frac{d}{dt} \left\{ \frac{1}{\alpha_I} (\tilde{w}_0^I)^2 + \frac{1}{\alpha_X} (\tilde{w}_0^X)^2 + \frac{1}{\alpha_Y} (\tilde{w}_0^Y)^2 \right\} dt \\ &\geq -\frac{1}{2} \left[\frac{1}{\alpha_I} (\tilde{w}_0^I)^2 + \frac{1}{\alpha_X} (\tilde{w}_0^X)^2 + \frac{1}{\alpha_Y} (\tilde{w}_0^Y)^2 \right]_{t=0} \\ &\equiv -\delta_1^2 (\text{const.}) \end{aligned} \quad (48)$$

For the second term in the same manner;

$$\begin{aligned} -\int_0^t \varepsilon^T \tilde{K}(y)v_{FB} dt &= -\frac{1}{\alpha_K} \sum_{i=1}^3 \sum_{j=1}^n \int_0^t \underbrace{\tilde{w}_{ij}^K \varepsilon_i \phi_j(y_i) v_{FBi} / \hat{k}_i(y_i)}_{-\dot{\tilde{w}}_{ij}^K / \alpha_K} dt \\ &\geq -\frac{1}{2\alpha_K} \sum_{i=1}^3 \sum_{j=1}^n [(\tilde{w}_{ij}^K)^2]_{t=0} \\ &\equiv -\delta_2^2 (\text{const.}) \end{aligned} \quad (49)$$

For the third term in the same manner;

$$\begin{aligned} -\int_0^t \varepsilon^T \tilde{f}_D(y) dt &= -\frac{1}{\alpha_D} \sum_{i=1}^3 \sum_{j=1}^n \int_0^t \underbrace{\tilde{w}_{ij}^D \varepsilon_i \phi_j(y_i)}_{-\dot{\tilde{w}}_{ij}^D / \alpha_D} dt \\ &\geq -\frac{1}{2\alpha_D} \sum_{i=1}^3 \sum_{j=1}^n [(\tilde{w}_{ij}^D)^2]_{t=0} \\ &\equiv -\delta_3^2 (\text{const.}) \end{aligned} \quad (50)$$

From Eqs. (48), (49), (50)

$$\int_0^t \varepsilon^T r dt \geq -(\delta_1^2 + \delta_2^2 + \delta_3^2) \equiv -\delta^2 \quad (51)$$

By learning rule (42)–(46), the nonlinear models (25), (26), (30)–(32) are obtained, and it is possible to tune gains in order to coincide y with y_M and compose the inverse dynamics.

In order to calculate the learning rules (42)–(46) practically by computer, the system should be converted to discrete system. Therefore, suppose $\hat{w}_{ij}^K(t) = \hat{w}_{ij}^K(kT) = \hat{w}_{ij}^K[k]$ ($k=0, 1, 2, \dots$), then $\dot{\hat{w}}_{ij}^K = (\hat{w}_{ij}^K[k+1] - \hat{w}_{ij}^K[k])/T$. For example, Eq. (42) is calculated as $\hat{w}_{ij}^K[k+1] =$

$\hat{w}_i^K[k] + T\alpha_{Ki}\epsilon_i[k]\phi_i(y_i[k])v_{FBI}[k]$. Other learning rules are calculated in the same manner.

Learning rules (42), (43) include basis functions $\phi_j(y_i)$. If those basis functions have a certain value coincidentally, the learning of $\hat{w}_i^K, \hat{w}_i^D$ interfere each other. Then, $\phi_j(y_i)$ is selected to have a certain value in a section of variable y_i , where the learning advances and have 0 in other sections, where the learning stops. Locating the basis functions caring for no superposition, the interference of the learning is prevented.

In addition to above characteristic, for the description of continuous and smooth nonlinearity, $\phi_j(y_i)$ should have such a shape that has a maximum value at its center, and the value of the basis functions decrease as departing from their own center. Because Eq. (24) satisfies such condition, it is selected as basis function.

