Cooperative Behavior of a Mechanically Unstable Mobile Robot for Object Transportation*

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Cooperative transportation of an object by two or more mobile robots requires each robot to exert an appropriate force to support and move the object, to move along the object, and to maintain the robot's attitude stably. Studies of cooperative transportation by multiple robots so far have scarcely considered the stability of the robot's attitude influenced by the force from the object. To take part in cooperative transportation, a mechanically unstable robot, such as a wheeled inverted pendulum, needs to control the force and follow the object while standing stably against the force in an integrated manner. We call this "cooperative behavior". To realize this behavior of a mechanically unstable robot, we built a control system to estimate the external force, maintain standing, and exert a specified force. Experiments were conducted on cooperative transportation between a human and the robot.

Key Words: Mechatronics and Robotics, Moving Robot, Observer, Cooperative Transportation, Wheeled Inverted Pendulum, Force Control, Cooperative Behavior, Stable Standing

1. Introduction

Cooperative transportation will make it possible for two or more mobile robots to transport a heavy, large or soft object which cannot be carried by one mobile robot. Cooperative transportation of an object by plural robots helps to reduce the required power of each robot and to downsize each robot. Realization of cooperative transportation requires each robot to have at least two kinds of abilities. Those are: not to drop or break the transported object and to maintain each robot's attitude stably against the force from the object while the robot transports it. To satisfy those requirements, each robot needs to have three different kinds of functions: the function to exert an appropriate force to support and move the object, the function to move along the object, and the function to maintain the attitude of itself stably. Therefore, the necessary condition for cooperative transportation is the behavior of the robot which allows it to play these three functions in an integrated manner. In this paper, we call this "cooperative behavior".

While the last kind of function, that is to maintain the attitude of the robot stably, in the functions of cooperative behavior is not much of a problem for a mechanically stable robot such as a four-wheeled robot because such a robot has a large stability margin against external forces, a mechanically unstable robot, which needs control to maintain its attitude stably such as a biped robot or an wheeled inverted pendulum, must adjust its attitude to prevent itself from falling down. Although the previous works on cooperative transportation by two or more mobile robots deal with the internal forces which occur from the interactions between the object and the robots during cooperative transportation, they assume that the force from the object does not influence the
stability of each robot and that the robot is always stable against the force because the robot is mechanically stable. Accordingly, they do not have to consider the problem of maintaining the attitude stably.

In our study, we consider cooperative transportation by mechanically unstable robots. In this case, the stability of each robot is strongly influenced by the force from the object and each robot has to maintain its attitude stably while it takes part in transporting the object. We chose a wheeled inverted pendulum as an instance of a mechanically unstable robot. There are some previous studies about stable control of wheeled inverted pendulums \(^{(16)}\), but few consider the disturbance by external forces. Small projection area to the ground of a wheeled inverted pendulum compared with a four-wheeled robot is one of its advantages, and it is hoped to derive the elucidation of cooperative behavior of human beings from the study of cooperative transportation by mechanically unstable robots.

In this paper, we aim to realize the above-mentioned cooperative behavior based on the force information from the transported object in cooperative transportation. To realize this behavior of a wheeled inverted pendulum, we first need a stable standing control of a wheeled inverted pendulum against an external force. We propose a method for estimating the external force, which is applied to the wheeled inverted pendulum, without adding any new sensors but using an observer considering the external force as one of the states of the system, and a position control method based on the estimated external force which enables the wheeled inverted pendulum to stand stably against the external force. Next, since the wheeled inverted pendulum needs to apply an appropriate force to the object for cooperative transportation, we propose a force control method for the wheeled inverted pendulum.

The experimental results, which were obtained using the proposed methods, confirmed that the external force applied to the wheeled inverted pendulum can be estimated, the wheeled inverted pendulum can stand stably against the external force, and the wheeled inverted pendulum can exert a specified force, and thus the effectiveness of the proposed methods was verified. Experiments were conducted on cooperative transportation between a human who takes the initiative in moving and the robot using the established force control system.

2. Modeling of Wheeled Inverted Pendulum

A wheeled inverted pendulum is a system which can be stabilized and driven by properly displacing its wheel axle which is the supporting point of the pendu-

![Vibration type rate gyroscope](image)

**Fig. 1** The structure of the wheeled inverted pendulum.

![Wheeled inverted pendulum](image)

**Fig. 2** The concept of cooperative transportation by two wheeled inverted pendulums.

lum. The structure of the wheeled inverted pendulum used in our study is shown in Fig. 1. This wheeled inverted pendulum \(^{(16)}\) does not have any sensors which directly measure the relative angle between the external environment and the pendulum, such as a potentiometer to obtain the inclination angle of the body. The wheeled inverted pendulum has a DC servo motor in the lower part of the body, and its driving torque is transmitted to the wheels via a reduction gear whose reduction ratio is 1:7. Since it is a coaxial two-wheeled robot which has two wheels attached at the ends of an axle, it has no steering function and its motion is limited in a vertical plane. It also has a rate gyroscope in the upper part of the body to measure the inclination angular velocity of the body and a rotary encoder on the motor axle to measure the rotational angle of the motor. The rotational angle of the motor corresponds to the relative angle between the body and the wheels.

