A Study on Stability Characteristics of Actively Controlled Externally Pressurized Air Journal Bearings*

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Results of theoretical investigations of the stability characteristics of an actively controlled externally pressurized air journal bearing are presented. The proportional and derivative control is used for the control algorithm of the active air bearing. The stability characteristics of the actively controlled bearing for zero steady-state eccentricity are investigated with the step jump method. The speed at onset of instability is increased for both proportional and derivative control of the bearing. The proportional control increased the stability threshold without affecting the whirl ratio. However, for the derivative control of the bearing, stability threshold increase is accompanied by a parallel reduction of the whirl ratio. Results show the active control of bearing can be adopted for the stability improvement of air journal bearing.

Key Words: Air Bearing, Whirl Instability, Active Control, Step Jump Method, Whirl Ratio

1. Introduction

Air journal bearings have been widely used for high-speed spindle systems because of their substantially low frictional loss and heat generation. Air journal bearings, however, have a self-excited instability, which is commonly called "whirl instability". Journal speed at the onset of this instability is the limiting factor of the journal operation. Extensive theoretical and experimental work has been carried out to suppress the whirl instability and extend journal operating speed. From such investigations on externally pressurized air journal bearings, the following stabilized bearings have been proposed: (1) bearings with flexible and damped supports like O-ring⁽¹⁾⁻⁽⁶⁾, (2) floating bush bearings^{(7),(8)}, (3) bearings with tangential air supply^{(9),(10)} and (4) lobe bearing⁽¹¹⁾. Even with these kinds of bearings, whirl instability still remains. Therefore, a need for effective ways to suppress whirl instability still exists. Because the whirl instability of a fluid film journal bearing is caused by the coupled force generated in

fluid film, the stability of a journal bearing can be improved if this coupled force is reduced by appropriate bearing motion relative to journal. The stabilization mechanism of an air bearing with flexible damped supports and a floating bush bearing can be considered as such kind of stabilization.

In the late 1980' development of micro-actuators opened up the possibility of active control of air journal bearings. Most of the investigations on actively controlled air journal bearings so far have focused on the improvement of the rotational accuracy. In 1990, Horikawa and Shimokobe⁽¹⁵⁾ showed that the rotational amplitude of the journal was reduced remarkably by active control of the bearing. In 1992, Mizumoto and Okuno⁽¹⁶⁾ found that the stiffness of an air bearing increases by the control of the exhaust resistance of a cavity on the bearing surface. However, there are no reports on the control of an air journal bearing for stability improvement.

In this paper an active air journal bearing, in which two directions (x, y in Fig. 1) of radial bearing motions are controlled, is proposed for stabilized bearing. The stability characteristics of the active air journal bearing are numerically investigated. The stability threshold and whirl ratio of the actively controlled air bearing was obtained for various control gains of proportional and derivative control

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actions.

Nomenclature

- a_s : control volume surface at source point
 A_s : a_s/R^2
 c : average radial clearance of bearing
 C_D : discharge coefficient of orifice, 0.85
 d : feeding hole diameter
 D : journal diameter
 f_{ij} : finite difference form of Reynolds equation for i, j node
 F_{ij} : j -directional film force response produced by i -displacement of journal
 G_{ij} : nondimensional force deviation, $\delta F_{ij}(t) \frac{c}{p_a R L}$
 h : film thickness
 H : nondimensional film thickness, h/c
 K_d : derivative control gain
 K_p : proportional control gain
 L : length of bearing
 l_s : control volume circumference at source point
 L_s : l_s/R
 m : mass of rotor
 M_1 : nondimensional mass of rotor, $\frac{m p_a}{2 \mu^2 L} \left(\frac{c}{R} \right)^5$
 M_2 : nondimensional mass of rotor, $\frac{m c \omega^2}{p_a L D}$
 $\vec{n}$: normal vector of control volume
 O_b : geometric center of bearing (x_b, y_b)
 O_j : geometric center of journal (x, y)
 p : pressure
 p_a : ambient pressure
 p_c : downstream pressure at source point
 P : nondimensional pressure, p/p_a
 P_s : nondimensional supply pressure
 q : mass flow rate
 Q : nondimensional mass flow rate, $q \sqrt{\Re T} / C_D \pi d h p_s$
 R : radius of journal
 $\Re$: gas constant
 t : time
 V_s : control volume include feeding hole
 $\vec{x}_u$: unit vector in circumferential direction
 x, y : coordinates system (in Fig. 1)
 X, Y : coordinates nondimensional with respect to X_{ref}, Y_{ref} : x, y directional reference input for active bearing
 γ : whirl ratio, ν/ω
 ξ : nondimensional coordinate in axial direction
 θ : coordinate in circumferential direction
 κ : specific heat ratio of gas
 Λ : bearing number, $6 \mu \omega R^2 / p_a c^2$
 $\vec{\Lambda}$: $\Lambda \cdot \vec{x}_u$
 Λ_t : feeding parameter, $24 \mu C_D \pi d \sqrt{\Re T} / p_a c^2$