Motion learning is defined as an achievement of objective motion control by obtaining the nonlinear models by estimating the parameters $w_i^K, w_i^D, w_i^0, w_i^x, w_i^y$ and tuning $\hat{J}(y), K_D, K_P, u_B$ using models thus obtained and constituting the inverse dynamics.

4.3 Condition for strictly positive real

Condition of Eq. (34) to be strictly positive real is given by positive real lemma. For simplicity, the constant matrices are given as the following equations;

$$A = aI \quad (52)$$

$$B = bI \quad (53)$$

$$C = cI \quad (54)$$

where a, b and c are positive real constants. Suppose T indicates a matrix diagonalizing the matrix $J(y)$, or $T^{-1}J(y)T = \text{diag}\{\mu_1, \mu_2, \mu_3\}$. Defining, ϵ, η and τ as $T\epsilon = \epsilon, T\eta = \eta$ and $T\tau = \tau$ respectively, Eqs. (34) and (36) are transformed;

$$\mu_i \ddot{\eta}_i + a \dot{\eta}_i + b \eta_i = -\tau_i \quad (55)$$

$$\epsilon_i = \eta_i + c \dot{\eta}_i \quad (56)$$

Then, the following state equation is gained;

$$\begin{bmatrix} \dot{\eta}_i \\ \ddot{\eta}_i \end{bmatrix} = A_E \begin{bmatrix} \eta_i \\ \dot{\eta}_i \end{bmatrix} + b_E(-\tau) \quad (57)$$

$$A_E = \begin{bmatrix} 0 & 1 \\ -b/\mu_i & -a/\mu_i \end{bmatrix} \quad (58)$$

$$b_E = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (59)$$

To these equations, Kalman-Yakobuvovich's theorem⁽⁹⁾ is applied. Therefore, using positive definite matrices $P(2 \times 2)$, $Q = \text{diag}\{q \quad q\} (q > 0)$, and $c = [1 \quad c]$, Lyapunov's equation is introduced as follows.

$$A_E^T P + P A_E = -Q \quad (60)$$

$$b_E^T P = c \quad (61)$$

By solving this equation; $c = \mu_i/a + \mu_i q/2a$ or

$$c > \max_{i=1,2,3} \left(\frac{\mu_i}{a} \right) \quad (62)$$

Fig. 3 The Block diagram of the control system

Fig. 4 The magnetic levitation and positioning system

The composed control structure is illustrated in Fig. 3. The proposed control structure is mainly composed of 3 components, which are the feedback system, the motion learning system and the inverse dynamic system. The feedback system stabilizes the motion system. The motion learning system obtains the motion model and tunes the feedback system to linearize the motion system. The inverse dynamics generate the input u to the motion system to cancel the linearized motion system. In this control structure, the control operation can be shared with the feedback system and the motion learning system to decrease the load on the processors.

5. Experiment of a Tracking Control

5.1 Experimental apparatus

The magnetic levitation and positioning system used in this experiment to ensure the efficiency of the proposed control method is explained. This experimental system is composed of a square steel table (80 mm x 80 mm x 2 mm, $m = 0.15$ kg), and located among electromagnets shown in Fig. 4. There are magnets to levitate and support the table along the vertical z -axis. The movable range of the table is $L[-1, +1]$ (mm). The control system is shared with feedback

Table 1 Constants

α_D	α_K	α_I	α_X	α_Y	a	b	c
750	75	3.75	0.015	0.015	18	600	2.

system and tuning system. The feedback (12) is executed by DSP (MTT LORY, clock 33 MHz) with the sampling rate of 5 kHz. The feedback gain tuning (27), (28), drift compensation (29) and parameter estimation by learning (42)–(46) are executed by PC (i486, clock 20MHz) with the sampling rate of 100 Hz. Both tasks are executed independently, however, during PC samples once, DSP samples 50 times. Therefore, PC's sampled data are able to be acquired by calculation of arithmetic mean of 50 data sampled by DSP. That is, the discrete data for the use of motion learning are given as $\bar{y}_i = \frac{1}{50} \sum_{l=1}^{50} y_i(l)$. This task filter out the signal error from the sampled data and prevent the divergence of motion learning system. The constants used in this experiment are shown in Table 1.

