Three-Dimensional Lattice Continuum Model of Cancellous Bone for Structural and Remodeling Simulation*

Taiji ADACHI**, Yoshihiro TOMITA**
and Masao TANAKA***

In this article, we discuss a mechanical model for structural and remodeling analysis/simulation of cancellous bone, that takes into account tissue microstructure and residual stress. A three-dimensional lattice continuum is used as a structural model of cancellous bone with trabecular architecture, and its mechanical behavior is investigated concerning the dependence of structural parameters on the apparent mechanical properties of the tissue. Assuming the local uniform stress state to be an optimal stress state realized at the remodeling equilibrium, a remodeling rate equation is proposed to express the stress regulation process at the microstructural level for the three-dimensional lattice continuum. In terms of the lattice continuum, a vertebral body is modeled based on quantitative measurements of the trabecular architecture of the cancellous bone, and a remodeling simulation is conducted under the conditions of repetitive bending with compression. By comparison of the obtained distributions of the residual stress and the volume fraction with the experimental observations, the validity of the proposed model in predicting the adaptive remodeling of cancellous bone using a three-dimensional lattice continuum is demonstrated.

Key Words: Biomechanics, Bone Remodeling, Adaptation, Lattice Continuum, Cancellous Bone, Residual Stress

1. Introduction

Internal structure and external shape of bone are continuously changing and being maintained by remodeling to adapt to mechanical environment. This adaptation mechanism has long been a subject of interest and investigation in bone mechanics(1,2). Based on experimental observations(3-9), much effort has been devoted to develop phenomenological models of bone remodeling(6-9). These models are mainly based on the conventional continuum, and relate macroscopic mechanical stimuli such as stress and strain to changes in the shape and apparent density of bone. Although the conventional continuum assumption is simple and useful, bone remodeling should be related to local mechanical stimuli at the microstructural level as is regulated by cellular activities(10,11). That is, to take mechanical considerations of bone remodeling into account, a new continuum model extended to include bone microstructure and the microscopic mechanical environment is required.

There have been a few attempts to take account of the effect of the microstructure of cancellous bone in continuum analysis, for example, by means of a cellular solid model(12), a fabric tensor(13), and a homogenization method(14,15). And also, a remodeling rate equation as a continuum that takes into account trabecular architecture was proposed(16) by developing an evolution equation of the fabric tensor of the cancellous bone. Furthermore, remodeling simulations applied directly to the changes in trabecular architecture considering mechanical stimuli at the trabecular level were proposed that addressed possible remodeling stimuli involving osteocyte(17) and

---

* Received 22nd February, 1999
** Department of Mechanical Engineering, Faculty of Engineering, Kobe University, 1-1 Rokkodai, Nada, Kobe 657-8501, Japan. E-mail: adachi@mech.kobe-u.ac.jp
*** Division of Mechanical Science, Department of Systems and Human Science, Graduate School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka, Osaka 560-8531, Japan
stress nonuniformity at the trabecular levels as a driving force of the remodeling. However, a continuum model for bone remodeling that includes both the mechanics considering bone microstructure and the remodeling relating to microscopic mechanical stimuli is essential. We have started preliminary studies on a phenomenological model for mechanical remodeling of bone with tissue microstructure, in which a two-dimensional lattice continuum is used as a model of cancellous bone with trabecular architecture and the nonuniformity of the natural state is taken into account. The validity of the proposed model has been demonstrated for the basic features of bone remodeling phenomena by comparison of simulation results with experimental observations in two dimensions. However, it is essential to extend this model to three dimensions for future application to actual remodeling phenomena of bones with complex trabecular architecture.

We discuss a three-dimensional lattice continuum as a model of cancellous bone with trabecular architecture as an extension of the two-dimensional model, and examine its mechanical properties which depend on the microstructural parameters. A phenomenological model of mechanical bone remodeling is described for the geometry of a lattice element in the lattice continuum by referring to the local mechanical conditions at the element level, and for the natural state of the element whose nonuniformity result in the residual stress in a three-dimensional lattice continuum. A lattice continuum model of a vertebral body under repetitive bending with compression based on quantitative measurements of the trabecular architecture is used in the remodeling simulation. To investigate the validity of the proposed model, the distributions of the apparent density and the residual stress obtained at the remodeling equilibrium are compared with the experimental results.

