Numerical Simulation and Experimental Verification of Wave Propagation in a Collapsible Tube*

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Propagation of isolated pressure wave through a thin-walled collapsible tube filled with a viscous incompressible fluid is calculated numerically. One-dimensional continuity equation, equation of motion including a wall friction term, and the tube law obtained experimentally are used as basic equations. The pressure waveforms at three locations along the collapsible tube calculated by the present method are compared with those obtained by the experiment. It is shown that the tube law obtained experimentally needs to be modified such that slope is discontinuous at a point where locally two-dimensional buckling of the collapsible tube occurs. The viscous effect is evaluated using an equivalent Womersley number α for the isolated wave. Incorporation of the wall friction term with high accuracy is necessary in the case of small α in one-dimensional flow calculation. The propagation velocity can be approximately calculated using the Moens-Korteweg velocity corresponding to a tube with circular cross-section.

Key Words: Biomechanics, Blood Flow, Collapsible Tube, Wave Propagation, Numerical Simulation, Tube Law, Buckling, Womersley Number, Moens-Korteweg Velocity

1. Introduction

Collapsible tubes are easily deformed due to change in transmural pressure. This property is similar to blood vessels so that the collapsible tube is often used as a model of blood vessel⁽¹⁾. Pulse waves are generated by pulsation of the heart and propagate through blood vessels. The flow generated in arteries is periodic starting and stopping flows, i.e., flow occurs suddenly from nearly quiescent state and suddenly stops after small duration of pulse flow⁽²⁾. One period of the pulse flow therefore can be approximated by

isolated pulse flow which is not influenced by flows before and after the pulse⁽³⁾. In the present research, the periodic pulse wave propagating through the artery is modeled by isolated pulse wave propagating through a collapsible tube.

A combined problem is necessary to be solved to obtain unsteady flow in the collapsible tube filled with a fluid. Ling et al.⁽⁴⁾ combine experimental data on local pressure, pressure gradient and radius for a tube having circular cross-section with radial component of the axi-symmetric Navier-Stokes equation. Liepsch et al.⁽⁵⁾ combine the linear Donnell equation for a cylindrical shell with the axi-symmetric Navier-Stokes equation in which convection term is neglected. Kamm et al.⁽⁶⁾ and Shimizu et al.⁽⁷⁾ combine static tube law, expressing relationship between transmural pressure and cross-sectional area, with one-dimensional flow equation in the longitudinal direction. In these reports, pre-buckling circular cross-section to post-buckling large deformation is treated. Matsuzaki et al.⁽⁸⁾ combine static equilibrium equation for elastic membrane supported by springs with one-dimensional flow equation in the longitudinal

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direction. Hayashi et al.⁽⁹⁾ combine equilibrium equation for upper and lower membranes of rectangular flow passage and static tube law with one-dimensional flow equation in the longitudinal direction.

In the present report, the basic equations similar to those of Kamm et al. are used, but a numerical method is used to analyze propagation of the pressure wave. Modification to the tube law around a buckling point, influence of the wall friction and propagation velocity of the pressure wave are calculated, and calculation results are compared with corresponding experimental results in order to verify the calculation method and to discuss the calculation results.

2. Numerical Calculation Method

One-dimensional mass and momentum conservation equations for viscous flows generated by propagation of pressure wave through a collapsible tube which is filled with a quiescent liquid at initial state are expressed by

$$\frac{\partial A}{\partial t} + \frac{\partial(Av)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + f(v) = 0 \quad (2)$$

where A is the cross-sectional area of the tube, v is the fluid velocity in the longitudinal direction, p is the internal pressure of the tube and the gravitational force is omitted. The fluid density is assumed to be constant owing to the fact that a liquid is filled in the collapsible tube whose compliance is large. In Eq. (2), f shows wall friction force expressed by⁽¹⁰⁾

$$f = \frac{8\nu}{R^2}v + \frac{2}{R}\sqrt{\frac{\nu}{\pi}} \int_0^t \frac{\partial v}{\partial \tau} \frac{d\tau}{\sqrt{t-\tau}} \quad (3)$$

where ν is the kinematic viscosity. Equation (3) is originally derived for unsteady laminar flow in a rigid straight tube with radius R , and the first term on the right side implies quasi-steady friction corresponding to the Poiseuille flow and the second term implies

additional unsteady friction. It is assumed that $v=0$ at $t=0$ and the radius is defined by $R=\sqrt{A/\pi}$.