- μ : absolute viscosity
 ν : whirling frequency of journal center
 τ : nondimensional time, νt
 ω : angular velocity of journal
 $(\dot{})$: time derivative of ()

2. Analysis

The governing equations for the stability characteristics of active air journal bearing include the Reynolds equation, which gives the pressure distribution within the bearing, as well as the equation of motion of the journal and bearing.

To determine the stability, both Reynolds equation and the equation of motion must be solved simultaneously since they are coupled through film thickness and pressure. The technique mentioned above is called nonlinear orbit method⁽¹³⁾. Principal advantages of this method are the completeness and detail of transient information. However, it requires a large number of computer calculations since the Reynolds equation must be solved at each time step.

In this paper we used the step-jump method which decouples the Reynolds equation and equation of motion by limiting the analysis to small deviation from an equilibrium state. In this method, the Reynolds equation is only solved during the process of generating the step responses. Therefore it offers a substantial savings in computing time. The nonlinear orbit method is used in some cases for comparison.

3. Reynolds Equation and Boundary Conditions

A schematic diagram of a double-row admission externally pressurized air journal bearing with the coordinate system used in the analysis is shown in Figure 1. The x, y axes are fixed in space, and the ξ, η coordinate system is fixed in the bearing with its origin at the bearing center. There are 12 feed holes uniformly distributed in each feeding plane. Assuming isothermal conditions and perfect gas law, Reynolds equation can be written in dimensionless form as

$$\frac{\partial}{\partial \theta} \left(P H^3 \frac{\partial P}{\partial \theta} \right) + \frac{\partial}{\partial \xi} \left(P H^3 \frac{\partial P}{\partial \xi} \right)$$

Fig. 1 Bearing and journal coordinate systems

$$= \Lambda \frac{\partial}{\partial \theta} (PH) + 2\Lambda \gamma \frac{\partial}{\partial \tau} (PH) \quad (1)$$

Figure 2 shows inflow and outflow at a source point region and grid around a source point. The pressure at the source point is obtained by solving the continuity equation for control volume surrounding the source point. From the continuity of inflow and outflow, the boundary conditions at the source point can be written as follows

$$\Lambda_i P_s H Q = \oint_{L_s} (2\vec{\Lambda} PH - H^3 \nabla P^2) \cdot \vec{n} dL + 4\Lambda \gamma \iint_{A_s} \frac{\partial (PH)}{\partial \tau} dA \quad (2)$$

And nondimensional mass flow rate, Q is given as below

$$Q = \left\{ \frac{2\kappa}{\kappa+1} \right\}^{1/2} \left\{ \frac{2}{\kappa+1} \right\}^{1/(\kappa-1)} \quad \text{for } \frac{P_c}{P_s} = \lambda \leq \left\{ \frac{2}{\kappa+1} \right\}^{\kappa/(\kappa-1)} \quad (3)$$

$$Q = \left\{ \frac{2\kappa}{\kappa-1} \right\}^{1/2} \lambda^{1/\kappa} \left(1 - \lambda^{(\kappa-1)/\kappa} \right)^{1/2} \quad \text{for } \frac{P_c}{P_s} = \lambda > \left\{ \frac{2}{\kappa+1} \right\}^{\kappa/(\kappa-1)} \quad (4)$$

Also, boundary conditions at both bearing ends and in the circumferential direction can be written as follows.

At both ends of the bearing

(a) Inherently compensated restrictor

(b) Flow around the source point

Fig. 2 Flow around the restrictor

$$P(\theta, \pm L/D) = 1 \quad (5)$$

In the circumferential direction

$$P(\theta, \zeta) = P(\theta + 2\pi, \zeta) \quad (6)$$

To solve the time-dependent nonlinear partial differential equation (1), the Newton-Raphson method is used. Equation (1) can be written in finite-difference form as

