Modified Gain Fuzzy Kalman Filtering Algorithm*

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Recently, we proposed a fuzzy Kalman filtering algorithm (FKF algorithm) which is formulated by embedding a set of parallel Kalman filters inside a fuzzy inference mechanism (FIM)⁽¹⁾⁻⁽⁴⁾. This paper proposes a version of FKF algorithm that directly determines the gain of each of the parallel Kalman filters. It fuzzifies both the modeling uncertainty of the dynamic system and the quality of each measured data to determine the Kalman gains. This can largely reduce the number of the parallel Kalman filters inside the FKF algorithm and the computation requirement. Monte Carlo simulations are conducted for observation and comparison. By adjusting the Kalman gains directly, the computation load is highly reduced with only slightly change in the filter performance.

Key Words: Kalman Filtering Algorithm, Modified Gain, Fuzzy Gaussian Stochastic Models

1. Introduction

Recently, we proposed a fuzzy Kalman filtering algorithm (FKF algorithm) which is formulated by embedding a set of parallel Kalman filters inside a fuzzy inference mechanism (FIM)⁽¹⁾⁻⁽⁴⁾. The FIM⁽³⁾ is designed to fuzzificate the system uncertainty into a set of fuzzy Gaussian stochastic models (FGSMs) and choose among which the best fit model. By including the fuzzy techniques⁽⁵⁾⁻⁽⁷⁾ in constructing the Kalman filtering algorithm (KF algorithm)⁽⁸⁾⁻⁽¹²⁾, many non-linear non-stationary system can be well approximated and well treated, the filtering performance can be improved and the possibility of occurring the filter divergence problem can be highly reduced. However, this is not an optimal algorithm, also it requires much more computation effort compare to the traditional KFs.

Reviewing the filter algorithms from the recursive least squares filters (RLSFs) which use the non-parametric model^{(13),(14)}, many versions of KFs which use the stochastic model⁽⁸⁾⁻⁽¹²⁾, and the FKF which use the

fuzzy stochastic models⁽¹⁾⁻⁽⁴⁾. We can see that the more complex in the model, the more flexibility can be get, however, the heavier in computation is required. How to improve the filtering performance and reduce the computation requirement are two important research directions.

In order to improve the filtering performance, we have investigated a new model called the "suboptimal neighborhood fuzzy Gaussian stochastic models" (SON-FGSMs)⁽⁴⁾. Using this SON-FGSMs, we can expect a nearly optimal filtering result, but the computation load is even worse.

In order to highly reduce the computation requirement, while keeping the filtering performance as nearly to that of the FKF with SON-FGSMs as possible. This paper proposes a method that uses fuzzy approach to judge both the modeling uncertainty of the dynamic system and the quality of each measured data to determine the Kalman gains directly for the parallel Kalman filters of the FKF algorithm.

In the following, I first describe the problem, the modified gain KF algorithm, and the modified gain FKF algorithm, then introduce the details of determining the adjusting factors of the Kalman gain, and then conduct the Monte Carlo simulations for observation and comparison. Finally give some comments and make the conclusions.

* Received 22nd September, 1997

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2. Preliminary

2.1 System description

Assume a dynamic system and the corresponding discrete time measurement equation are given as following.

$$\dot{x}_*(t) = f(x_*(t), u(t), t) + w_*(t) \quad (1)$$

$$z(k+1) = H(k+1)x_*(k+1) + v_*(k+1) \quad (2)$$

where $k=0, 1, 2, \dots$, $t=kT$, T is the sampling period of the measurement system. $x_* \in R^n$, $u \in R^l$, and $z \in R^m$ are the state, control, and measurement vectors respectively. $f(\cdot)$ is the dynamic functions of the controlled system. $H \in R^{m \times n}$ is the measurement matrix, $v_* \sim N(0, R)$ is the m -dimensional vector of measurement noise, $w_* \sim N(b_*, Q_*)$ is the n -dimensional vector of uncertainty with the unknown bias b_* , and the unknown covariance matrix Q_* . It is assumed that the initial state $x_*(0)$ is unknown.