2. Three-Dimensional Lattice Continuum Model of Cancellous Bone

A three-dimensional lattice continuum is proposed here as a continuum model for cancellous bone with discrete trabecular architecture, as shown in Fig. 1. The mechanics of the model are described based on couple stress theory.

2.1 Lattice structure model

A three-dimensional lattice continuum, as shown in Fig. 2, is a continuous model consisting of a discrete system of linear elastic bar and beam elements rigidly interconnected at right angles to each other. In general, it can be observed that trabeculae are interconnected to each other arbitrarily at skew angles.

As more general continuum modeling of the trabecular bone, this orthogonal lattice continuum model could be extended to a skew lattice continuum model. In this study, we assume the orthogonality of the lattice based on the observation that the mean intercept length of the cancellous bone can be fit into ellipsoids measured by the fabric tensor that indicates the orthotropy of the trabecular architecture. And it is also based on the fact that the mechanical properties of cancellous bone obtained by testing usually reveals the orthotropy. Once the orthotropy of the mechanical properties of cancellous bone is assumed, we can use the orthogonal lattice continuum to represent the orthotropic macroscopic mechanical behavior of cancellous bone as a continuum.

We consider a lattice unit as shown in Fig. 3 that consists of three members with rectangular cross sections, the principal axes of which are parallel to the coordinate axes \( x_i \) \((i = 1, 2, 3)\). The lattice interval and cross-sectional area of the member \( i \) along the axis \( x_i \) are denoted by \( L_i \) and \( A_i = W_i W_n \), respectively, where \( W_i \) is the width of the member \( i \) in the...
direction of the axis $x_i$. Each member is assumed to behave as an isotropic elastic material with Young's modulus $E_i$ and shear modulus $G_i$. In this work, the summation convention is not applied to Latin indices such as $i, j$, and $k$.

2.2 Stress and strain of lattice continuum

The macroscopic stress tensor $T_{ij}$ and couple stress tensor $\mu_{ij}$ are defined as

$$T_{ij} = \frac{N_{ij}}{L_i L_j}, \quad \mu_{ij} = \frac{M_{ij}}{L_i L_j},$$

by averaging the axial force $N_{ii}$, shearing force $N_{ij}$, torsional moment $M_{ii}$, and bending moment $M_{ij}$ acting on the member $i$ as shown in Fig. 3. Thus, the macroscopic stress and couple stress tensors are related to the microscopic forces and moments. Assuming that a lattice member behaves as a bar with a axial stiffness $E_i A_i$ for $N_{ii}$ and torsional rigidity $G_i J_i$ for $M_{ii}$, and as a beam with bending stiffness $E_i I_{ii}$ for $N_{ij}$ and $M_{ij}$, the macroscopic strain tensor $\gamma_{ij}$ and curvature tensor $\kappa_{ij}$ are defined as

$$\gamma_{ij} = \frac{N_{ij}}{E_i A_i}, \quad \kappa_{ij} = \frac{1}{2\eta} \left( \frac{L_i^2 N_{ii} + L_j^2 N_{jj}}{E_i I_{ii}} \right),$$

$$\gamma_{ij} = \frac{M_{ij}}{G_i J_i}, \quad \kappa_{ij} = \frac{M_{ij}}{E_i I_{ij}},$$

by averaging the deformation of the lattice members over the lattice unit, where $I_i$ and $J_i$, respectively, are the moment of inertia and the polar moment of inertia written in terms of the member width $W_{ii}$ as in the conventional theory of elasticity.

2.3 Constitutive equation of lattice continuum

In couple stress theory\(^{[24]}\), the microrotation vector is assumed to be identical to the macrorotation vector of the continuum. That is, the constitutive equation relates the strain tensor $\gamma_{ij}$ and the curvature tensor $\kappa_{ij}$ to the symmetric part $\sigma_{ij}$ of the stress tensor $T_{ij}$ and the deviatoric part $\mu_{ij}$ of the couple stress tensor $\mu_{ij}$. Eliminating $N_{ii}$ and $M_{ij}$ from Eqs. (1) and (2), the constitutive equation for the three-dimensional lattice continuum is given by

$$\sigma_{ii} = E_i \gamma_{ii}, \quad \mu_{ij} = 4G_i \kappa_{ij},$$

$$\sigma_{ij} = 2G_i \gamma_{ij}, \quad \mu_{ij} = 4G_i \kappa_{ij},$$

where $E_i, G_i, \gamma_{ii}, L_{ii},$ and $L_{ij}$ are the apparent material constants written as

