Friction of Sliding Leadscrews under Hydrodynamic Lubrication*

(1st Report, Establishment of Clearance Shape Model and Its Analysis Based on Infinitely Short Bearing Theory)

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Because sliding leadscrews used in precision positioning systems are directly driven by motors with higher speed, it becomes necessary to evaluate more accurately their friction losses to estimate the required motor power or the amount of heat generated. It has been proved experimentally that the interface between the screw shaft and the nut achieves a hydrodynamic condition under higher rotational speed. However, a method of calculating the friction has not yet been established. This study aims at proposing a theoretical formula to describe the frictional characteristics of sliding leadscrews under the hydrodynamic condition and at verifying it experimentally. In this report, lead errors of a screw shaft and a nut are measured accurately and discussed to formulate a clearance shape model with respect to the drunkenness error of the nut. The infinitely short bearing theory is applied to the shape model in order to derive some dimensionless numbers.

Key Words: Machine Element, Friction, Lubrication, Sliding Leadscrew, Hydrodynamic Bearing, Clearance Shape, Infinitely Short Bearing, Dimensionless Number

1. Introduction

As the demand for high-speed positioning has increased, sliding leadscrews used in precision positioning mechanisms are coming to be directly driven by motors at higher speed than usual^{(1),(2)}. Because sliding leadscrews have more friction losses than ball screws, it is necessary to evaluate the friction in order to estimate the required motor power or the amount of heat lost. In particular, frictional losses under a high-speed condition have an influence on the specifications of the positioning systems. However, a method of evaluating the frictional properties of the sliding leadscrews has not been established so far.

Some experimental results on the frictional properties under lubrication have been reported in the past⁽³⁾⁻⁽⁶⁾. They show that the interface between the

screw shaft and the nut achieves a hydrodynamic condition at a higher rotational speed. As a typical example, Fig.1 shows the author's experimental result for a screw shaft and a nut with a metric trapezoidal screw thread. The figure shows the coefficient of friction, μ , and the rate of separation, τ , when rotational speed N of the screw shaft and oil viscosity η are changed under a mean contact

Fig. 1 Friction of trapezoidal leadscrew

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pressure of p_0 . The rate of separation τ means the condition of electrical contact between the screw shaft and the nut: $\tau=0$ means that the screw shaft and the nut have intimate metallic contact, $\tau=1$ means that they are separated by an oil film. The horizontal axis in the figure represents a dimensionless variable called "bearing constant". As N and η are increased, μ decreases initially, and then increases along a straight line after taking a minimum value. The property is similar to the Stribeck curve of sliding bearings. The screw shaft and the nut begin to part near the minimum point of μ , and finally they are separated by an oil film to achieve the hydrodynamic condition. This study aims at constructing a general theory to describe the frictional properties of the sliding leadscrews under the hydrodynamic condition at the higher rotational speed, and at verifying the proposed theory experimentally.

It is possible to apply Reynolds's theory to the problem of hydrodynamic lubrication, if the profile of the clearance between the frictional surfaces (or shape of oil film) is given. In the case of sliding leadscrews, though the hydrodynamic condition is confirmed experimentally, the essential factors that contribute to the hydrodynamic condition are not clear. Therefore, the problems have never been formulated. It has been impossible to make quantitative discussion on the actual condition of the clearance between the screw shaft and the nut, because it is difficult to measure the dimensions of screw threads precisely. In particular, the measurement of internal screw threads has been difficult. However, recently, the accuracy of three-dimensional coordinate measuring machines has been markedly improved to overcome such difficulties⁽⁷⁾. In this study, the lead errors

of a screw shaft and a nut, which are the major factors affecting the clearance shape between the external and internal threads, are measured in detail to establish the clearance shape model between their flanks. Moreover, by applying the infinitely short bearing theory to the clearance shape model, some dimensionless numbers ruling the hydrodynamic friction of sliding leadscrews are derived. Not only does the normalized approach by the dimensionless numbers make it possible to estimate the frictional properties of any sliding leadscrews at higher speed, it also gives a rational guideline for the proposed method of modifying the lead of the screw by giving an intentional cyclic error⁽⁸⁾. In the 2nd report, the theoretical results in this report are verified experimentally.