The tube law denoting static relation between the transmural pressure p_{tr} and the cross-sectional area A is obtained from our experiment and expressed as

$$A = A(p_{tr}), \quad p_{tr} = p - p_e \quad (4)$$

where p_e is an external pressure of the tube. Let the initial constant internal pressure be p^* and the initial constant transmural pressure be p_{tr}^* . The disturbance pressure $\tilde{p}$ is defined by

$$\tilde{p} = p - p^* = p_{tr} - p_{tr}^* \quad (5)$$

where the external pressure of the tube is assumed to be unchanged. Applying Eqs. (4) and (5) to Eqs. (1) and (2) gives the following equation.

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} = F \quad (6)$$

where

$$Q = \begin{bmatrix} \tilde{p} \\ v \end{bmatrix}, \quad E = \begin{bmatrix} (\tilde{p} + \rho c^2)v \\ \frac{\tilde{p}}{\rho} + \frac{v^2}{2} \end{bmatrix}, \quad (7)$$

$$F = \begin{bmatrix} \tilde{p} \frac{\partial v}{\partial x} + \rho v \frac{\partial c^2}{\partial x} \\ -f \end{bmatrix} \quad (8)$$

$$c = \sqrt{\frac{A}{\rho} \frac{dp_{tr}}{dA}}$$

c is identical to the Moens-Korteweg velocity⁽¹¹⁾ if the cross section of the tube is circle.

Equation (6) is solved numerically using the two-step Lax-Wendroff method⁽¹²⁾ to give $\tilde{p}(x, t)$, $v(x, t)$, $A(x, t)$, and then $p(x, t)$ is calculated by Eq. (5).

3. Experimental Method

Figure 1 shows schematic diagram of the experimental apparatus. The collapsible tube used in the experiment is made of natural rubber with the Young modulus $E=9.72 \times 10^5$ Pa and the Poisson ratio $\nu=0.5$. The collapsible tube with circular cross-section is 16 mm in inner-diameter, 1 mm in thickness and 3840

Fig. 1 Experimental apparatus

mm in length. The experiment is performed under the condition that this tube is stretched in the longitudinal direction by 25% in length and is horizontally supported. The stretch is given because of the fact that arteries in a body are naturally under a condition of longitudinal stretch, maximum value of which is about 40%⁽¹³⁾. A rigid tube located upstream is longitudinally connected to the collapsible tube and a rubber tube located further upstream is longitudinally connected to the rigid tube. An isolated pressure wave is generated by applying an external pulse force to the rubber tube with a hammer. Immediately after the pressure wave is generated, pressures at the static pressure taps of Q_1, Q_2, Q_3, Q_4 in Fig. 1, being spaced at equal interval of 400 mm, are measured with semiconductor pressure transducers through vinyl tubes. The vinyl tube filled with silicone oil is 5 mm in inner-diameter, 1 mm in thickness, 600 mm in length and of high rigidity. The distance from the location where the external pulse force is applied to the location $x=0$ in Fig. 1 where the rigid and collapsible tubes are connected is 350 mm. The distance from $x=0$ to the location of Q_1 is 300 mm. A rigid tube is connected to downstream end of the collapsible tube and the distance from the location of Q_4 to this downstream end location is taken as large as 3300 mm. Influence of reflection waves from this downstream end and from right edge of the downstream rigid tube can therefore be eliminated by taking short duration of the measurement. The collapsible tube is immersed in the same liquid as that in the collapsible tube to remove influences of self-weight of the collapsible tube and of hydro-static pressure. The liquid level in a reservoir tank located most downstream is changed to adjust the initial transmural pressure. Two kinds of liquids are used, water and glycerin solution with mass fraction of 60%. The density and kinematic viscosity of the water are 998.2 kg/m^3 and $1.002 \times 10^{-6} \text{ m}^2/\text{s}$, respectively. The density and kinematic viscosity of the glycerin solution are 1158 kg/m^3 and $17.27 \times 10^{-6} \text{ m}^2/\text{s}$, respectively.