$$E_i = E_i S_i, \quad \gamma_{ii} = -\frac{G_i J_i}{L_i^2 L_j},$$

$$G_i = \frac{L_i L_j}{L_i L_j (E_i L_i L_j + E_j L_j L_i)},$$

$$L_{ii} = \frac{L_i^2}{2}, \quad L_{ij} = \frac{L_i L_j}{4\sqrt{3}} \sqrt{1 + \frac{E_i L_i L_j}{E_j L_j L_i}},$$

using the material constants $E_i, G_i$ and the structural parameters $L_i, W_{ii}$. The member cross-sectional area ratio $S_i$ is defined as the ratio of the cross-sectional area $A_i (= W_{ii} L_{ii})$ of the lattice member to the cross-sectional area $L_i L_j$ of the lattice unit perpendicular to the axis $x_i$ and is written as

$$S_i = \frac{A_i}{L_i L_j} = \frac{W_{ii} L_{ii}}{L_i L_j} = \kappa_i \eta_i,$$

using the member width ratio $\kappa_i (= W_{ii}/L_i)$. These structural parameters can be determined by quantitative measurements of the trabecular architecture of a cancellous bone\(^{[20]}\). Thus, 18 independent material constants determine the mechanical behavior of the three-dimensional lattice continuum.

3. Mechanical Behavior of Lattice Continuum

In this section, we investigate the dependence of the structural parameters in the constitutive equation (3) for the proposed lattice continuum on the mechanical behavior. To clarify the basic mechanical features, we focus on the case of a cubic lattice unit that consists of lattice elements of uniform material with square cross-sections. That is, the parameters are simplified as $E_1 = E_2 = E_3 = E$, $L_1 = L_2 = L_3 = L$, and $W_{ii} = W_{jj} = W$. 3.1 Anisotropy of elastic properties

Consider the apparent elastic modulus $E(\Theta_{ia}) = 1/\tilde{C}_{iia}(\Theta_{ia})$ under the condition of uniaxial tension in the direction of the axis $\tilde{x}_a$, where $\Theta_{ia}$ is the directional cosine of axis $\tilde{x}_a$ from $x_a$, and $\tilde{C}_{iia}$ is the component of the compliance tensor, the inverse of the stiffness tensor $E_{iia}$, in the $\tilde{x}_a$ coordinate system. Normalizing by the elastic modulus $E$ of the material itself, the apparent elastic modulus of the lattice continuum is written as

$$E(\Theta_{ia}) = \left( \begin{array}{ccc}
\frac{1}{\eta_{1a}} \Theta_{1a} + \frac{1}{\eta_{2a}} \Theta_{2a} + \frac{1}{\eta_{3a}} \Theta_{3a} \\
\frac{\eta_{1a} + \eta_{2a}}{\eta_{1a} \eta_{2a}} \Theta_{1a} \Theta_{2a} + \frac{\eta_{1a} + \eta_{3a}}{\eta_{1a} \eta_{3a}} \Theta_{1a} \Theta_{3a} \\
\frac{\eta_{1a} + \eta_{3a}}{\eta_{1a} \eta_{3a}} \Theta_{1a} \Theta_{3a} + \frac{\eta_{2a} + \eta_{3a}}{\eta_{2a} \eta_{3a}} \Theta_{2a} \Theta_{3a}
\end{array} \right)^{-1},$$


JSME International Journal
using the member width ratio  \( \eta_i (= W_i/L_i) \).

In Fig. 4(a), the apparent elastic modulus \( E(\Theta_{\alpha})/E \) is plotted for member width ratios of \( \eta_1 = \eta_2 = \eta_3 = \eta = 0.4, 0.5, 0.6, \) and 0.7, which correspond to member cross-sectional area ratios of \( S_i = 0.16, 0.25, 0.36, \) and 0.49, respectively. When the member width ratio \( \eta_i \) is dependent on the individual axis, the apparent elastic modulus exhibits orthogonal anisotropy as shown in Fig. 4(b), in which \( \eta_1 \) and \( \eta_3 \) are fixed at 0.8 and 0.5, and \( \eta_2 \) has values of 0.6, 0.45, and 0.3, so that the member cross-sectional area ratios, \( S_1 = \eta_1 \eta_2 \), are \( S_1 = 0.4, S_2 = 0.3, 0.225, \) and 0.15, and \( S_3 = 0.48, 0.36, \) and 0.24. The ratio \( E/E \) becomes a maximum and its value coincides with the member cross-sectional ratio

Fig. 4  Directional dependence of elastic modulus ratio \( E/E \)

Fig. 5  Elastic modulus ratio \( E/E \) vs. volume fraction \( V_f \)

\( S_i \) in the direction of the principal axis of the lattice. The minimum value is obtained in the oblique direction to the lattice element, and the anisotropy becomes significant as \( \eta_2 \) decreases. These results qualitatively agree with experimental observations\(^{(19)}\).