2. Measurement of Lead Errors and Discussion

2.1 Lead errors and clearance between flanks

Figure 2 shows the relationship among the geometric parameters of metric trapezoidal screw threads. The external thread and the internal thread each have two flanks for the engagement of the screw thread. The flanks on the side where the axial load is supported are called pressure flanks, and the flanks on the opposite side are called clearance flanks. Consequently, the engagement of the screw thread is formed by these four flanks; two pressure flanks and two clearance flanks on each external and internal thread.

These four flanks are led along a helix of the screw thread. There are several methods of describing the coordinates along the helix. The right part of Fig. 2 shows the expanded view of a helix with pitch P wound around a cylinder with diameter d_p . Using φ for the angular coordinate about the center axis and y for the axial coordinate, the helix is defined as a

Fig. 2 Geometric parameters of metric trapezoidal screw threads

Fig. 3 Lead error and clearance

curve satisfying $y = (P/2\pi) \times \varphi$, while thread number i or length of circular arc x_c is defined uniquely corresponding to φ . In this paper, x is defined as the coordinate along the helix with a lead angle of β .

The helix of an actual screw thread has lead error, and each of the four flanks forming the screw engagement has its own cumulative lead error. To generate some hydrodynamic pressure under a relative slide between the flanks, the clearance of h_{ex} between the pressure flanks, or h_{bx} between the clearance flanks, must have a wedge shape in the x (or φ) direction. The relationship between h_{ex} or h_{bx} and φ is derived from the cumulative lead error of each flank, as shown in Fig. 3. If the cumulative lead errors of the external thread are Δl_e (at pressure flank) and Δl_b (at clearance flank), and if those of the internal thread are ΔL_e and ΔL_b , then h_{ex} and h_{bx} are described as follows.

$$\begin{aligned} h_{ex} &= E_1 + \Delta L_e - \Delta l_e \\ h_{bx} &= E_2 + \Delta l_b - \Delta L_b \end{aligned} \quad (1)$$

where E_1 or E_2 means the clearance between the pressure flanks or the clearance flanks, at $\varphi=0$, which is determined by the difference in the actual pitch diameter and relative axial position between the external and internal threads.

Drunkness error is defined as a residual after subtracting the cumulative pitch error from the cumulative lead error. The drunkness error means the fluctuation of lead within a pitch, which is obtained by eliminating the inclination from the cumulative lead error. Usually, the drunkness error and the cumulative pitch error of a long screw shaft are measured at different arrangements, and they are explained separately.

2.2 Measurement of lead error

The specimen used in the experiment shown in Fig. 1 is measured here to discuss the actual condition of lead error. The dimensions of the specimen are

Fig. 4 Dimensions of specimen

(a) Cumulative pitch error

(b) Drunkenness error

Fig. 5 Lead error of screw shaft

shown in Fig. 4. The screw shaft is made of SCM415 (with heat treatment after carburized), and its external screw thread is finished by lapping after thread grinding. The nut is made of BC6 (bronze casting), and its internal screw thread is finished by lapping after turning.

Figure 5 shows the cumulative pitch error and the drunkness error of the screw shaft measured by a lead measuring machine with a laser interferometer⁽⁹⁾. Figure 5(a) shows the cumulative pitch error and its approximation line, and Fig. 5(b) shows the drunkness error within five leads near the left end. Because the measurement of drunkness generally includes the effects of runout caused by bending or deflection of the specimen, the figure proves that the drunkness error of the screw shaft is less than 1 μm.

Figure 6 shows the cumulative lead error of the

Fig. 6 Lead error of nut

nut measured by a three-dimensional coordinate measuring machine (Zeiss UMM500+RT5). The cyclic errors by drunkenness are superposed, with the same phase at both flanks, on straight lines the gradient of each of which is $-7 \mu\text{m}/(10 \text{ threads})$. The period of drunkenness corresponds to one lead and the total amplitude is $4 \mu\text{m}$.

