Numerical Analysis of Flow and Fiber Orientation in Slit Channel Flow of Short Fiber-Polymer Melt Mixture*

Kazunori KATO**, Youxin ZHANG**
and Naoto OTAKE**

In injection molding of short fiber-polymer melt mixture, the velocity field of the mixture changes due to interaction between fibers, when fiber volume fraction increases. Interaction between fibers and a mold wall also has an effect on the mixture flow property, especially when fiber length is of the same order as the mold cavity thickness. In this paper, an iso-thermal flow of mixture through a slit channel and fiber orientation distribution in that flow are analysed numerically in consideration with fiber-fiber interaction and fiber-mold wall interaction, and the following results are obtained: (1) In rather short distance from entrance of a slit channel, fibers are rearranged rapidly and the flow becomes approximately steady after that. (2) When fiber volume fraction increases, the mixture flow tends to become plug-like one and pressure loss increases. (3) A layer of low fibers volume fraction is generated along mold wall due to fiber-mold wall interaction, and the pressure loss decreases as fiber length increases.

Key Words: Composite Material, Viscous Flow, Injection Molding, Short Fiber-Polymer Mixture, Fiber-Mold Wall Interaction, Slit Channel Flow, Three Dimensional Fiber Orientation Distribution

1. Introduction

In the flow of injection molding of short fiber-polymer melt mixture, apparent viscosity of the mixture increases considerably, because of interaction between fibers. Furthermore, interaction between a fiber and a mold wall makes influence on mixture flow property and fiber orientation, when fiber length is not very short, comparatively with the thickness of a mold cavity. In injection molding, the analysis of material flow and fiber orientation distribution is often made. In that situation, it becomes necessary to make clear the behavior of fibers in consideration with the above interactions to estimate the viscosity of the mixture precisely.

The authors discussed about flow property and

fiber orientation distribution near mold wall in a simple shear flow of a mixture, already⁽¹⁾. In this paper, a slit channel flow has been dealt with, and influence of interaction between fibers and that between a fiber and a mold wall on mixture flow property, on pressure loss in a channel and on fiber orientation distribution have been made clear. Here, Tucker's model^{(2),(3)} is used as the interaction model between fibers. Furthermore, flow analysis has been performed by using fiber distribution in this paper and the viscosity expression for concentrated fiber mixture, proposed by Batchelor⁽⁴⁾ and Shaqfeh^{(5),(6)}, in which effect of fiber length on flow property are not taken into consideration. Results are compared with those by the authors' analysis.

2. Method of Analysis

Two dimensional, iso-thermal flow of Newtonian fluid through a slit channel is dealt with, where thickness and length of the channel is designated by $2H$ and L , respectively, and fluid viscosity is by η . Cartesian coordinates X, Y, Z are set as shown in Fig. 1 and fiber orientation is expressed by θ, ϕ (Fig. 1). d and l

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** Department of Mechanical Engineering and Science, Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro-ku, Tokyo 152-8552, Japan. E-mail: kkato@mech.titech.ac.jp

Fig. 1 Schematic diagram of fiber orientation and slit flow

denote fiber diameter and length.

2.1 Motion of fiber

2.1.1 Force from fluid to fiber Drag force along the perpendicular direction to a fiber is expressed by the following equation:

$$D = C_d q S, q = \frac{1}{2} \rho \Delta V^2 \quad (1)$$

where S : area of a fiber of unit length, projected on the parallel plane to the fiber ($=d$), ρ : fluid density, and ΔV : perpendicular component of relative velocity of fluid to a fiber. Drag force along a fiber length is neglected^{*1}. In injection molding, Reynolds number concerning a fiber, $Re (= \rho d \Delta V / \eta)$, is much less than 1 and $C_d \propto 1/Re$ approximately. Then

$$D = 1/2 k_d \eta \Delta V, k_d: \text{constant} (\approx 10.0) \quad (2)$$

X -component of the force and moment components associated with θ and ϕ are designated by F^{θ} , M_{θ}^{θ} and M_{ϕ}^{θ} and they are calculated by integrating drag force of Eq. (2) over fiber length.

2.1.2 Force from mold wall When a fiber gets contact with a mold wall, the normal velocity component of the contact end of the fiber V_Y vanishes. The normal reaction force from a mold wall F_Y^w is determined from this condition. Friction between a fiber and a mold wall is neglected. Then, moments M_{θ}^w and M_{ϕ}^w are expressed as functions of F_Y^w . X -component of fiber velocity is obtained from equilibrium equation along X -direction.