### 3.2 Dependence of apparent elastic modulus on volume fraction

Based on the experimental data, the apparent elastic modulus \( \bar{E} \) of the cancellous bone is usually related to the volume fraction \( V_f \) or the apparent density \( \rho \) by a power law\(^{(13,30)}\). When the properties of the trabecular material are homogeneous, the volume fraction \( V_f \) is proportional to the apparent density \( \rho \). The exponent \( n \) of the power law

\[
\bar{E} = E V_f^n \tag{7}
\]

has been reported to be approximately 2 for small volume fractions from experimental observations\(^{(12)}\).

For the lattice continuum in this study, the apparent elastic modulus \( E(\Theta_{\alpha})/E \) is written as

\[
\frac{\bar{E}(\Theta_{\alpha})}{E} = \left( \Theta_{11} + \Theta_{12} + \Theta_{13} \right) \eta^2 - 2(\Theta_{11} \Theta_{12} + \Theta_{11} \Theta_{13} + \Theta_{12} \Theta_{13}) \eta + \Theta_{11} \Theta_{12} + \Theta_{11} \Theta_{13} + \Theta_{12} \Theta_{13}, \tag{8}
\]

and the volume fraction \( V_f \) is written as

\[
V_f = 3\eta^2 - 2\eta^4, \tag{9}
\]

for the case \( \eta_1 = \eta_2 = \eta_3 = \eta \). Thus, \( \bar{E}/E \) is related to \( V_f \) by the member width ratio \( \eta \).

For the cases \( (a) \ \Theta_{11} = 1 \) and \( (b) \ \Theta_{11} = \Theta_{12} = \Theta_{13} = 1/\sqrt{3} \), the apparent moduli \( E/E \) are plotted in Fig. 5 as solid lines against the volume fraction \( V_f \) (0.1 \( \leq V_f \leq 0.5 \)). These curves are approximated by Eq. (7) with the exponents \( n = 1.17 \) and 2.26, respectively, although Eqs. (8) and (9) do not yield a simple power law. These two lines indicate the upper and lower bounds of the elastic modulus \( \bar{E}/E \) with respect to the orientation of uniaxial stretching with a fixed volume fraction. Comparing these to the curve for the empirically obtained value of \( n = 2 \) shown in the figure.
by a broken line, the lattice continuum is reasonable as a mechanical model of cancellous bone in terms of the exponents \( n \) of the power law in Eq. (7).

4. Mechanical Bone Remodeling Taking Residual Stress into Account

The basic concept of mechanical bone remodeling, in which residual stress is taken into account\(^ {19,20,21,22} \), is applied to the three-dimensional lattice continuum representing the cancellous bone described in the previous section. The remodeling rate equation alters the width of the lattice elements resulting in changes in the structural parameters and natural states, which regulate the local stress in the lattice element.

4.1 Remodeling rate equation

Structural change due to remodeling of cancellous bone is caused by cellular activities at the trabecular surface\(^ {11} \). This implies that consideration of the local mechanical environment is essential in trabecular remodeling when the remodeling is considered to be a stress/strain regulation process adapting to the mechanical environment. In some mathematical models of bone remodeling the stress is used, and in others the strain is used as the argument in the rate equation. When no residual stress or strain remain in the bone tissue, the stress and strain are proportional to each other within the elastic range, and the only difference between the two arguments is the proportionality constant. This situation, however, does not occur, because the structure of cancellous bone has self-equilibrium stress as reported previously\(^ {21,22} \). In the case of soft tissue, in which the existence of residual stress is well known, the normal state at remodeling equilibrium is described by several hypotheses\(^ {23,24} \) which have been proposed to characterize the optimality of the stress and strain distributions in living tissue. We adopt the uniform stress hypothesis on the trabecular surface at remodeling equilibrium.