2.3 Discussion

In both cases of the screw shaft and the nut, the cumulative lead error is composed of the linear inclination and the cyclic drunkenness error. The linear inclinations are different between the screw shaft and the nut, i.e., $-1 \mu\text{m}/(10 \text{ threads})$ for the screw shaft and $-7 \mu\text{m}/(10 \text{ threads})$ for the nut. Therefore, the clearance determined by Eq.(1) has a wedge shape with an expansion ratio of $6 \mu\text{m}/(10 \text{ threads})$. On the other hand, the cyclic drunkenness error of the nut has a total amplitude of $4 \mu\text{m}$ and a period of one lead, while the drunkenness of the screw shaft is negligible. Therefore, when the screw shaft and the nut are engaged, the clearance between their flanks varies periodically, so that the clearance has the wedge shape at each part of the cycle where the local slope is negative.

The hydrodynamic pressure is dominated by the gradient of the clearance in the sliding direction. It is clear that the local gradient within a cycle of drunkenness is steeper than the linear inclination. Moreover, the linear inclination over the entire range of the nut length is not a constant cause of the hydrodynamic pressure, because it is easily changed by thermal deformation with temperature difference between the screw shaft and the nut. On the other hand, the cyclic wedge is slightly affected by the thermal deformation, so that it is a constant and dominant cause of the hydrodynamic pressure.

On the basis of the above discussion, the cyclic drunkenness error of the nut is considered to be the most dominant geometric factor affecting the hydrodynamic load capacity. The cyclic drunkenness

errors of the internal screw threads, ΔL_e and ΔL_b , are formulated using cosine functions as follows.

$$\left. \begin{aligned} \Delta L_e &= A_x \left(\cos \frac{n}{m} \varphi - 1 \right) = A_x (\cos \theta - 1) \\ \Delta L_b &= A_x \left(\cos \frac{n}{m} \varphi - 1 \right) = A_x (\cos \theta - 1) \end{aligned} \right\} \quad (2)$$

where m is the number of threads in the nut, and n is the number of cycles of drunkenness in the nut, so that the period of drunkenness is given as $\varphi = 2\pi m/n$ rad (if the period is one lead as shown in Fig. 6, then $m=n$). θ in Eq.(2) is a newly introduced variable which assumes 2π for one period of drunkenness. It is related to φ as follows.

$$\theta = \frac{n}{m} \varphi \quad (3)$$

Substituting Eq.(2) into Eq.(1) and letting $\Delta l_e = \Delta l_b = 0$, the following equations are obtained.

$$\left. \begin{aligned} h_{ex} &= A_x (\cos \theta - 1) + E_1 \\ h_{bx} &= A_x (1 - \cos \theta) + E_2 \end{aligned} \right\} \quad (4)$$

3. Establishment of Clearance Shape Model for Screw Threads

In the formulation of Eq.(4), the following are assumed.

(1) Both flanks of internal threads have periodic lead errors with a period of $\varphi = 2\pi m/n$ rad (Eq.(2)).

(2) The lead error of the screw shaft is much less than that of the nut, so that it is negligible.

(3) The simple pitch diameter of the nut is constant over the entire range of nut length, so that the difference in pitch diameter between the screw shaft and the nut is constant.

(4) The angle error of the thread ridge is negligible.

(5) Both axes of the screw shaft and the nut are concentric with each other. Moreover, the following are assumed for the discussion.

(6) The screw shaft and the nut have a rigid body.

(7) Viscous friction at the root of the thread groove is neglected.

h_{ex} and h_{bx} in Eq.(4) represent the clearances observed in the cross section including central axes. The true clearance, that is the shortest distance between the flanks, is observed in the cross section B' which is perpendicular to the thread ridge, as shown in Fig. 7, and h_e and h_b are given as follows.

$$\left. \begin{aligned} h_e &= h_{ex} \cos \alpha' \cos \beta \\ h_b &= h_{bx} \cos \alpha' \cos \beta \end{aligned} \right\} \quad (5)$$

where α' means the half-angle of the thread ridge observed in the cross section perpendicular to the thread ridge, and is related to α as follows⁽¹⁰⁾.

$$\tan \alpha' = \tan \alpha \cos \beta \quad (6)$$

The following equations are derived from Eqs.(4) and (5).