2.1.3 Interaction between fibers Tucker and others⁽²⁾ proposed a model of fibers interaction, referring to Brownian motion. In this model, angular velocities are expressed as follows:

$$\dot{\theta}_B = -\frac{C_I [\dot{\gamma}]}{\phi} \frac{\partial \phi}{\partial \theta} \quad (3)$$

^{*1} According to detailed discussion, the drag force along fiber length is not very small. However, its effect on flow properties corresponds to that of the drag force of Eq. (1), but it is increased by some amount, and it does not change the following discussions

$$\dot{\phi}_B = -\frac{1}{\sin^2 \theta} \frac{C_I [\dot{\gamma}]}{\phi} \frac{\partial \phi}{\partial \phi} \quad (4)$$

where $[\dot{\gamma}]$ is the magnitude of shear rate. According to Tucker's experiments, interaction coefficient C_I is expressed as

$$C_I = 0.000391 \left(\frac{l}{d} \right)^2 c \quad (0.08 < c < 0.16) \quad (5)$$

Moment components M_{θ}^B , M_{ϕ}^B from fluid to a fiber corresponding to Eqs. (3) and (4) can be obtained in a similar way in 2.1.1.

Fiber orientation distribution function ϕ is defined as follows:

$$\phi = \frac{\Delta N}{N \Delta \Omega} \quad (6)$$

where ΔN : number of fibers in incremental spacial angle $\Delta \Omega$, and N : total number of fibers in a unit volume.

The variation of ϕ in time increment Δt can be calculated in consideration that ΔN fibers in $\Delta \Omega$ rotate by $\dot{\theta} \Delta t$ and $\dot{\phi} \Delta t$ in $\phi = \text{constant}$ and $\theta = \text{constant}$ planes, respectively.

2.2 Modification of fluid velocity

It is assumed that velocity changes slowly along X -direction. It results the following equilibrium equation in X direction, in which U is X -component of fluid velocity.

$$\frac{\partial}{\partial Y} \left(\eta \frac{\partial U}{\partial Y} \right) = -\frac{\partial P}{\partial X} + f_x(Y) \quad (7)$$

In Eq. (7), $U(Y)$ becomes parabolic expression of Y , when η is constant and body force f_x vanishes.

Procedure of numerical analysis is as follows: fluid velocity is assumed first and fiber motion is analysed by using Eq. (7), where f_x is set at zero. In this solution, forces and moments are applied from fluid to fibers. Then, reaction forces and moments from fibers to fluid are caused and they make fluid velocity change. These reaction forces are treated as a body force f_x in Eq. (7) and fluid velocity and fiber motion analysis are iterated.

In a slit channel flow, fiber orientation changes in translation of a mixture in time increment Δt . Furthermore, fluid velocity varies with respect to Y . Then, fiber concentration and fiber orientation in the section at down-stream by ΔX from an observing section are given as the modified one of those in the observing section, in which their changes during time increment $\Delta X/U(Y)$ are taken into account.

3. Results about Fiber Orientation and Velocity and Pressure Distributions

3.1 Fiber orientation and velocity distribution in standard processing condition

Fiber distribution is not in two dimensional space ($\theta \neq 90^\circ$), though flow is two dimensional one. At the

Fig. 2 Contours of fiber orientation distribution, $\xi/l=1/2$, $l=2$ mm, $c_0=0.1$, $nl^3=204$

entry of a slit channel, fiber distribution is given in the following way: fiber volume fraction (c) is uniform ($c=c_0$, c_0 : initial value) and fiber orientation distribution is random at an arbitrary coordinate Y , first. Then, fibers, which interact with a mold surface, are removed and volume fractions of fibers in the other orientation distribution are increased so that the total fiber volume fraction at each portion may reach c_0 and become uniform. Viscosity η is set at 1 000 Pa·s.

3.1.1 Fiber orientation distribution Orientation distribution function $\psi(\theta, \phi)$ depends on position. ψ in the layer at the distance $l/2$ from a mold wall ($\xi/l=1/2$, ξ : distance from a mold wall), where most fibers in the region near a mold wall concentrate in this layer, is shown in Fig. 2. In the initial state, ψ is approximately uniform. However, fibers converge to some orientation state rapidly ($\theta=90^\circ$, $\phi=165^\circ$) and becomes approximately steady (Figs. 2(a), (b)). Orientation distribution function ψ can be represented by a typical section of the contour, say, the section of $\theta=90^\circ$ plane. Such sections at various values of X are shown in Fig. 3. It was found that velocity distribution becomes approximately steady at $X=5\sim 20$ mm.