$$f_{ij}(P_{kl}) = \left\{ P_{i+1/2} H_{i+1/2}^3 \frac{P_{i+1j} - P_{ij}}{\Delta \theta_i} - P_{i-1/2} H_{i-1/2}^3 \frac{P_{ij} - P_{i-1j}}{\Delta \theta_{i-1}} \right\} + T \left\{ P_{ij+1/2} H_{ij+1/2}^3 \frac{P_{ij+1} - P_{ij}}{\Delta \zeta_j} - P_{ij-1/2} H_{ij-1/2}^3 \frac{P_{ij} - P_{ij-1}}{\Delta \zeta_{j-1}} \right\} + \Lambda \{ P_{i+1/2} H_{i+1/2} - P_{i-1/2} H_{i-1/2} \} - \Lambda (\Delta \theta_i + \Delta \theta_{i-1}) \left\{ P_{ij} \frac{\partial H_i}{\partial \tau} + \frac{H_i}{\Delta \tau} (P_{ij} - P_{ij}^*) \right\} \quad (7)$$

where

$$T = \frac{\Delta \theta_i + \Delta \theta_{i-1}}{\Delta \zeta_j + \Delta \zeta_{j-1}},$$

P_{ij}^* = Pressure at previous time step

The Jacobian of f_{ij} at P_{kl} is given by

$$\frac{\partial f_{ij}}{\partial P_{kl}} = -A_{ij} \delta_{ik} \delta_{jl} + B_{ij} \delta_{i+1k} \delta_{jl} + C_{ij} \delta_{i-1k} \delta_{jl} + D_{ij} \delta_{ik} \delta_{j+1l} + E_{ij} \delta_{ik} \delta_{j-1l} \quad (8)$$

where

$$A_{ij} = \left\{ \left(\frac{H_{i+1/2}^3}{\Delta \theta_i} + \frac{H_{i-1/2}^3}{\Delta \theta_{i-1}} \right) + T \left(\frac{H_i^3}{\Delta \zeta_j} + \frac{H_i^3}{\Delta \zeta_{j-1}} \right) \right\} P_{ij} + \frac{\Lambda}{2} (H_{i+1/2} - H_{i-1/2}) + \Lambda (\Delta \theta_i + \Delta \theta_{i-1}) \left(\frac{\partial H_i}{\partial \tau} + \frac{H_i}{\Delta \tau} \right)$$

$$B_{ij} = \left(\frac{H_{i+1/2}^3}{\Delta \theta_i} P_{i+1j} - \frac{\Lambda}{2} H_{i+1/2} \right)$$

$$C_{ij} = \left(\frac{H_{i-1/2}^3}{\Delta \theta_{i-1}} P_{i-1j} - \frac{\Lambda}{2} H_{i-1/2} \right)$$

$$D_{ij} = T \frac{H_i^3}{\Delta \zeta_j} P_{ij+1}$$

$$E_{ij} = T \frac{H_i^3}{\Delta \zeta_{j-1}} P_{ij-1}$$

δ_{ij} : Kronecker's delta

The pressure increment of the each iteration step is

$$\Delta P_{ij} = - \left(\frac{\partial f_{ij}}{\partial P_{kl}} \right)^{-1} f_{ij}(P_{kl}) \quad (9)$$

and pressure of the next iteration step is

$$(P_{ij})_n = (P_{ij})_{n-1} + \Delta P_{ij} \quad (10)$$

This process is repeated until the pressure between two successive iterations satisfies the following convergence criterion.

$$\frac{\sum_{i=0}^m \sum_{j=0}^l |(P_{ij})_n - (P_{ij})_{n-1}|}{\sum_{i=0}^m \sum_{j=0}^l |(P_{ij})_n|} \leq 10^{-6} \quad (11)$$

The number of mesh subdivisions in the circumferential direction and axial direction is 96 and 14, respectively, with variable size.

4. Equation of Motion

Equations of translational motion of rigid rotor are

$$\begin{aligned} m\ddot{x} &= \iint p(\theta, z) R \cos \theta d\theta dz \\ m\ddot{y} &= \iint p(\theta, z) R \sin \theta d\theta dz \end{aligned} \quad (12)$$

In this active bearing, the center position of bearing (X_b, Y_b) is made to change actively by the center position of journal (X, Y) as follows

$$\begin{aligned} X_b &= K_p(X_{ref} - X) - K_d(\dot{X}) \\ Y_b &= K_p(Y_{ref} - Y) - K_d(\dot{Y}) \end{aligned} \quad (13)$$

where X_{ref}, Y_{ref} are reference input.

In this study static equilibrium position of journal center is used for reference input.

5. Step Jump Method

The procedure for investigating the stability characteristics of actively controlled air journal bearing by step jump method is described in this section.