4. Calculation Results and Discussion

4.1 Calculation conditions

Calculation region is from the point Q_1 in Fig. 1 to a sufficiently downstream point ($x=4000 \text{ mm}$). Pressure waveform measured at Q_1 is used as the left boundary condition and undisturbed state is specified as the right boundary condition. The time-marching calculation is stopped before the pressure wave reaches the right boundary to remove reflection waves from there. The initial conditions are $\bar{p}=0, v=0$ and $A=A^*$, where A^* is an initial cross-sectional area of the collapsible tube.

The solid line in Fig. 2 shows the tube law of the collapsible tube obtained from the experiment⁽¹⁴⁾ and the broken line shows the compliance per unit tube-length (C/L) which is reciprocal of differentiation of the tube law with respect to A . The numerical calculations of the pressure wave propagation are performed using the solid circle ($p_{tr}^*=0 \text{ Pa}$) and the open circle ($p_{tr}^*=1960 \text{ Pa}$) as two kinds of initial conditions. Calculation results are compared with corresponding results measured experimentally. The thin solid line in Fig. 3 is magnified plot of the tube law around the solid and open circles in Fig. 2. The small circles on the thin solid line in Fig. 3 denote measured data of the tube law and the thin solid line is an interpolation curve of them. Measurement errors are

Fig. 2 Tube law and tube compliance

Fig. 3 Magnification of original and modified tube laws near $p_{tr}=0$

included in the data on the tube law, so that smooth interpolation curve of $A - dp_{tr}/dA$ is not obtained even if higher-order interpolation curve is used. The dp_{tr}/dA is a dominant factor relevant to the propagation velocity of the pressure wave, as is shown in Eq. (8). Oscillations in the pressure waveform often occurred when the numerical calculation is performed using unsmooth $A - dp_{tr}/dA$ relation. In order to overcome this difficulty, smoothing is carried out after $A - dp_{tr}/dA$ relation is calculated at data points by fourth-order central difference. The thin solid line in Fig. 4 shows magnified plot of $dp_{tr}/dA - p_{tr}$ curve thus obtained around the solid and open circles.

4.2 Calculation results in the case of using original tube law

The chain lines in Fig. 5 (for water) and Fig. 6 (for glycerin solution) show calculation results of change in disturbance pressure with time, where dp_{tr}/dA in Eq. (8) is calculated from the thin solid line in Fig. 4. The upper figures of Figs. 5 and 6 are for $p_{tr}^* = 0$ Pa. The lower figures of Figs. 5 and 6 are for $p_{tr}^* = 1960$ Pa, where the chain lines almost overlap the solid lines which will be described later. In Figs. 5 and 6, the broken lines show measured results and $x = 0.3, 0.7, 1.1, 1.5$ m are distances from the upstream end location of the collapsible tube to the locations of Q_1, Q_2, Q_3, Q_4 , respectively. The four levels of the abscissas with $x = 0.3, 0.7, 1.1, 1.5$ m denote the pressure of $\bar{p} = 0$ or $p_{tr} = p_{tr}^*$. The pressure waveform denoted by the broken line at $x = 0.3$ m is treated as input waveform, and the calculated and measured pressure waveforms are compared at $x = 0.7, 1.1, 1.5$ m. It is shown that discrepancy between the chain and broken lines for the water is smaller than that for the glycerin solution. In the upper figures of Figs. 5 and 6, the calculated results denoted by the chain lines show that the leading edge of pressure waveform steepens as the wave propagates from $x = 0.7$ to 1.5 m. In Fig. 5, the

Fig. 4 Magnification of original and modified dp_{tr}/dA near $p_{tr}=0$

chain lines show that the propagation velocity around the trailing edge decreases as the wave propagates.