As was discussed before⁽¹⁾, fibers move away from the mold wall and concentrate at the layer, apart from the mold wall by half of fiber length ($\xi/l=1/2$), because of interaction between a fiber and a mold wall. Fiber volume fraction near a mold surface decreases.

Number of fibers in some designated orientation

Fig. 3 Fiber orientation distribution over ϕ , $\theta=90^\circ$, $\xi/l=1/2$, $l=2$ mm, $c_0=0.1$, $nl^3=204$

Fig. 4 Product of fiber orientation distribution and volume fraction at various positions apart from mold surface by ξ , $X=50$ mm, $l=2$ mm, $c_0=0.1$, $nl^3=204$, ξ : distance from mold surface

Fig. 5 Distribution of fiber volume fraction (c) along cavity thickness, average fraction $c_0=0.1$

is given as the product of fiber volume fraction c and orientation distribution function ψ . It is shown in Fig. 4. In a layer near the middle plane of a slit channel, shear rate $\dot{\gamma}$ is rather small and the length of transient region before steady state X_a becomes long. The length X_a at $\xi/l=3/4$ is twice that at $\xi/l=1/2$.

3.1.2 Fiber volume fraction and velocity distribution Distribution of fiber volume fraction $c(Y)$ depends on apparent viscosity distribution and it makes influence on velocity. Fiber volume fraction c

Fig. 6 Fluid velocity profile along cavity thickness, $H=2$ mm, $c_0=0.1$, $l=2$ mm, $nl^3=204$

Fig. 7 Distribution of main variants along flow direction, $|dP/dX|$: Pressure gradient, $U_{\max}$: Velocity in middle plane, $\Delta\phi$: Width of ϕ corresponding to $\psi=1/2$ $\psi_{\max}$, $c_0=0.1$, $l=2$ mm, $H=2$ mm, $nl^3=204$

is shown in Fig. 5. At the entry of a channel, c distributes uniformly. However, it decreases in the region near a mold wall ($\xi/l < 1/4$) and increases in the layer of $\xi/l = 1/2$. In the middle plane of a channel, initial orientation distribution remains, because $\partial U/\partial Y = 0$. Distribution of c becomes approximately constant after passing the transient region ($0 \leq X \leq 10$ mm).

From additional discussions about the results of flows of short fiber length mixture, it was found that distribution of c becomes uniform in the region, apart from mold wall by more than fiber length ($\xi/l \geq 1$).

Velocity distribution over the channel thickness is shown in Fig. 6. In the region, a little down stream from channel entry ($X=10$ mm), apparent viscosity decreases with increasing fiber orientation concentration (Fig. 2) and velocity gradient $\partial U/\partial Y$ increases. Then velocity distribution becomes approximately that of a plug flow.

3.1.3 Pressure gradient in flow direction For practical purpose, estimation of pressure loss is important. Distribution of pressure gradient in the flow direction $|dP/dX|$ is shown in Fig. 7.

Velocity at the middle plane ($U_{\max}$) and half value

Fig. 8 Effect of nl^3 on fiber orientation distribution ($\theta=90^\circ$), $c_0=0.1$, $l=2$ mm, $H=2$ mm, $X=50$ mm, $\xi/l=1/2$

widths of ϕ distribution at $\xi/l=1/2$ and $3/4$, ($\Delta\phi$), are plotted in Fig. 7. These variables approximately converges to constant values at the distance of 5 times fiber length or so ($X_a=10$ mm). Length of transient flow region is rather long in the layer near middle plane. However, pressure gradient converges rapidly, because it depends mainly on the shear flow property near a mold wall.

3.2 Fiber volume fraction and flow property

Fiber conditions are determined by length (l), diameter (d) and number in unit volume (n). Fiber diameter does not make influence on fiber-fluid interaction (see Eq. (2)). Then, number of independent conditions are two.

By the way, there are two important parameters concerning fibers. One is nl^3 , which is number of fibers in a cube of l^3 (n : number of fibers in unit volume). The other is fiber length l . This determines thickness of the low fiber volume fraction layer on a mold wall and it also makes great influence on velocity distribution and pressure gradient. These two conditions are changed and their effects on flow properties are discussed.