2.2 The modified gain Kalman filter

In order to design a Kalman filter for the dynamic system of Eqs.(1) and (2), we can model the dynamic system and the corresponding measurement equations as following

$$x(k+1) = \Phi(k+1, k)x(k) + \Psi(k+1, k)u(k) + [b(k) + w(k)] \quad (3)$$

$$z(k+1) = H(k+1)x(k+1) + v(k+1) \quad (4)$$

with the initial state assumed to be

$$x(0) \sim N(x_0, P_0)$$

where $x \in R^n$ is the modeled state vector, $\Phi \in R^{n \times n}$, $\Psi \in R^{n \times l}$ are the related transition matrices, $b \in R^n$ is the bias vector, $w \sim N(0, Q)$, and $v \sim N(0, R)$ are the n -, and m -dimensional uncorrelated Gaussian white sequences respectively. Here b , Q , and R can be pre-assigned or on line estimated.

For the modeled dynamic system described in Eqs.(3) and (4), define: $Z_k = [z(1), z(2), \dots, z(k)]$, $\hat{x}(k|k) = E[x(k)|Z_k]$, $\tilde{x}(k|k) = x(k) - \hat{x}(k|k)$, $P(k|k) = E[\tilde{x}(k|k)\tilde{x}^T(k|k)]$, $P(k+1|k) = E[\tilde{x}(k+1|k)\tilde{x}^T(k+1|k)]$, and f_k as the adjusting factor for the Kalman gain, in which $E[\cdot]$ is the expectation operation. Then we can construct the following modified gain Kalman filter:

Algorithm I: The modified gain Kalman filtering algorithm:

State Prediction

$$\hat{x}(k+1|k) = \Phi(k+1, k)\hat{x}(k|k) + \Psi(k+1, k)u(k) + b(k) \quad (5.a)$$

Covariance Matrix Prediction

$$P(k+1|k) = \Phi(k+1, k)P(k|k)\Phi^T(k+1, k) + Q(k) \quad (5.b)$$

Weighting Matrix Update

$$K(k+1) = f_k P(k+1|k) H^T(k+1) [H(k+1) \times P(k+1|k) H^T(k+1) + R(k+1)]^{-1} \quad (5.c)$$

State Update

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[z(k+1) - H(k+1)\hat{x}(k+1|k)] \quad (5.d)$$

Covariance Matrix Update

$$P(k+1|k+1) = [I - K(k+1)H(k+1)]P(k+1|k) \quad (5.e)$$

2.3 The modified gain fuzzy Kalman filter

The block diagram of the MG-FKF is shown in Fig. 1.

In which:

F: The fuzzification procedure

R: The membership degree renew procedure

D: The defuzzification procedure

KF_i: The parallel Kalman filters

r_{av}: The residual average estimation

K: The Kalman gain of the best fit Kalman filter

f_{ki}: The adjusting factor for the corresponding Kalman gain

r: The residuals

r_l: The long term average of the latest *l* residuals

r̄: The exponential weighted average of the residuals.

Δr: The short term variation of the residuals

R, Q: The covariance matrices of the measurement uncertainty and the modeling uncertainty respectively.

f⁻, f⁺: The truth value before and after the renew procedure respectively.

x, x̂: The state vector before and after the defuzzification procedure respectively.

P, P̂: The state covariance matrix before and after the defuzzification procedure respectively.

Algorithm II: The MG-FKF algorithm:

1) The residual average estimation: Estimate three average values of the residual sequence.

2) Fuzzification: Use the estimated residual sequence to determine the **f_{ki}** and the corresponding membership degrees.

3) Parallel Kalman filtering: Put each one of the **f_{ki}** into a modified gain Kalman filter, forms a set of Parallel Kalman filters. And attach a membership

Fig. 1 The block diagram of the MG-FKF algorithm on FGSM

degree f_i^- to each channel i .

4) Membership degree renew: For each channel i of the parallel Kalman filter, use the new residuals and the f_i^- to evaluate the new f_i^+ .