4.3 Calculation results in the case of using modified tube law

Contrary to the measured pressure waveforms, the steepening of the leading edge of the waveform and decrease in the propagation velocity around the trailing edge appear only in the calculation results. These phenomena are caused by decrease in the propagation velocity in the region of small disturbance pressure. The propagation velocity is approximately determined by Eq. (8), as will be shown below. Therefore, the decrease in the propagation velocity is caused by the fact that dp_{tr}/dA and A in Eq. (8)

Fig. 5 Pressure waveforms for water (solid line: using modified tube law, chain line: using original tube law)

Fig. 6 Pressure waveforms for glycerin solution (solid line: using modified tube law, chain line: using original tube law)

become small in the region where $\bar{p}$ or $p_{tr} - p_{tr}^*$ is small.

The measured data of the tube law around the region where $\bar{p}$ is nearly equal to zero are shown by the small circles on the thin solid line in Fig. 3. These data were not obtained by measuring local cross-sectional area of the tube, but calculated by dividing inner volume of the tube by the tube length. The inner volume was obtained by measuring an inner volume at zero transmural pressure and then calculating change in the inner volume by reading change in water level in a perpendicular glass tube connected to the collapsible tube when the transmural pressure is varied. The tube law shown in Fig. 2 or 3 is therefore one obtained

by averaging local tube laws at every cross-sections of the collapsible tube. The portion of thin solid line with large change in slope is modified into the thick solid line as shown in Fig. 3. The solid and open circles in Figs. 3 and 4 then move to the solid and open squares, respectively. The slope discontinuity in the thick solid line in Fig. 3 means a buckling point of the collapsible tube corresponding to two-dimensional deformation in which the tube geometry is uniform in the longitudinal direction. When p_{tr} is decreased to the buckling pressure, the cross-section of the tube changes from circle to a shape with n -lobe mode, resulting in discontinuity in the tube law⁽¹⁵⁾. The slope discontinuity can disappear in our experimental method, because the tube used was not perfectly uniform in the longitudinal direction, resulting in longitudinally non-uniform buckling pressure and because both ends of the tube are circularly clamped, resulting in no buckling at these cross-sections. The modification to the tube law shown in Fig. 3 means that locally two-dimensional buckling occurs at each cross-section. The modification except around the buckling point is not necessary, because length to diameter ratio of the tube used in the experiment of measuring the tube law is as large as thirty.

The modification curve is determined by extrapolation of each of the two curves, that is, one is larger and the other is smaller than the buckling pressure. In Fig. 3, the broken line shows a theoretical curve of $A - p_{tr}$ (see Appendix) in the region where the circular cross-section does not buckle, and the double circles on the broken line show the buckling points for $n=2, 3$ ⁽¹⁵⁾. The modification curve in Fig. 3 is plotted reasonably in view of the two-dimensional buckling, because the thick solid line agrees well with the broken line in the region of $p_{tr} > 0$ and because the point where slope is discontinuous in the thick solid line agrees well with the point of $n=2$.