Resorption and deposition on the trabecular surface due to remodeling lead to changes in the width and orientation of the lattice members\(^ {25} \). In this study, orthogonality of the lattice and the structural orientation are assumed at the remodeling equilibrium, and attention is focused on the width of the lattice members and their natural state which are maintained by remodeling to adapt to changes in the mechanical environment. That is, the stress due to the axial force \( N_u \) acting on the lattice member is only used as the mechanical stimulus and the remodeling rate equation is written as a function of the normal component of the effective stress

\[
T_s^u = \frac{N_u}{A_i} = \frac{T_{ul} L_s}{A_i} = \frac{T_{ul}}{S_i},
\]

Approaching remodeling equilibrium at which the stress distribution is uniform, the nonuniformity of the effective stress \( T_s^u \) acts as the driving force for remodeling, and the remodeling rate equation of the remodel width \( W_u \) is written as

\[
-\frac{1}{W_u} \frac{\partial W_u}{\partial t} = -R_i \nabla^2 T_s^u,
\]

using the Laplace operator \( \nabla^2 \), which is a measure of the nonuniformity of the effective stress distribution, where \( R_i \) is the positive remodeling rate constant. This expression is a kind of rate equation based on the principle of optimal operation\(^ {25} \), and is a locally governed rule seeking the local equilibrium state under normal mechanical operation, without referring to the goal or optimal stress at remodeling equilibrium prescribed a priori. Changes in the member width \( W_u \) in Eq. (11) lead to changes in the apparent material constants in the constitutive equation (3).

4.2 Change in local natural state by remodeling

The natural length of the lattice member, that is the length in the individual stress-free state, may be altered by changes in the member width \( W_u \) due to remodeling as illustrated in Fig. 6. This change in natural state is reflected in the constitutive equation in terms of the initial strain \( \gamma_0 \). We consider a lattice member with cross-sectional area \( A_i \) and initial strain \( \gamma_0 \), and its cross-sectional change by remodeling under the strain \( \gamma_u \) due to an external load \( P \) as shown in Fig. 6 (b) to (c). It is expected that the new tissue shown by the hatched region in Fig. 6 (c) cannot refer to the natural state \( \gamma_0 \) of the old tissue under the strain \( \gamma_u \) in Fig. 6 (a), and the natural state of the new tissue may be different from that of the old tissue. As the simplest case in this context, it is assumed that the new tissue is formed in its own natural state with neither initial stress nor strain\(^ {21} \). Parallel combina-

![Fig. 6 Change in natural length of a lattice element due to remodeling](image-url)
tion of the old tissue with cross-sectional area $A_i$ and the new tissue with cross-sectional area $(\partial A_i/\partial t) dt$ during the infinitesimal time interval $[t, t + dt]$ determines the change in the natural state of the lattice member to $\gamma_i^{(t + dt)}$ as shown in Fig. 6(d). Then, the rate of initial strain $\gamma_i^{(t)}$ is written as

$$\frac{\partial \gamma_i^{(t)}}{\partial t} = -\frac{1}{A_i} \frac{\partial A_i}{\partial t} (\gamma_i^{(t)} - \gamma_i^{(0)}).$$

Replacing $\gamma_i^{(t)}$ in Eq. (3) by $(\gamma_i^{(t)} - \gamma_i^{(0)})$, we obtain the constitutive equation taking into account the change in the natural state. This means that the distribution of the local natural state becomes nonuniform due to remodeling, and residual stress may remain in the tissue due to the statical indeterminacy of the structure even in the unloaded state.

5. Vertebral Model as a Lattice Continuum

To simulate the remodeling of cancellous bone using the proposed remodeling rate equation of the lattice continuum, a vertebral body is modeled as a lattice continuum with reference to the quantitative characteristics of the trabecular architecture of the vertebral cancellous bone.

5.1 Measurement of quantitative characteristics of vertebral trabecular architecture

Specimens of cancellous bone 5 mm thick were excised from mature bovine coccygeal vertebra as shown in Fig. 7. The bone marrow was removed by boiling in 1% KOH solution for 40 minutes. Measurements were carried out on a transverse section 10 mm from the distal end as shown in Fig. 7(a), and on a coronal section at the center as shown in Fig. 7(b). The sectioned surface of the specimen was stained with black ink and observed using a CCD-camera. An image of a square region 5 mm by 5 mm was obtained by image processing with a spatial resolution of 300 x 300 pixels. The mean intercept length was measured using the black-and-white image of bone and marrow parts on the inspection lines at angular intervals of 1 degree, and the fabric ellipse was determined by fitting the obtained data plots of the mean intercept length to the ellipse.