Fig. 7 Section of thread ridge

Fig. 9 Profile of clearance

Fig. 8 Clearance shape model

$$\begin{aligned} h_e &= \{A_x(\cos \theta - 1) + E_1\} \cos \alpha' \cos \beta \\ h_b &= \{A_x(1 - \cos \theta) + E_2\} \cos \alpha' \cos \beta \end{aligned} \quad (7)$$

Let us define the following parameters.

$$\begin{aligned} A &= A_x \cos \alpha' \cos \beta \\ h_{2e} &= E_1 \cos \alpha' \cos \beta - 2A_x \cos \alpha' \cos \beta \\ h_{2b} &= E_2 \cos \alpha' \cos \beta \end{aligned} \quad (8)$$

Using these parameters, h_e and h_b are expressed as follows.

$$\begin{aligned} h_e &= A(1 + \cos \theta) + h_{2e} \\ h_b &= A(1 - \cos \theta) + h_{2b} \end{aligned} \quad (9)$$

Consequently, Eq. (9) gives the clearance between pressure flanks, h_e , with a minimum clearance of h_{2e} , and the clearance between clearance flanks, h_b , with a minimum clearance of h_{2b} , both of which are cosine curves, as shown in Fig. 8. Furthermore, defining parameters c and e as follows,

$$\begin{aligned} h_{2e} + h_{2b} &= 2c \\ h_{2b} - h_{2e} &= 2e \end{aligned} \quad (10)$$

the following expressions are obtained.

$$\begin{aligned} h_e &= A(1 + \cos \theta) + (c - e) \\ h_b &= A(1 - \cos \theta) + (c + e) \end{aligned} \quad (11)$$

where e means the deviation of the thread position as shown in Fig. 8, and c is equal to h_{2e} (or h_{2b}) when $e = 0$.

Dividing both members of Eq. (11) by c and defining the following variables,

$$H_e = \frac{h_e}{c}, H_b = \frac{h_b}{c}, \lambda = \frac{A}{c}, \varepsilon = \frac{e}{c}, \quad (12)$$

the following expressions are obtained.

$$\begin{aligned} H_e &= \lambda(1 + \cos \theta) + (1 - \varepsilon) \\ H_b &= \lambda(1 - \cos \theta) + (1 + \varepsilon) \end{aligned} \quad (13)$$

Equation (13) gives a dimensionless expression for the clearance between pressure flanks or clearance flanks using c as representative length. In this study, ε is the rate of deviation, and λ is the amplitude ratio. For example, Fig. 9 shows the profiles of clearances H_e and H_b when $\lambda = 0.1$.

The parameter c is interpreted for the screw

thread as follows. From Eq.(11),

$$h_e + h_b = 2c + 2A = c_{p0} \sin \alpha', \quad (14)$$

where c_{p0} means the difference in simple pitch diameter (or clearance at simple pitch diameter) between the external thread and the internal thread. Moreover, from Eq.(14), the following equation is derived.

$$c = \frac{c_{p0}}{2} \sin \alpha' - A = \frac{c_p}{2} \sin \alpha' \quad \left(c_p = c_{p0} - \frac{2A_x}{\tan \alpha'} \right) \quad (15)$$

where c_p is given after subtracting the pitch diameter equivalence for the lead error with amplitude A_x from the clearance at simple pitch diameter, c_{p0} . Namely, c_p means the difference in virtual pitch diameter between the external thread and the internal thread (or clearance at virtual pitch diameter)⁽¹¹⁾.

4. Application of Infinitely Short Bearing Theory

4.1 Load capacity

It is assumed that the clearance between the thread ridge and the thread groove, for one period of drunkenness error as shown in Fig. 8, is filled with lubricant oil with viscosity η . When a relative velocity of v is given between them, hydrodynamic pressures, p_e and p_b , are generated at the pressure flank and the clearance flank, respectively. Integrations of the pressure distributions, p_e and p_b , give the load capacities, W_e and W_b . When the nut, on which the axial load of F is applied, is rotated by torque $T (= U_f d_p / 2$, where U_f is the circumferential force on the pitch line), the load acting on the thread ridge in section B' is $F / \cos \beta$, as shown in Fig. 7. Dividing the load by the number of drunkenness cycles, n , the equilibrium of forces for one cycle of drunkenness is formulated by the following equations.

$$W_a \cos \alpha' = \frac{F}{n \cos \beta} \quad (16)$$

$$W_a = W_e - W_b$$

W_a in Eq.(16) is called the resultant load capacity.