3.2.1 Effect of nl^3 Fiber volume fraction c_0 and fiber length l are set constant and only nl^3 is changed. Orientation distribution function $\psi(\theta, \phi)$ is shown in Fig. 8, where θ is set constant ($\theta=90^\circ$). The peak value of ψ decreases and the half value width of it $\Delta\phi$ increases, when nl^3 increases. It results from the fact that interaction coefficient C_I increases with increasing nl^3 . This is introduced from Eq. (5) and the relation $\pi/4 \cdot (l/d)^2 = nl^3$. Velocity distribution becomes approximately that of a plug flow, when nl^3 increases (Fig. 9).

Distribution of pressure gradient $|dP/dX|$ is shown in Fig. 10, where flow rate Q is set constant. $|dP/dX|$ increases with increasing nl^3 . Rapid change is observed only in the region near entry ($X \leq 10$ mm).

3.2.2 Effect of fiber length Fiber volume

Fig. 9 Effect of nl^3 on fluid velocity distribution, $c_0=0.1$, $l=2$ mm, $H=2$ mm

Fig. 10 Effect of nl^3 on pressure gradient along flow direction, $l=2$ mm, $H=2$ mm, $c_0=0.2$

Fig. 11 Distribution of fiber volume fraction for different fiber length l , $c_0=0.05$, $H=2$ mm, $nl^3=408$, $X=50$ mm

fraction near mold wall decreases and fibers concentrate at the layer of $\xi/l=1/2$ (Fig. 5). Similar results are obtained, when fiber length l is changed. Results are shown in Fig. 11, where c_0 , nl^3 and Q are set constant. Distribution of velocity U is shown in Fig. 12. The region, where U changes steeply, exists only in the region near a mold wall and it corresponds to the low fiber volume fraction layer (Fig. 11). The thickness of it decreases with decreasing l .

Now, the effect of fiber length on velocity distri-

Fig. 12 Effect of fiber length l on fluid velocity, $c_0=0.05$, $H=2$ mm, $nl^3=408$, $X=50$ mm

Fig. 13 Effect of fiber length on pressure gradient, $c_0=0.05$, $H=2$ mm, $nl^3=408$

bution is discussed. When fiber length l is short ($l=1$ mm), the thickness of low fiber volume fraction layer becomes small and c distributes uniformly in most part in mold cavity. Therefore, velocity distribution becomes parabollic. On the contrary, it becomes a plug flow, when l is long ($l=2$ mm). Consequently, it is concluded that a plug flow is caused by the effect of fiber-mold wall interaction and that it does not occur in a flow of very short fiber mixture, even when fiber volume fraction c_0 is large.

Pressure gradient $|dP/dX|$ is shown in Fig. 13. It decreases with increasing fiber length. The reason is that thickness of low fiber fraction layer increases. Transient region near entry is limited in the region of $X \leq 10$ mm.

4. Discussions

Theoretical researches about stress strain-rate relation of concentrated fiber mixture, proposed by Batchelar⁽⁴⁾ and Shaqfeh^{(5),(6)}, are well-known, whereas very few experiential results have been reported up to now. So the author's result will be compared with Shaqfeh's theoretical result.

Shaqfeh proposed the following equation:

$$\sigma = \eta_f \left(\langle PPPP \rangle - \frac{1}{3} \delta \langle PP \rangle \right) : E + 2\eta_s E \quad (8)$$

Fig. 14 Effect of fiber volume fraction and orientation on apparent viscosity of fiber-fluid mixture (Shaqfeh's model)

$$\eta_f = \frac{8\pi n l^3 \eta_s}{3[\ln(1/c) + \ln \ln(1/c)]} \quad (9)$$

where $\mathbf{P}$: unit vector along fiber (Fig. 1), $\boldsymbol{\sigma}$: stress tensor, $\mathbf{E}$: strain-rate tensor, n : number of fibers in unit volume, and $\eta_s = 1000 \text{ Pa}\cdot\text{s}$. When fiber orientation distribution is known in advance in a slit channel flow, velocity distribution can be calculated from Eq. (8) and equilibrium equation (Eq. (7)). Fiber orientation distribution in the authors' results are used in Eq. (8), and the result is compared with velocity distribution in the authors' solution.