The fabric ellipses at radial positions of 2 and 4 mm are magnified by a factor of 4.5, and shown in Fig. 8(a) for the transverse section. The principal axes of the fabric ellipses, that represent the orientation of the trabecular architecture, almost coincide with the radial and circumferential directions of the vertebra. The principal diameters of the fabric ellipses, which represent the characteristic lengths of the trabecular architecture, are slightly longer in the circumferential direction than in the radial direction. Figure 8(b)

Fig. 7 Trabecular architecture of cancellous bone in bovine coccygeal vertebra

Fig. 8 Distribution of fabric ellipses in vertebra

JSME International Journal

shows the fabric ellipses for the central coronal section with a magnification factor of 3.0. In this cross section, the principal directions of the fabric ellipses approximately align along the cephalocaudal direction, their principal diameters are relatively small near the cortical bone, and the ratio of the principal diameters is large in the cephalocaudal direction.

5.2 Vertebral body model

A bovine coccygeal vertebral body is simplified as a circular solid cylinder of cancellous bone, a hollow cylinder of cortical bone, and circular discs for the growth plate and end-plate at the distal and proximal ends, as shown in Fig. 9. Referring to the above observations, the model dimensions of the vertebral body are determined as follows. The radius of the boundary between the cancellous and cortical bones and that of the outer surface of the cortical bone are $r_1=8.0$ mm and $r_2=9.0$ mm; the thickness of the growth plate and the end-plate are $\delta_c=0.2$ mm and $\delta_e=1.8$ mm; and the resultant length in the cephalocaudal direction is $2L=40.0$ mm. Since the observed orientations of the trabecular architecture approximately coincide with the radial, circumferential, and cephalocaudal directions in the vertebral body as shown in Fig. 8, the orientation of the lattice structure is assumed to align along the $r$, $\theta$, and $z$ directions of the cylindrical coordinate system. The lattice intervals $L_i$ are determined as $L_r=0.60$ mm, $L_\theta=0.50$ mm, and $L_z=1.10$ mm, respectively, from the average diameters of the fabric ellipses in the transverse and coronal sections.

The elastic modulus of the lattice member material is determined as $E_i=4.32$ GPa by substituting the experimentally obtained apparent elastic modulus $E_a=1.21$ GPa in the cephalocaudal direction and the average member cross-sectional area ratio $S_a=0.28$, determined from the observed volume fraction, into Eq. (3). Assuming Poisson's ratio to be $\nu=0.3$, the shear modulus of the member material is determined as $G_i=2.92$ GPa. The cortical bone, end-plate, and growth plate are assumed to behave as isotropic elastic materials with elastic moduli of $E_c=E_k=4.32$ GPa, and $E_G=E_E/300$, and Poisson's ratios of $\nu_G=0.3$ and $\nu_c=0.48$.

6. Remodeling Simulation of Vertebral Body

To examine the validity of the proposed model as a phenomenological model for mechanical bone remodeling, a remodeling simulation for a vertebral body is carried out under a repetitive bending moment with constant compressive load. The obtained distributions of the apparent bone density and the residual stress are compared with the experimental results for bovine coccygeal vertebrae.

6.1 Remodeling simulation model

The remodeling simulation is conducted using a finite element method, formulated for couple stress theory. Assuming symmetry with respect to the plane $z=0$, the cranial half ($z>0$) is discretized as shown in Fig. 10 using rectangular hexahedral elements consisting of five tetrahedra (35). The cortical and cancellous bone between the growth plates is discretized into 9 layers in the cephalocaudal direction along the $z$ axis, and each layer in the transverse plane is discretized into 60 hexahedral elements for the cancellous bone and 24 elements for the cortical bone. Both the growth plate and the end-plate are considered to be a
single layer, consisting of 84 hexahedral elements.