From equilibrium state, the rotor is instantaneously stepped in the one of the degrees of freedom. The gas film pressure distribution, after a sudden small displacement, is obtained from the assumption that the step jump is an isothermal process. With the rotor held at that position, and the pressure distribution after a step jump as initial condition of the gas film, the Reynolds equation is integrated forward in time until the pressure distribution reaches the new steady state. The fluid film force response produced by the step jump displacement of the rotor is given by

$$F_{ij}(t), i, j = x, y \quad (14)$$

The first subscript refers to the step jump direction of rotor, and the second subscript refers to the direction of the force generated.

The force deviation from equilibrium due to unit step displacement of rotor is

$$\delta F_{ij}(t) = \frac{1}{\Delta x_i} (F_{ij}(t) - F_{eq}) \quad (15)$$

Where F_{eq} is the generalized equilibrium force, and Δx_i is the step size taken in generating the response. The force deviation is nondimensionalized as

$$G_{ij}(t) = \delta F_{ij}(t) \frac{c}{p_a R L} \quad (16)$$

$$\begin{aligned} M_2 \delta \ddot{X} &= \delta \bar{F}_{xx} + \delta \bar{F}_{yx} \\ M_2 \delta \ddot{Y} &= \delta \bar{F}_{xy} + \delta \bar{F}_{yy} \end{aligned} \quad (17)$$

where $M_2 = \frac{mc\omega^2}{p_a R L}$

From the step jump response, dimensionless form of force generated within the air film is

$$\begin{aligned} \delta \bar{F}_{xi} &= \delta X(0) G_{xi}(T) + \int_0^T \delta \dot{X}(\tau) G_{xi}(t-\tau) d\tau \\ &\quad - \delta X_b(0) G_{xi}(T) - \int_0^T \delta \dot{X}_b(\tau) G_{xi}(t-\tau) d\tau \end{aligned}$$

$$\begin{aligned} \delta \bar{F}_{yi} &= \delta Y(0) G_{yi}(T) + \int_0^T \delta \dot{Y}(\tau) G_{yi}(t-\tau) d\tau \\ &\quad - \delta Y_b(0) G_{yi}(T) - \int_0^T \delta \dot{Y}_b(\tau) G_{yi}(t-\tau) d\tau \end{aligned} \quad (18)$$

, $i = x, y$

The locus of journal center was calculated with Eqs.(17) and (18). Also the nonlinear orbit method, which solve the Reynolds equation and equation of motion simultaneously, is used for some cases to check the step jump method.

6. Results and Discussion

The bearing configuration chosen has a length-to-diameter ratio L/D of 1, with two circumferential feeding planes each located a distance $3L/8$ from the center plane of the bearing. The stability of active air bearing operating at zero steady-state eccentricity, particularly susceptible to whirl instability, is investigated for feeding parameter Λ_t 0.85, 1.7 and supply pressure P_s 3, 5. Figure 3 shows force responses of the bearing for various Λ due to x -direction step jump displacement of journal. Because the journal operating at zero eccentricity, from symmetry it holds that $G_{yy}(\tau) = G_{xx}(\tau)$, $G_{yx}(\tau) = -G_{xy}(\tau)$. Time step $\Delta\tau$ for step jump response calculation was set 3.14×10^{-3} , so

(a) $P_s=3, \Lambda_t=0.85$

(b) $P_s=5, \Lambda_t=0.85$

(c) $P_s=5, \Lambda_t=1.7$

Fig. 3 Step responses for various bearing number

there are 2000 iterations per revolution of rotor. The $|G_{xx}|$, magnitude of force generating along the direction of rotor displacement, is proportional to the supply pressure and feeding parameter. The G_{xy} , motivate force of whirl instability, becomes smaller as increasing the P_s and Λ_t . But as Λ gets larger dependency of G_{xy} on P_s and Λ_t is reduced.

The stability of the rotor-bearing system is determined from the orbital locus of journal center. The rotor-bearing system is stable when the orbital locus of the journal center converges to a point within bearing circle or draws limit cycle. Otherwise, journal center diverges, the bearing is considered unstable⁽¹⁷⁾.

Figure 4 gives comparison of the critical masses of rotor calculated with step jump method and nonlinear orbit method for various rotational speed of rotor. There is good agreement between step jump method and the nonlinear orbit method, the differences are less than 2%.