5) Defuzzification: Maximize over all membership values of f_i^+ to select the best fitted Kalman filter, and takes the output of this best fitted Kalman filter as the FKF output.

3. The Determination of the Kalman Gains

From Eq. (5) we can see that the over all effect of determining the modeling uncertainty and measurement uncertainty is the determination of the Kalman gain. Why do we not simply determine the Kalman gain directly?

3.1 The fuzzification of the modeling uncertainty

Here, I use the same approach as that of SON-FGSMs to Fuzzificate the Covariance Matrix $Q^{(4)}$. This method first determines the Jazwinski's adaptive covariance matrix $Q_J^{(10)}$, and uses the modified Mehra's Optimal Test^{(4),(11)} to determine the bands of fuzzification, then builds the related membership functions and the fuzzy inference rules.

Assume that the difference between the estimated Q_J and the unknown optimal Q_* can be determined by $[N_{out}]_{avg}$ and $\text{var}[\hat{\rho}_k^e]_{avg}$ alone. We can build the related membership functions as shown in Fig. 2.

In which $[N_{out}]_{avg}$ is the average value of $[N_{out}]_{ii}$, which is the number of times that all the i -th diagonal elements $\rho_1, \rho_2, \dots, \rho_k$ of the sequential estimated Q_J s lie outside the $\pm(1.96/N^{1/2})$ bounds. (The 95% confidential limits of the modified Mehra's Optimal Test^{(4),(11)} here N is the sequence length); $\text{var}[\hat{\rho}_k^e]_{avg}$ is the average variance of $\text{var}[\hat{\rho}_k^e]_{ii}$ of all the i -th diagonal elements $\rho_1, \rho_2, \dots, \rho_k$ of Q_J s.

The membership functions for $[N_{out}]_{avg}$ and $\text{var}[\hat{\rho}_k^e]_{avg}$ are shown in Fig. 2. In which the parameters $s_i, \sigma_i, i=1, 2, 3$, etc. and the linguist variables NPZ, VPB, ..., etc. are all heuristically determined⁽⁵⁾⁻⁽⁷⁾.

For a given Q_J , if either the value of $[N_{out}]_{avg}$ or $\text{var}[\hat{\rho}_k^e]_{avg}$ is abnormal, it indicates that the size of Q_J

Fig. 2 The membership functions of $[N_{out}]_{avg}$ and $\text{var}[\hat{\rho}_k^e]_{avg}$

is either too large or too small. According to this concept two sets of the membership functions f_k^1 and f_k^2 of f_k should be established such that one is in the smaller direction, the other is in the larger direction.

3.2 The modification of the measured data

Here, I use the concept of "The Quality of the measured data⁽¹⁾⁻⁽⁴⁾" to modify the covariance matrix R , and show the membership functions of Δr in Fig. 3.

For a given measured data, when the corresponding Δr (the variation of the residual), is too large, it means that "The possibility that 'The measured data is abnormal' is great". We can treat this case by enlarging the covariance matrix R . In contrary, if Δr is small or zero, that means the measured data is normal, we can set the covariance matrix R unchanged.

The covariance matrix R represents the accuracy of the measurement instrument. Usually, we have to calibrate the instrument before using it, so we can have the calibrated matrix R before hand. The enlargement of the covariance matrix R for a measured data means that we trust less on this measured data and more on the prediction.

3.3 The determination of the adjusting factors of Kalman gain

According to the arguments in sub-sections 3.1 and 3.2, and refer to the membership functions defined by Figs. 2 and 3, we can form the following fuzzy rule sets:

IF $[N_{out}]_{avg}$ is NPX , and $\text{Var}[\hat{\rho}_k^e]_{avg}$ is NPY ,
and Δr is MAZ THEN f_k^1 and f_k^2

In which NPX, NPY , and MAZ are the linguist variables, f_k^1 and f_k^2 are two adjusting factors of Kalman gains. Refer to Fig. 2, and Fig. 3, for each given variable $[N_{out}]_{avg}$, $\text{Var}[\hat{\rho}_k^e]_{avg}$ and Δr belongs to two membership functions, so that in this approach, we will have sixteen Kalman gains in total. So that we will have sixteen parallel Kalman filters in the FIM of the FKF algorithm.