The solid lines in Figs. 5 and 6 show calculation results obtained by using the modified tube law. In the upper figures of Figs. 5 and 6, the solid lines are closer to the measured results than the chain lines are, because the steepening disappears around the leading edge of waveform and because the propagation velocity increases around the trailing edge. On the other hand, in the lower figures of Figs. 5 and 6, the influence of the modification to the tube law is small due to the fact that p_{tr}^* ($=1960$ Pa) locates near the upper boundary of the modification region. Although the modification will become very small in Fig. 2, the influence of the modification on the pressure waveform is large. This is because the modification is applied to the region where the slope of the tube law changes largely. These calculation results show that

in the numerical calculation of the pressure wave propagation through a collapsible tube, the tube law obtained from the experiment is necessary to be modified such that the slope is discontinuous at the buckling point.

4.4 Influence of wall friction

In Fig. 7, the chain lines show calculated pressure waveform at $p_{tr}^* = 1960$ Pa for the water (upper figure) and for the glycerin solution (lower figure) in the case where the wall friction term f in Eq. (2) is supposed to be zero. The solid lines in Fig. 7 are reproduction of the calculated pressure waveforms in the case of $f \neq 0$ denoted by the solid lines in Figs. 5 and 6. The difference between the solid and chain lines in the upper figure of Fig. 7 is small, but the difference is

Fig. 7 Pressure waveforms (solid line: having wall friction, chain line: having no wall friction)

large in the lower figure. Therefore, the effect of the wall friction is small for the water, but the effect is large and to be taken into account for the glycerin solution.

In Fig. 8, the solid lines show the calculated velocity, the chain lines the calculated pressure, and the broken lines the calculated negative pressure-gradient ($-\partial \tilde{p}/\partial x$) at $p_{tr}^* = 1960$ Pa for the water (upper figure) and the glycerin solution (lower figure). The boundary layer thickness δ on the tube wall for the pulse velocity shown in Fig. 8 is approximated by $\delta \sim \sqrt{\nu \Delta T}$ based on the Stokes boundary layer⁽¹⁶⁾, where ΔT denotes the pulse width. The velocity waveform shown in Fig. 8 can be treated as periodic one in pulsatile flows, as described in the introduction. The duration of velocity with nearly zero magnitude is ΔT to $2\Delta T$ in the pulsatile velocity waveform of human arteries⁽²⁾. The angular velocity is then approximated by $\omega \sim 1/\Delta T$ to give

$$\alpha \approx R^* \sqrt{\frac{\omega}{\nu}} \sim \frac{R^*}{\sqrt{\nu \Delta T}} \sim \frac{R^*}{\delta} \quad (9)$$

where α denotes equivalent Womersley number based on the tube radius at initial state, the pulse width and

Fig. 8 Velocity, pressure waveforms and negative pressure gradient

the kinematic viscosity. The meaning of α is an approximated value of the ratio of the initial tube radius to the laminar boundary layer thickness developed during ΔT . Therefore, the influence of fluid viscosity is large for small α , but inviscid flow approximation can be used for large α . According to the solid lines in Fig. 8, $\Delta T \approx 0.2$ s for both water and glycerin solution. The values of α calculated by Eq. (9) are 16 for the water and 3.8 for the glycerin solution. Calculation results for oscillatory flow in a rigid straight tube with circular cross-section show that the longitudinal velocity profile is almost uniform except near the wall for $\alpha \sim 16$ and that the velocity profile is similar to the Poiseuille flow for $\alpha \sim 3.8$ ⁽¹³⁾. The effect of viscosity is dominant for $\alpha < 3$ and inviscid flow approximation can be used for $\alpha > 10$ ⁽¹³⁾.

The role of the equivalent Womersley number in Eq. (3) in the numerical calculation is that influence of the wall friction becomes small with increasing α due to small ν and ΔT as shown in Eq. (9), and vice versa. Therefore, in the case of large α , one-dimensional inviscid approximation is possible and influence of assumption for the wall friction is small. On the other hand, in the case of small α , highly accurate assumption for the wall friction is necessary, because the assumption has great influence on the viscous one-dimensional flow.