In the case of a practical sliding leadscrew, the length in the sliding direction is much greater than the width of the contact area; thus the special feature of the sliding surface is its extremely narrow shape. For example, in the case of the leadscrew shown in Fig. 4, the length of one lead along the x axis is $\pi d_p / \cos \beta = 144$ mm, while the length along the z axis (width) is $L = 4$ mm, so that the slenderness ratio reaches 36. Therefore, the infinitely short bearing theory is applicable to solve the distribution of p_e and p_b .

The basic formula for p_e follows the Reynolds equation, where the term of pressure gradient in the sliding direction is neglected^{(12),(13)}.

$$\frac{\partial}{\partial z} \left(\frac{h_e^3}{\eta} \cdot \frac{\partial p_e}{\partial z} \right) = 6v \frac{\partial h_e}{\partial x} \quad (17)$$

Integrating Eq.(17) twice with respect to z , and let-

ting $p_e = 0$ at $z = \pm L/2$, we obtain

$$p_e(x, z) = \frac{3\eta v}{h_e^3} \cdot \frac{\partial h_e}{\partial x} \left(z^2 - \frac{L^2}{4} \right). \quad (18)$$

Equation (18) gives the parabolic distribution along the z axis, as shown in Fig. 8, so that p_e has a positive value where $(\partial h_e / \partial x)$ is negative. Using dimensionless numbers defined as follows:

$$h_e = c H_e, \quad x = \frac{d_p}{2 \cos \beta} \cdot \frac{m}{n} \theta, \quad z = \frac{L}{2} \zeta, \quad (19)$$

then Eq.(18) is normalized as follows.

$$p_e(\theta, \zeta) = \frac{3\eta v L^2}{2c^2 d_p} \cdot \frac{n}{m} \cos \beta \cdot \frac{1}{H_e^3} \cdot \frac{\partial H_e}{\partial \theta} (\zeta^2 - 1) \quad (20)$$

Similarly, by substituting H_b for H_e of Eq.(20), p_b is given. Substituting Eq.(13) into Eq.(20) and letting $\zeta = 0$, the maximum pressure p_{0e} along the z axis is given as follows.

$$\begin{aligned} p_{0e}(\theta) &= \frac{3\eta v L^2}{2c^2 d_p} \cdot \frac{n}{m} \cos \beta \cdot g_e(\theta) \\ p_{0b}(\theta) &= \frac{3\eta v L^2}{2c^2 d_p} \cdot \frac{n}{m} \cos \beta \cdot g_b(\theta) \end{aligned} \quad (21)$$

where

$$\begin{aligned} g_e(\theta) &= -\frac{1}{H_e^3} \cdot \frac{\partial H_e}{\partial \theta} = \frac{\lambda \sin \theta}{\{\lambda(1 + \cos \theta) + (1 - \epsilon)\}^3} \\ g_b(\theta) &= -\frac{1}{H_b^3} \cdot \frac{\partial H_b}{\partial \theta} = \frac{-\lambda \sin \theta}{\{\lambda(1 - \cos \theta) + (1 + \epsilon)\}^3} \end{aligned} \quad (22)$$

The load capacity per unit length along the x axis at θ , w_e (at pressure flank) and w_b (at clearance flank), are given by

$$\begin{aligned} w_e(\theta) &= \frac{L}{2} \int_{-1}^1 p_e(\theta, \zeta) d\zeta = \frac{\eta v L^3}{c^2 d_p} \cdot \frac{n}{m} \cos \beta \cdot g_e(\theta) \\ w_b(\theta) &= \frac{L}{2} \int_{-1}^1 p_b(\theta, \zeta) d\zeta = \frac{\eta v L^3}{c^2 d_p} \cdot \frac{n}{m} \cos \beta \cdot g_b(\theta) \end{aligned} \quad (23)$$

In Eq.(21) or (23), the distributions of $p_{0e}(\theta)$ or $w_e(\theta)$ at the pressure flank, and $p_{0b}(\theta)$ or $w_b(\theta)$ at the clearance flank are determined by functions $g_e(\theta)$ and $g_b(\theta)$ in Eq.(22), respectively. Figure 10 shows $g_e(\theta)$ and $g_b(\theta)$ when $\lambda = 0.1$. As ϵ increases, g_e increases while g_b decreases. Assuming that the load capacity is neglected in the range where the pressure is negative, the resultant load capacity W_a is obtained by integrating Eq.(23) as follows.