For the initial condition at $t=0$, the member cross-sectional area ratios are set to $S_r = S_a = S_x = 0.1$ for the cancellous bone, that is, the member widths are set to $W_{rr} = W_{aa} = 0.16$ mm, $W_{aa} = W_{xx} = 0.19$ mm, and $W_{rr} = W_{xx} = 0.35$ mm. No initial strain $\gamma_{rr} = \gamma_{aa} = \gamma_{xx} = 0$ is imposed for the cancellous bone at $t=0$. The cortical bone, growth plate, and end-plate are assumed to have conventional elasticity throughout the remodeling simulation.

Figure 10 shows the boundary conditions for a compressive load of $P=600$ N and a repetitive bending moment of $M=300$ N-m, acting on the end-plate surface at $z=20.0$ mm facing the intervertebral disc, where the axis and direction of bending change in a cyclic manner. The remodeling rate constant is set as $R_t = 0.2$ mm$^2$/MPa$\cdot$day, in which $\Delta t$ represents a unit time such as a day.

6.2 Stress regulation by remodeling

Obtained distributions of the member cross-sectional area ratio $S_x$ on the central transverse section at $z=0$ and the central coronal sections at $\theta=0$ and $\pi$ are shown in Fig. 11 for $t=100$. In this figure, the range of $0.1 \leq S_x \leq 0.4$ is represented by a gray scale with 11 grades, where the growth plate is shown in white and the end-plate and the cortical bone are shown in black. The distribution of the effective stress $\sigma_{rr}$ along the radial direction on the central transverse section at $z=0$ is shown in Fig. 12(a), and that of $\sigma_{rr}$ along the cephalocaudal direction for the outer cancellous bone is shown in Fig. 12(b). At $t=0$, as shown by the broken line in Fig. 12(a), the average value of the effective stress $\sigma_{rr}$ under $P+M$ and $P-M$ has a maximum magnitude in the central region, and is small near the cortical bone. This nonuniform distribution of $\sigma_{rr}$ activates the remodeling process and the distribution becomes almost flat at $t=100$, as shown by the solid line in Fig. 12(a). The resulting distribution of the member cross-sectional area ratio $S_x$ is shown in Fig. 11. This shows that the region with relatively large area ratio, that is high volume fraction, forms a circular cone from the base facing the end-plate to the centroid of the body, which is found to be a triangular region in the coronal section of the vertebral body as shown in Fig. 7(b). As for the effective stress $\sigma_{rr}$, its magnitude increases, as shown by the solid line in Fig. 11(b), as a result of the generation of negative initial strain in the radial direction.

6.3 Residual stress caused by remodeling

As a result of remodeling, the initial strain $\gamma_{rr}$
distributes nonuniformly, so that a residual stress $\sigma^r$ remains in the vertebral body due to the statical indeterminacy of the structure even when all the external loads are removed. In the cephalo-caudal direction, a positive residual stress $\sigma^r_z$ remains in the cancellous bone and a negative stress remains in the cortical bone as shown in Fig. 12 (a). In the radial direction, a positive residual stress $\sigma^r_r$ remains in the outer cancellous bone as shown in Fig. 12 (b), and, in the circumferential direction, a negative residual stress $\sigma^r_\theta$ remains in the cortical bone. These residual stresses are induced by remodeling, that is, they contribute to make the effective stresses uniform under external loads.

We have observed residual stress in the cancellous-cortical bone systems of bovine coccygeal vertebrae as summarized in the Appendix. The same experiment is performed numerically in order to examine the residual stress caused in a remodeling simulation. In the actual experiment, strain gauges are attached to the cortical surface at the center in the cephalo-caudal direction as shown in Fig. 13 (a). In the experimental procedure, the end-plate is removed first, and the cancellous bone is subsequently removed. In the same way, the strain changes are calculated at each stage of the numerically simulated removal operation. The strain changes obtained are listed in Table 1 with those obtained experimentally. When the end-plate is removed, the strain change in the cephalo-caudal direction, $\Delta e^{\text{ca}}_z$, is larger than that in the circumferential direction, $\Delta e^{\text{ca}}_\theta$. When the the cancellous bone is removed, the strain change in the cephalo-caudal direction, $\Delta e^{\text{ca}}_z$, is smaller than $\Delta e^{\text{ca}}_\theta$. The resultant strain changes, $\Delta e^{\text{ca}}$, in the cephalo-caudal direction are smaller than those, $\Delta e^{\text{ca}}$, in the circumferential direction. These qualitative characteristics determined numerically agree reasonably well with those observed experimentally, and the resultant strain changes in both the cephalo-caudal and

![Diagram](image)