The perpendicular line segments in Fig. 8 show locations of maximum velocity. The decrease in the height of the segment with time for the glycerin solution is more rapid than that for the water owing to larger wall friction of the former. The fluid accelerates ($\partial v / \partial t > 0$) due to the ordinary pressure gradient ($-\partial \bar{p} / \partial x > 0$) and it decelerates ($\partial v / \partial t < 0$) due to the adverse pressure gradient ($-\partial \bar{p} / \partial x < 0$), as shown in Fig. 8. The foot of the perpendicular line almost agrees with the location where the pressure gradient is zero for the water, but the foot locates on the left of the location where the pressure gradient is zero for the glycerin solution. The latter means that the phase lag of the velocity behind the pressure gradient decreases, and this tendency is similar to that in the oscillatory flow in a rigid straight tube with circular cross-section, but the influence of large f in Eq. (2) generates this tendency for the glycerin solution.

The chain lines in Fig. 9 show calculation results for the glycerin solution at $p_{ir}^* = 1960$ Pa in the case where only the quasi-steady wall friction expressed by the first term on the right side of Eq. (3) is taken into account and where the unsteady wall friction expressed by the second term is omitted. The solid lines in Fig. 9 are reproduction of the calculation results shown by the solid lines in the lower figure of Fig. 6, where both quasi-steady and unsteady wall

Fig. 9 Pressure waveforms (solid line: having steady and unsteady wall frictions, chain line: having steady wall friction only)

friction terms are included. In Fig. 9, the propagation velocity and the maximum pressure of the chain lines agree more closely with those of the broken lines, but the solid lines have more similar waveform to the broken lines. It is known from the above considerations that the unsteady wall friction term is not of high accuracy in calculating propagation of the pressure wave in the collapsible tube, although it is reported⁽¹⁷⁾ that the unsteady wall friction is needed in calculating damping fluctuation pressure of an oil hammer with high accuracy. On the other hand, when the unsteady friction term is omitted for the water, difference of the results from the solid lines in the upper figure of Fig. 7 is negligible.

4.5 Discussion about turbulent transition

The parameters relevant to turbulent transition are maximum Reynolds number $\bar{Re}$ and Womersley number α , and the transition condition is given by $\bar{Re} > 250\alpha$ based on measurement results for blood flows in arteries of a variety of animals⁽¹⁸⁾. The turbulent transition occurs around maximum velocity or followed deceleration region, and local Reynolds number of the maximum velocity is larger than critical Reynolds number for steady flow⁽¹⁹⁾. The maximum Reynolds numbers obtained from the calculation results shown in Fig. 8 are $\bar{Re} = 10700$ for the water and $\bar{Re} = 460$ for the glycerin solution, because of no experimental data on the velocity. The value of 250α is calculated using α obtained in section 4.4 to give $\bar{Re} > 4000$ for the water and $\bar{Re} < 950$ for the glycerin solution. The turbulent transition condition is satisfied for the water. However, the wall friction is

chosen as that of laminar flow expressed by Eq. (3). This selection is based on the following considerations. The flow in arteries can be approximated by periodic starting and stopping flows, and before starting the flow is not perfectly quiescent but still includes small disturbances⁽²⁾. However, a finite time is required for the small disturbances to develop into large disturbances, and the laminar to turbulent transition does not occur in a very short time⁽²⁰⁾. In our experiment, the pulse velocity propagates through completely quiescent fluid so that the transition to turbulence will not occur. The measured pressure waveform does not include turbulent fluctuations, but the turbulent flow, even if exists, would not be detected due to the fact that the pressure waveform is measured with the pressure transducer through a long tube in which fluctuations with high frequencies are attenuated. There are so few reports⁽²¹⁾ about the turbulent transition in starting flows that the transition point and the wall friction after the transition are further problems to be researched.