$$W_a = W_e - W_b = \frac{\eta v L^3}{c^2} C_w, \quad (24)$$

where C_w (load constant) is given by

$$\begin{aligned} C_w &= \frac{1}{2} \left(\int_0^\pi g_e(\theta) d\theta - \int_\pi^{2\pi} g_b(\theta) d\theta \right) \\ &= \frac{1}{2} \left(\left[\frac{1}{2H_e^2} \right]_0^\pi - \left[\frac{1}{2H_b^2} \right]_\pi^{2\pi} \right) \\ &= \frac{\epsilon}{(1 - \epsilon^2)^2} - \frac{\epsilon(1 + 2\lambda)}{\{(1 + 2\lambda + \epsilon)(1 + 2\lambda - \epsilon)\}^2} \end{aligned} \quad (25)$$

$C_w = 0$ in Eq.(25) when $\epsilon = 0$, while the resultant load capacity $W_a = 0$ in Eq.(24). This means the condition of $W_e = W_b$ is realized, so that the condition of $\epsilon = 0$ is neutral.

Fig. 10 Profile of function $g_e(\theta)$

4.2 Coefficient of friction

Only the shear stress by the Couette flow between flanks is taken into account in the study of the friction force on the moving element in Fig. 8. The shear stress τ_s at the pressure flank is given as follows.

$$\tau_s = \frac{\eta v}{h_e} = \frac{\eta v}{c} \cdot \frac{1}{H_e} \quad (26)$$

The friction force at pressure flank, f_e , is given by

$$f_e = \frac{L d_p}{2 \cos \beta} \cdot \frac{m}{n} \int_0^{2\pi} \tau_s d\theta = \frac{\eta v L}{2c} \cdot \frac{d_p}{\cos \beta} \cdot \frac{m}{n} \int_0^{2\pi} \frac{d\theta}{H_e} \quad (27)$$

The friction force at clearance flank, f_b , is obtained similarly, so that the total friction f_a is given as follows.

$$f_a = f_e + f_b = \frac{\eta v L}{c} \cdot \frac{d_p}{\cos \beta} \cdot \frac{m}{n} C_f, \quad (28)$$

where C_f (friction constant) is given by

$$C_f = \frac{1}{2} \left(\int_0^{2\pi} \frac{d\theta}{H_e} + \int_0^{2\pi} \frac{d\theta}{H_b} \right) = \frac{1}{2} \left(\int_0^{2\pi} \frac{d\theta}{\lambda(1+\cos \theta) + (1-\epsilon)} + \int_0^{2\pi} \frac{d\theta}{\lambda(1-\cos \theta) + (1+\epsilon)} \right)$$

$$= \frac{\pi}{\sqrt{(1-\epsilon)(1-\epsilon+2\lambda)}} + \frac{\pi}{\sqrt{(1+\epsilon)(1+\epsilon+2\lambda)}} \quad (29)$$

The apparent coefficient of friction μ is formulated by Eqs. (24) and (28) as follows.

$$\mu = \frac{f_a}{W_a} = \frac{c}{L^2} \cdot \frac{d_p}{\cos \beta} \cdot \frac{m}{n} \cdot \frac{C_f}{C_w} \quad (30)$$

4.3 Conversion into screw thread parameters

It is assumed that the leadscrew has metric trapezoidal screw threads. W_a , v and L are related to the mean contact pressure at the screw thread, p_0 , the rotational speed of screw shaft, N , and the thread pitch P , respectively, as follows.

$$W_a = \frac{\pi d_p}{\cos \beta} \cdot \frac{m}{n} L p_0, \quad v = \frac{\pi d_p}{\cos \beta} N, \quad L = \frac{P}{2 \cos \alpha'} \quad (31)$$

Rearranging after substituting Eqs. (31) and (15) into Eqs. (24) and (30), the following dimensionless numbers, S and M , are obtained.

$$\left. \begin{aligned} S &= \left(\frac{d_p}{c_p} \right)^2 \left(\frac{P}{d_p} \right)^2 \left(\frac{n}{m} \right) \frac{\eta N}{p_0} = \frac{\sin^2 \alpha' \cos^2 \alpha'}{C_w} \\ M &= \left(\frac{d_p}{c_p} \right) \left(\frac{P}{d_p} \right)^2 \left(\frac{n}{m} \right) \mu = \frac{2 \sin \alpha' \cos^2 \alpha'}{\cos \beta} \cdot \frac{C_f}{C_w} \end{aligned} \right\} \quad (32)$$

The frictional property under the hydrodynamic condition between the screw shaft and the nut with the cyclic lead error as in Eq. (2) is described by the relationship between the dimensionless numbers S and M . Moreover, if the values of λ and ϵ are given, the values of S and M are determined by Eqs. (25) and (29), and the right side of Eq. (32).