4.1 Apparent viscosity in Shaqfeh's model

From Eq. (8), apparent viscosity $\eta_a (= \tau / \dot{\gamma})$ depends on c and fiber orientation, in which the first term, corresponding to the effect of fiber orientation, is proportional to nl^3 (Eq. (9)). Then, relationship between η_a and c in various fiber orientation are obtained and shown in Fig. 14. Here, nl^3 is set at 51. Results reveal that effect of fiber vanishes when fibers are parallel to the mold surface ($\phi = 0^\circ$) and that it becomes maximum when $\phi = 45^\circ$.

According to the authors' results, fiber orientation concentrates at $\phi = 165^\circ$, so the increase of viscosity due to fiber orientation is not so large as that of random distribution mixture. Effect of nl^3 is large, because $\eta_f \propto nl^3$ in Eq. (9), whereas effect of c is not so large.

4.2 Comparison with Shaqfeh's results

Velocity distribution calculated by using Shaqfeh's viscosity model is shown in Fig. 15, in which nl^3 is varied as a parameter and other processing conditions and fiber conditions are set same as those in Fig. 9. Velocity distribution becomes similar to that of a plug flow, when nl^3 increases, and such property coincides with that in the authors' result (Fig. 9).

Coincidence of effect of fiber length l on velocity field in the result, obtained by using Shaqfeh's viscosity model, with that in the author's result is also

Fig. 15 Velocity distribution obtained by using Shaqfeh's constitutive equation and authors' result about fiber orientation distribution, $X=50 \text{ mm}$, $c_0=0.1$, $l=2 \text{ mm}$, $H=2 \text{ mm}$

Fig. 16 Comparison of authors' results of pressure gradient with Shaqfeh's on various nl^3 , $c_0=0.1$, $l=2 \text{ mm}$, $X=50 \text{ mm}$

confirmed.

Pressure loss, or pressure gradient $|dP/dX|$, in a channel flow is important in practical problems, eg. viscosity measurement. An example of pressure gradient in the flow of Shaqfeh's material is plotted in Fig. 7 by solid square marks. It takes a large value, but converges rapidly to some constant value. Difference between this result and the authors' result (solid circles in Fig. 7) comes from the difference of effect of fiber orientation distribution on apparent viscosity in Shaqfeh's viscosity model and Tucker's fiber interaction model. When fiber orientation distributes randomly, viscosity increases in Shaqfeh's model, whereas it does not change in Tucker's model, in which no fiber interaction occurs. This problem should be discussed in detail in consideration with variation of fiber orientation around the channel entry and it is beyond the scope of the present paper.

Converged values of pressure gradients in those examples, whose velocity distributions were discussed in Fig. 15, are plotted by open circles in Fig. 16. The authors' results are by solid circles. Both results agree with each other. Similar comparison is performed in Fig. 17, in which fiber length l is varied. Good agreement of both results in wide ranges of

Fig. 17 Comparison of authors' results of pressure gradient with Shaqfeh's on various fiber length l , $c_0=0.1$, $nl^3=408$, $X=50$ mm

fiber conditions proves that the simple method by the use of Shaqfeh's viscosity model can be applied for approximate analysis of pressure gradient, though fiber orientation distribution must be assumed in advance.

5. Conclusions

Velocity and pressure distributions and fiber orientation distribution in two dimensional flow through a slit channel are analysed in consideration with interactions between fibers and between a fiber and a mold wall. Results are summarized as follows:

(1) Velocity, pressure gradient and fiber orientation distributions approximately converge to some constant values after passing through a short transient region. Length of the transient region X_a is about 10 mm, when cavity thickness $2H=4$ mm.

(2) Velocity distribution $U(Y)$ becomes that of a plug flow, as the fiber interaction parameter nl^3 increases. Here, fiber volume fraction c and fiber length l are constant and l is not very short, comparatively to cavity thickness $2H$. Pressure gradient $|dP/dX|$ increases with increasing nl^3 .

(3) Fiber volume fraction c decreases in the region, near mold wall ($\xi/l \leq 1/4$, ξ : distance from a mold wall), because of interaction between a fiber and

a mold wall. When l is large, thickness of that region increases and $|dP/dX|$ decreases.

(4) It was found that velocity and pressure gradient distributions, which are calculated by using Shaqfeh's viscosity model and by assuming fiber orientation distribution, agree with those in the authors' solution in wide ranges of nl^3 and l . It means that the former method can be used for approximate analysis of a flow.

Consequently, it was found that interaction between a fiber and a mold wall gives great influence on flow property and fiber orientation. Detailed experiments should be made for the proof of the present results, though it is necessary that various experimental techniques may be developed for exact measurement of phenomena.

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