Figure 5 shows stability limits and whirl ratios of active air bearing for various proportional control gains as a function of Λ . The stability limits for fixed bearing is proportionate to supply pressure and feeding parameter. As reported by Fleming et al.⁽¹⁴⁾, when the bearing is fixed and operating at zero steady-state eccentricity the nondimensional critical mass of rotor, M_2 , does not vary with Λ . The critical mass of rotor is proportionally increased with control gain. But for a given K_p critical mass does not vary with Λ as the case of bearing is fixed ($K_p=0$). When the K_p is 1, bearing moves equal distance opposite to rotor about reference point, the M_2 is twice that of fixed bearing with stiffness doubling effect. However the whirl ratio does not affected by proportional control of bearing, whirl ratio remains same as the bearing was fixed ($K_p=0$). Stability improvements of proportional control results from bearing stiffness increasing effect.

Figure 6 shows stability characteristics of der-

Fig. 4 Comparison between step jump method and nonlinear orbit method ($\Lambda_t=0.85$, $P_s=5$)

ivatively controlled active air bearing for various control gain as a function of Λ . The critical mass M_2 was increased as increasing the control gain. The stability improvement, unlike the proportional control of bearing, is varying with Λ . For a given derivative control gain, M_2 is increased as Λ is increased. Whirl ratio is reduced as the derivative control gain increased.

Figure 7 shows stability limits with dimensionless rotor mass M_1 , which depends only on the properties of the bearing and lubricant. Curves for larger values of K_d gain demonstrates a minimum critical mass at some bearing number (ex. $\Lambda=6$ for $K_d=0.5e-4$). Beyond that bearing number stability limits increases with increasing the bearing number, so it is possible to make whirl instability free system with derivative control of bearing.

Figure 8 shows the x -directional time history of journal and bearing center for proportional control and derivative control when they draw limit cycle. The phase difference between journal and bearing center is 180° for proportional control and 90° for derivative control. And it can be seen, as mentioned previously, derivative control of bearing reduce the whirling frequency.

(a) $P_s=3$, $\Lambda_t=0.85$

(b) $P_s=5$, $\Lambda_t=0.85$

(c) $P_s=5$, $\Lambda_t=1.7$

Fig. 5 Proportional control of bearing

Fig. 6 Derivative control of bearing

Fig. 7 Stability map with dimensionless mass M_1 ($\Lambda_t=0.85, P_s=3$) Fig. 8 Trajectory of journal and bearing center ($\Lambda_t=0.85, \Lambda=4.8, P_s=3, M_2=2.9$)

Table 1 Whirling amplitude of journal and bearing

Λ		1.65	3.30	4.95	6.60	8.25
Proportional control	Aj	0.0495	0.0475	0.0448	0.0420	0.0391
	Ab	0.0179	0.0177	0.0180	0.0186	0.0197
	Ab/Aj (= Kp)	0.3624	0.3725	0.4020	0.4429	0.5028
Derivative control	Aj	0.0487	0.0452	0.0409	0.0363	0.0330
	Ab	0.0022	0.0041	0.0055	0.0065	0.0074
	Ab/Aj	0.0459	0.0908	0.1354	0.1800	0.2237

Table 1 compares the proportional and derivative control with amplitude of journal and bearing center for various bearing numbers. As shown in the Table 1, amplitude of bearing center for proportional control is given product of control gain and amplitude of journal center. The amplitude of journal and bearing of derivative control are less than that of proportional control. It seems that the derivative control is more effective way of stability improvement method than

the proportional control.

Figure 9 shows stability limits of proportional-derivative controlled bearing. As K_d increased the stability limit curves for K_p intersect each other. For larger K_d , as Λ is increased K_p aggravates the stability. Figure 10 shows whirl ratio of proportional-derivative controlled bearing. For a given derivative control gain whirl ratio is increased as K_p control gain gets larger.

Fig. 9 Stability limits of proportional-derivative control ($\Lambda_t=0.85$, $P_s=5$)

Fig. 10 Whirl ratio of proportional-derivative control ($\Lambda_t=0.85$, $P_s=5$)

7. Conclusion

The stability characteristics of an actively controlled externally pressurized air journal bearing, which is operating at zero steady state eccentricity, are theoretically investigated using step jump method. For the proportional and derivative control of bearing the following results were obtained.

1. The whirl instability of air journal bearing can be suppressed effectively by active control of bearing.
2. For proportional control of bearing, the critical mass M_2 is increased linearly to the control gain but does vary with Λ . The whirl ratio does not affected by proportional control of bearing, it remains as bearing is fixed.
3. For derivative control of bearing, M_2 is increased with increasing the control gain and the amount of improvement is become larger as Λ is increased. And derivative control of bearing reduces the whirl ratio.

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