Comparing the right sides of S and M in Eq. (32), the following relation is obtained under the condition that $\epsilon=0$ and $\lambda=0$.

$$M = \frac{4\pi}{\sin \alpha' \cos \beta} S \quad (33)$$

Equation (33) gives the relationship between S and M under the condition of $\epsilon=\lambda=0$, that is called the neutral plane condition. Equation (33) corresponds to Petroff's equation for journal bearings⁽¹⁴⁾.

5. Behavior of Basic Dimensionless Numbers

Figure 11 shows C_w and C_f calculated by Eqs. (25) and (29) with various ϵ and λ . It also shows S and M calculated by Eq. (32). Both C_w and C_f increase monotonically as ϵ increases. The increase in C_f is gentle, whereas C_w increases steeply when ϵ is larger than 0.8. M is proportional to C_f/C_w , while the increase in C_w is steeper than C_f ; therefore, M decreases monotonically.

Figure 12(a) shows the relationship between $(1-\epsilon)$ and S , while Fig. 12(b) shows that between M and S . The broken line in Fig. 12(b) represents the neutral plane condition given by Eq. (33). The $M-S$

Fig. 11 Behavior of dimensionless parameters

(a) $(1-\varepsilon)$ - S relation (b) M - S relationFig. 12 ε and M as a function of S (in case of $\cos \beta = 1$ and $\alpha = 15^\circ = \alpha'$)

relation in Fig. 12(b) shows that M shifts to a lower value as λ increases, and that the difference in M for various λ becomes greater as S decreases. The theoretical curves approach to the straight line of the neutral plane condition at large S . Moreover, from Fig. 12(a), it is considered that the increase in λ facilitates the formation of an oil film because $(1-\varepsilon)$ is equivalent to h_{2e}/c in Eqs.(10) and (12).

The experimental results of Fig. 1 are converted

into the relationship between M and S plotted in Fig. 12(b). The value of c_p in Eq.(32) is determined by actual measurement as $c_p = 0.07$ mm. The value of λ for the screw specimen is calculated by Eq.(12) as $\lambda = 0.21$ with the value of $2A_x = 4$ μ m in Fig. 6. The plots of experimental data approach the theoretical curves under the hydrodynamic condition, in the range where the Stribeck curve increases. However, experimentally determined M values are slightly larger than the theoretical curve of $\lambda = 0.2$. The tendency is considered to be caused by the effect of higher order modes in the surface profile (higher order modes of lead error in Fig. 6 or surface roughness), and the influence of elastic deformation.

Equation (32) and Fig. 12 give the frictional property described by the relationship between the dimensionless numbers, M and S , under the following conditions: nominal dimensional parameters of d_p , c_p and P , and driving conditions of η , N and p_0 . If the propriety of Eq.(32) is verified, it becomes possible to estimate the hydrodynamic friction under any condition. In the second report, some experimental results on several specimens with different nominal dimensions are compared with the theoretical results in this report, and the propriety of the proposed theory is verified in detail.

6. Conclusions

The frictional property under the hydrodynamic condition is discussed analytically with respect to the drunkenness error of the nut. The conclusions reached are as follows.

(1) On the basis of measurement of the cumulative lead error of the screw shaft and the nut, it is considered that the cyclic drunkenness error of the nut is the dominant cause of the hydrodynamic pressure between the screw shaft and the nut, so that the clearance shape model between the screw shaft and the nut is established with respect to the drunkenness error.

(2) The infinitely short bearing theory is applied to the established clearance shape model, so that the basic dimensionless numbers are formulated to describe the frictional property of the sliding lead-screw (with trapezoidal screw threads) under hydrodynamic lubrication.

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