3.4 The MG-FKF algorithm

Following the aforementioned procedure in determining the adjusting factors of Kalman gain, we can

Fig. 3 The membership functions of Δr

construct the MG-FKF algorithm of Algorithm II.

4. Simulation and Comparison

4.1 Problem statement

Three KFs are designed for comparison. The first one is a Simple KF, that use constant Q and R matrices, the second one is the SON-FKF, the 3rd one is the MG-FKF.

Assume a target trajectory is created by the following discrete time equations.

$$\begin{aligned} x(k+1) &= x(k) + Tv_x(k) + w_1(k) \\ v_x(k+1) &= v_x(k) + Ta_x(k) + w_2(k) \\ y(k+1) &= y_0 + \sin(0.2k) \sin(0.05k)y^{1/2}(k) \\ &\quad + Tv_y(k) + w_1(k) \\ v_y(k+1) &= v_y(k) + Ta_y(k) + w_2(k) \end{aligned} \quad (6)$$

and the following measurement equation.

$$z(k+1, k) = H(k+1)X(k+1) + v(k+1) \quad (7)$$

In which x, y, v_x, v_y, a_x, a_y are the positions, velocities, and accelerations in the x, y direction respectively, $a_x = a_y = 0.02 \text{ m/sec}^2$, $v_x(0) = 2 \text{ m/sec}$, $v_y(0) = 0 \text{ m/sec}$, $x(0) = y(0) = 50 \text{ m}$, $T = 0.1 \text{ sec}$ is the sampling rate, k is the time index. w_1, w_2 are Gaussian noises in position and velocity respectively, both of them are zero mean with standard deviation equal to 2 m, and 2 m/sec respectively. $y_0 = 50 \text{ m}$ is a constant. For the measurement equation, $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $x = [x \ v_x \ y \ v_y]^T$, and v is the measurement uncertainty assumed to be zero mean with standard deviation equal to 10 m.

For comparison, the data created by Eq. (6) is interpolated to create another data set with the sampling rate doubled. Then two measurement data sets are taken by Eq. (7) with different sample rates $T_1 = T = 0.1 \text{ secs}$, and $T_2 = \frac{1}{2}T = 0.05 \text{ secs}$ respectively. The measured data set of T_1 is used for all three KFs, The measured data set of T_2 is used for the MG-FKF in the 2nd simulation example only.

In three FKFs the following discrete time linear equations are used.

$$X(k+1) = \Phi(k+1, k)X(k) + b(k) + w(k) \quad (8)$$

$$z(k+1, k) = H(k+1)X(k+1) + v(k+1) \quad (9)$$

$$k = 0, 1, 2, \dots$$

with zero guessed initial value of $X(0)$, and a large P_0 .

where, $\Phi = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $b(k)$ and $w(k)$ are the

modeling uncertainty, $b(k)$ is the bias, and $w(k)$ is assumed to be zero mean with covariance matrix $Q(k)$. For the Simple KF case, both $b(k)$ and $Q(k)$ are constants and pre-setted, and for the SON-FKF, and MG-FKF cases, they are on line estimated. The

related parameters for the membership functions for the MG-FKF are listed in the Appendix A.

Simulations are conducted to find the recursive estimate $\hat{X}(K+1|K+1)$ s of $X(K+1)$ and the related root mean squares of the estimated errors.

4.2 Simulations Results

The simulation results are shown in Fig. 4 to Fig. 6 respectively. The following performance measure is adopted for performance comparison

$$J = \sqrt{\frac{1}{n} \sum_i^n (x_i - \hat{x}_i)^2} \quad (10)$$

In which x_i are the original target states, while $\hat{x}_i$ are the estimated states.