4.6 Propagation velocity

Let the cross-sectional area of the collapsible tube express by

$$A = A^* + \tilde{A} \quad (10)$$

where $\tilde{A}$ denotes disturbance cross-sectional area. Linearization of Eq. (4) with respect to the disturbance quantities gives

$$\tilde{p} = \left(\frac{dp_{tr}}{dA} \right)^* \tilde{A} \quad (11)$$

By substituting Eqs. (5), (10) and (11) into Eqs. (1) and (2) with $f=0$ and by neglecting second order terms of the disturbance quantities, the wave equations for $\tilde{A}$, $\tilde{p}$ and v together with the following equation are obtained

$$\tilde{p} = \rho c^* v \quad (12)$$

where

$$c^* = \sqrt{\frac{A^*}{\rho} \left(\frac{dp_{tr}}{dA} \right)^*} \quad (13)$$

denotes inviscid propagation velocity of small disturbance through the initial quiescent state. Eqs. (11) and (12) show relationships among the small disturbance quantities so that $\tilde{p}$ and v are in phase as shown in Fig. 8.

Figure 10 shows comparison between c^* and propagation velocity u calculated by the maximum pressure points of the waveforms shown in Figs. 5 and 6. For both water and glycerin solution, u is larger than c^* because of the fact that the disturbance velocity v is not added to c^* . The relative errors of c^* compared with u are 10.3% for the water and 7.0% for the glycerin solution at $p_{tr}^*=0$ Pa, where the error is larger than that at $p_{tr}^*=1960$ Pa. In the present calculation, the approximate propagation

Fig. 10 Propagation velocity

velocity can be obtained by c^* for both water and glycerin solution, because the order of v is one order in magnitude smaller than that of c^* . The order in magnitude of v can be estimated by c^* , $\tilde{p}_{tr}$ obtained experimentally and Eq. (12). The values of v calculated by Eq. (12) are 0.75 m/s for the water and 0.58 m/s for the glycerin solution, where $\tilde{p}$ is obtained by using maximum value of the broken line at $x=0.3$ m in the lower figures of Figs. 5 and 6. These values of v agree fairly well with the maximum values of solid lines in Fig. 8.

5. Conclusions

The propagation of isolated pressure wave through a collapsible tube filled with water or glycerin solution as a model of artery is calculated numerically. The following conclusions are obtained by comparing calculation results with corresponding experimental data.

(1) In the numerical calculation, it is necessary to assume that local two-dimensional deformation in the cross-section occurs around the buckling pressure. Therefore, the tube law obtained experimentally needs to be modified such that slope of the tube law becomes discontinuous at the buckling point.

(2) In the case of the equivalent Womersley number $\alpha \sim 16$ for the water, viscous effect is almost negligible. In the case of $\alpha \sim 3.8$ for the glycerin solution, the viscous effect needs to be taken into account.

(3) When the value of α is small and the viscous effect is not negligible, highly accurate estimation of the wall friction in one-dimensional calculation is necessary, since the velocity profile in a cross-section varies largely.

(4) The propagation velocity of the pressure wave in the present experiment can be obtained by using c^* . The relative errors of the propagation velocity are 10.3% for the water and 7.0% for the glycerin solution.

Appendix

Let the neutral radius be R_1 and the thickness be h_1 of a collapsible tube which is stretched longitudinally by 25%. When a transmural pressure above the buckling transmural pressure is imposed on this tube, the radius changes to R_2 and the thickness to h_2 . By applying Hooke's law with respect to the cylindrical coordinates (r, θ, z) to deformation of the tube, the following relations between stress σ and strain ε are obtained.

$$\left. \begin{aligned} \varepsilon_r &= \frac{1}{E} [\sigma_r - \kappa(\sigma_\theta + \sigma_z)] \\ \varepsilon_\theta &= \frac{1}{E} [\sigma_\theta - \kappa(\sigma_z + \sigma_r)] \\ \varepsilon_z &= \frac{1}{E} [\sigma_z - \kappa(\sigma_r + \sigma_\theta)] \end{aligned} \right\} \quad (14)$$