Fig. 13 Specimen of bovine coccygeal vertebra for residual stress release experiment

<table>
<thead>
<tr>
<th></th>
<th>End-plate $\Delta e_z$</th>
<th>Cancellous bone $\Delta e_\theta$</th>
<th>Total $\Delta e = \Delta e_z + \Delta e_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta e^{\text{ca}}_z$</td>
<td>33.3</td>
<td>53.0</td>
<td>86.3</td>
</tr>
<tr>
<td>$\Delta e^{\text{cr}}_z$</td>
<td>22.7</td>
<td>96.9</td>
<td>119.6</td>
</tr>
<tr>
<td>$\Delta e^{\text{ca}}_\theta$</td>
<td>75.4</td>
<td>10.4</td>
<td>85.8</td>
</tr>
<tr>
<td>$\Delta e^{\text{cr}}_\theta$</td>
<td>8.8</td>
<td>113.5</td>
<td>122.3</td>
</tr>
</tbody>
</table>

circumferential directions, $\Delta e^{\text{cr}}_z$ and $\Delta e^{\text{cr}}_\theta$, quantitatively agree with the experimental values, although the individual magnitudes at each stage does not. Therefore, the basic validity of the proposed phenomenological model is demonstrated through results obtained using a vertebral model with a simple shape.

7. Conclusions

The adaptation mechanism of bone to its mechanical environment has long been a subject of interest and investigation in bone mechanics. In this work, based on the concept that bone remodeling is regulated by local mechanical stimuli on a microstructural level related to cellular activities, a model of mechanico-bone remodeling of cancellous bone is proposed as a regulation process of microstructural level stress using a three-dimensional lattice continuum. An adaptive remodeling simulation is conducted for a vertebral body.

A three-dimensional lattice continuum is proposed as a mechanical model of cancellous bone with trabecular architecture, which is used to represent the mechanical hierarchy of the bone tissue microstructure in the study of the continuum mechanics. The dependence of the structural parameters in the constitutive equation on the apparent mechanical properties are investigated in terms of its anisotropy and the relationship between the elastic modulus and the apparent density. The results show that the model is reasonable as a mechanical model of cancellous bone.

Assuming the local uniform stress state to be the optimal stress state at remodeling equilibrium, a remodeling rate equation is proposed as a regulation process of the stress at the lattice element level by changing the width of the lattice elements which results in changes in the structural parameters and in the natural length of the lattice elements represented in terms of the initial strain in the constitutive relation. Based on geometrical observations of the trabecular architecture of cancellous bone, a bovine vertebral body is modeled as a lattice continuum, and a remodeling simulation is conducted under repetitive bending with compression to examine the validity of the proposed model. The obtained distributions of the
volume fraction and the residual stress at remodeling equilibrium show reasonable agreement with those observed in actual vertebrae. These simulation results lead to the conclusion that the proposed model for mechanical remodeling of bone with tissue structure using a three-dimensional lattice continuum taking into account the residual stress can be applicable to predict the adaptive remodeling of bone phenomenologically.

Acknowledgment

This work was financially supported in part by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science, Sports and Culture of Japan. The support by Japan Society for the Promotion of Science (Postdoctoral Fellowships for Research Abroad 1998) is gratefully acknowledged. The authors also thank Mr. O. Matsui for helpful assistance.

Appendix: Observation of Residual Stress in Vertebral Body

Experimental observation of the residual stress in a vertebral body\(^{21}\) is summarized here. The most cranial vertebrae from fresh bovine tails about 2 years old were used as specimens. Two biaxial waterproof strain gauges were bonded to the cleaned cortical surface and positioned symmetrically with respect to the sagittal plane between the spinous process and the transverse process on the half-way plane in the cephalocaudal direction, as indicated with crosses (R and L) in Fig. 13. The principal axes of the gauges were arranged along the cephalocaudal and circumferential directions. Strain changes are measured after 1 hour in a saline bath after each of the following operations.

1. Reference state of the strain was determined.
2. Both cranial and caudal end-plates were removed carefully by cutting the growth plates with a handsaw.
3. Cancellous bone shown by horizontal hatching in Fig. 13(b) was removed using a light-duty rotary cutter.

The measured strains induced in the cephalocaudal and the circumferential directions, \(\Delta e_x\) and \(\Delta e_y\), are averaged over 7 specimens as listed in Table 1\(^{21}\).

References