Figure 4 shows the original target trajectory (solid line) and the related noise corrupted measured data. ('+' for the measured data set of T_1 , and '*' for the measured data set of T_2). Using the measured data set of T_1 , Fig. 5 shows the target trajectory (solid line), and the three estimated trajectories, ('o' for the Simple KF, '+' for the SON-FGSs, and '*' for the

Fig. 4 The original target trajectory and the measured data

Fig. 5 Comparison of the estimation results of three FKFs

MG-FKF). The 'root mean squares value of the estimation errors' are: $J_1=849.0601$ (Simple-KF), $J_2=14.9655$ (SON-FKF), and $J_3=14.8352$ (MG-FKF). Figure 6 shows the comparison of the estimation errors. ('+' for the SON-FGSMS, Using the measured data set of T_1 , and '*' for the MG-FKF, using the measured data set of T_2), in which $J_2=14.9655$ (SON-FKF), and $J_3^*=10.7873$ (MG-FKF).

Figure 5 shows that when the difference between the dynamic equation of the original target trajectory and that used for the KF algorithms is large, the estimation performance of the SON-FKF and MG-FKF is much better than that of the Simple KF. Figure 6 tries to show that for the same sampling rate, the proposed MG-FKF has much less computation load than SON-FKF. So I can use a higher sampling rate for MG-FKF to improve the estimation performance.

5. Discussion and Conclusion

5.1 Discussion

Because our proposed FKF algorithm requires relative loose of pre-required assumptions such as the studied system should be linear, the stochastic processes should be stationary etc., it can easily out-performance those of other traditional Kalman filters. However, this FKF algorithm is not an optimal algorithm, also the structure of the fuzzy Kalman filter is much more complex than that of the corresponding traditional Kalman filter, simplification in structure and computation without sacrificing performance is worth to be investigated.

By virtue of the sub-optimal neighborhood expansion, and the a posteriori possibility optimization, the performance of this FKF algorithm is nearly optimal, but the sub-optimal neighborhood expansion and parallel process during fuzzy Kalman filtering need

Fig. 6 The estimation error comparison of SON-FKF and MG-FKF

much more computation.

By directly determining the Kalman gains of the parallel KFs inside the FIM, we can largely reduce the number of the parallel KFs and the computation requirement.

5.2 Conclusion

This paper presents a method for constructing the modified gain FKF algorithm, that can highly reduce the computation requirement but also keep the filter performance nearly optimal.

The Monte Carlo simulations are conducted for observation and comparison. The results can well agree with the expectation.

There are a lot of research areas open for further study, such as the determination of the quality of the measured data, the signal processing techniques in handling data in the form of fuzzy stochastic models, the methodology simplification, the training methods for FIMs, the areas of applications etc..

Acknowledgement

This work is supported by the National Science Council of the Republic of China (Contract No. NSC 84-2512-S-035-004).

Appendix A

The membership functions and the table of the related variables that are used for computer simulations are listed as following:

I. For determining the adjusting factor of measurement uncertainty

Refer to Fig. A-1, in which, s_r the standard deviation of the expected residuals, Δr the difference between the estimated residual and the long term

Fig. A-1 The membership functions for measurement uncertainty

Fig. A-2 The membership functions for the bias of the modeling uncertainty

Fig. A-3 The membership functions for covariance matrix of the modeling uncertainty

Table A-1 The table of parameters

$a_r=3$	$b_r=2$	$c_r=1.2$
$a_b=3$	$b_b=2$	$c_b=1.2$
$a_{q1}=3$	$b_{q1}=5.8$	$c_{q1}=8.7$
$a_{q2}=0.04$	$b_{q2}=0.3$	None

residual average, and f_r the adjusting factor of measurement uncertainty.

II. For determining the adjusting factor of bias

Refer to Fig. A-2, in which, r_i the long term residual average, s_b is the estimated standard deviation of r_i , and f_b the factor used to modified the bias vector.

III. For determining the adjusting factor of the covariance matrix of the modeling uncertainty

Refer to Fig. A-3, in which, s_q is the estimated standard deviation of the element of matrix Q , and σ_q is the estimated standard deviation of the element of ρ_k .

The parameters used for membership functions are listed in Table A-1

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