In these equations, $\varepsilon_z=0$ holds by assuming the plane strain state in the cross-section, $\sigma_\theta = p_{tr}R_2/h_2$ are obtained from equilibrium of the hoop stress and the transmural pressure, and $\sigma_r=0$, $\varepsilon_r = (h_2 - h_1)/h_1$, $\varepsilon_\theta = (R_2 - R_1)/R_1$ are used based on thin-wall approximation. Substitution of these equations into Eq.(14) gives R_2 and h_2 together with $A (= \pi(R_2 - h_2/2)^2)$. The theoretical curve in Fig. 3 shows relationship between p_{tr} and A thus obtained.

References

- (1) Pedley, T.J., *The Fluid Mechanics of Large Blood Vessels*, (1980), Cambridge Univ. Press.
- (2) Mills, C.J., Gabe, I.T., Gault, J.H., Mason, D.T., Ross, J. Jr., Braunwald, E. and Shillingford, J.P., *Cardiovasc. Res.*, Vol. 4 (1970), p. 405-417.
- (3) Okino, H., Sugawara, M. and Matsuo, H. (eds.), *Mechanics and Basic Measurements of Cardiovascular System*, (in Japanese), (1980), Koudansha.
- (4) Ling, S.C. and Atabek, H.B., *J. Fluid Mech.*, Vol. 55, pt. 3 (1972), p. 493-511.
- (5) Liepsch, D., Seemann, A. and Siekmann, J., *J. Biomech.*, Vol. 18, No. 9 (1985), p. 685-694.
- (6) Kamm, R.D. and Shapiro, A.H., *J. Fluid Mech.*, Vol. 95, pt. 1 (1979), p. 1-78.
- (7) Shimizu, M. and Tanida, Y., *J. Fluid Mech.*, Vol. 127 (1983), p. 315-339.
- (8) Matsuzaki, Y., Ikeda, T., Kitagawa, T. and Sakata, S., *Trans. ASME, J. Biomech. Eng.*, Vol. 116 (1994), p. 469-476.
- (9) Hayashi, S., Hayase, T. and Kawamura, H., *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese), Vol. 62, No. 594, B (1996), p. 556-563.
- (10) Achard, J.L. and Lespinard, G.M., *J. Fluid Mech.*, Vol. 113 (1981), p. 283-298.
- (11) Fung, Y.C., *Biodynamics: Circulation*, (1984), Springer.
- (12) Roache, P.J., *Computational Fluid Dynamics*, (1976), Hermora Pub.
- (13) Nichols, W.W. and O' Rourke, M.F., *McDonald's Blood Flow in Arteries*, 3rd ed., Edward Arnold, (1990).
- (14) Yoneyama, N., M. Thesis in Kansai Univ., (in Japanese), (1984).
- (15) Tadjbakhsh, I. and Odeh, F., *J. Math. Anal. Appl.*, Vol. 18 (1967), p. 59-74.
- (16) Schlichting, H., *Boundary-Layer Theory*, 7th ed., (1979), McGraw-Hill.
- (17) Zielke, W., *Trans. ASME, J. Basic Eng.*, Vol. 90 (1968), p. 109-115.
- (18) Parker, K.H., *Cardiovascular Flow Dynamics and Measurements*, Hwang, N.H.C. and Normann, N.A. (eds.) (1977), p. 633-663, Univ. Park Press.
- (19) Nerem, R.M. and Seed, W.A., *Cardiovasc. Res.*, Vol. 6 (1972), p. 1-14.
- (20) Nerem, R.M., *Cardiovascular Flow Dynamics and Measurements*, Hwang, N.H.C. and Normann, N.A. (eds.) (1977), p. 191-215, Univ. Park Press.
- (21) Maruyama, T., Kato, Y. and Mizushima, T., *J. Chem. Eng. Jpn.*, Vol. 11, No. 5 (1978), p. 346-353.