5.2 The Result of experiment and discussion

The result of the experiment is shown and discussed here. The objective trajectory y_M is a square of which the side is 1 mm and the center is located at the origin of the coordinates. First, the objective trajectory used in this experiment is shown in Fig. 5. The number of basis functions n is 6 and the steepness factor σ is 6.0. The initial values of $\hat{w}_i^k$ is selected so that initial $\hat{k}_{pi}(y_i) = 400$ and $\hat{w}_i^p = 0$. The initial value of $\hat{w}_0^l = 0.45$, $\hat{w}_0^x = 0$ and $\hat{w}_0^y = 0$. Figure 6 shows that the objective trajectory and the motion trajectory of the table before motion learning. This shows the trajectory of the table largely deviates from the objective before motion learning due to the influence of drift force f_D . Figure 7 shows the behavior of errors during motion learning. As motion learning starts, the trajectory of the table rapidly begins to coincide with the objective trajectory and the tracking performance is much improved. Figure 8 shows that the table trajectory almost completely coincides with the objective trajectory after motion learning. This indicates that the model of the motion system has been obtained accurately and that tuning feedback gains are executed successfully.

6. Conclusion

In this study, aiming an improvement of control performance, reduction in burden of the processor and increase in speed for an uncertain motion system by sharing tasks, a control method is proposed as 1) Obtaining a nonlinear model present at drive system, 2) Linearization of the motion system using obtained

Fig. 5 The objective trajectory

Fig. 6 The tracking trajectory before motion learning

Fig. 7 The tracking errors during motion learning

Fig. 8 The tracking trajectory after motion learning

nonlinear model, 3) Realization of high performed motion control by constitution of inverse dynamics of linearized motion system. In this study, motion learning is defined as above control method. A tracking control is experimented in Magnetic Levitation System using proposed control method. The result shows that the trajectory of the rigid body that deviated from the objective trajectory before motion learning coincided with the objective trajectory after motion learning, and the performance of tracking and response were improved. We conclude that the proposed control method is useful for a rigid body motion control system by the proposed motion learning method.

References

- (1) Tsuji, J., Omori, H. and Sano, A., Adaptive Control Incorporating Neural Network, Trans. Soc. Inst. Cont. Eng., (in Japanese), Vol. 30, No. 3 (1994), p. 295-302.
- (2) Sanner, R.M, and Slotine, J.J.E. IEEE Trans. on Neural Network, Vol. 3, No. 6 (1992), p. 837-863.
- (3) Hao, S., Yang, Z. and Tsuji, T., Adaptive Nonlinear Control for a Magnetic Levitation System, Trans. Soc. Inst. Cont. Eng., (in Japanese), Vol. 32, No. 1 (1996), p. 87-96.
- (4) Kemal, M. and Narendra, K.S., The International Journal of Robotics Research, Vol. 15, No. 6 (1996), p. 592-610.
- (5) Tomizuka, M., On the Theory and the Application of the Adaptive Control System, (in Japanese), (1981), Ohm Pub. Co., Ltd.
- (6) Ando, T. and Iwatsubo, T., Adaptive Control for a Nonlinear Control System by Gain Self-Tuning and It's Application to a Magnetic Levitation System, Trans. Soc. Inst. Cont. Eng., (in Japanese), Vol. 34, No. 8 (1998), p. 1019-1024.
- (7) Popov, V.M., Automation and Remote Control, 24 (1963), p. 1-23.
- (8) Landau, I.D. Automatica, Vol. 10, No. 4 (1974), p. 353-379
- (9) Kanai, K., Robust Adaptive Control, Soc. Inst. Cont. Eng. (in Japanese), (1990).
- (10) Nakano, K., An Introduction to Neurocomputing, (in Japanese), (1990), Corona Pub. Co., Ltd.