The concept of cooperative transportation of an object held by two wheeled inverted pendulums is presented in Fig. 2. While transporting an object
cooperatively, each inverted pendulum exerts an appropriate force on the object for cooperative transportation. From the opposite point of view, it sustains the same magnitude of an external force and has to stand stably against the force. Therefore, we consider a model of the wheeled inverted pendulum which is subjected to an external force. Even though the external force can be applied to the body, at any point, of the wheeled inverted pendulum, we assume for simplicity that the external force is applied horizontally to the body, at the axle height, of the wheeled inverted pendulum (Fig. 3). Only the horizontal component of the external force \( F_e \) appears in the equations of motion. In this model, a clockwise rotation is regarded as positive for the body and the wheels, and the inclination angle of the body is 0 degree when the body stands up vertically. The equations of motion of this model are as follows:

\[
\begin{align*}
(m_1 l^2 + J_1 + n^2 J_m) \ddot{\theta}_1 + (m_1 rl \cos \theta_1 - n^2 J_m) \ddot{\theta}_2 \\
+ f_s (\dot{\theta}_1 - \dot{\theta}_2) - m_1 gl \sin \theta_1 = -nKu, \\
(m_1 rl \cos \theta_1 - n^2 J_m) \dot{\theta}_1 \\
+ ((m_1 + m_2) r^2 + J_2 + n^2 J_m) \dot{\theta}_2 \\
- f_s (\dot{\theta}_1 - \dot{\theta}_2) - m_1 rl \sin \theta_1 \cdot \frac{\dot{\theta}_1}{\dot{\theta}_2} = nKu + Fr, 
\end{align*}
\]

(1)

(2)

where, \( m_1, m_2, J_1, J_2, J_m \) are respectively the masses of the body and the wheels, and the moments of inertia of the body, the wheels, and the motor rotor, \( \theta_1 \) the inclination angle of the body, \( \dot{\theta}_1 \) the rotational angle of the wheel, \( l \) the distance between the wheel axle and the center of gravity of the body, \( r \) the radius of the wheel, \( f_s \) the resistance in the driving system, \( K_t \) the torque constant of the motor, \( n \) the reduction ratio of the gear, \( F_e \) the external force applied to the wheeled inverted pendulum, \( u \) the motor current, and \( g \) the gravitational acceleration. The values of some parameters are shown in Table 1.

### Table 1 Parameters of the experimental system

| \( m_1 \) | 1.0043 [kg] |
| \( m_2 \) | 0.1736 [kg] |
| \( J_1 \) | 2.1564 \times 10^{-2} [kg m^2] |
| \( J_2 \) | 4.70 \times 10^{-6} [kg m^2] |
| \( l \) | 0.188 [m] |
| \( r \) | 0.03 [m] |
| \( f_s \) | 2.0 \times 10^{-4} [N m/(rad/s)] |
| \( J_m \) | 1.862 \times 10^{-6} [kg m^2] |
| \( K_t \) | 3.43 \times 10^{-2} [N m/A] |
| \( n \) | 7 |

### 3. Control Methods

#### 3.1 Stable standing control against the external force

A control system needs to be built to enable a wheeled inverted pendulum to stand stably against the external force in the case that an external force is applied to the wheeled inverted pendulum. Applying the moving control method for a wheeled inverted pendulum on an unknown rough terrain surface\(^{(10)}\), the external force is estimated by an observer regarding it as one of the states of the system and we build a control system which can let the robot stand stably against the external force using the estimated value of the force. The inclination angle of the body \( \theta_1 \), which is an unknown state, is also obtained from the estimation by the observer, rather than from the integration of the inclination angular velocity of the body which causes the integration error\(^{(10)}\).

#### 3.1.1 Estimation of the external force by the observer

Since the inclination angle of the body, \( \dot{\theta}_1 \), and the inclination angular velocity, \( \dot{\theta}_1 \), are near-zero values when the wheeled inverted pendulum is standing up vertically, we can approximate, \( \dot{\theta}_1 = 0 \), \( \sin \dot{\theta}_1 = \dot{\theta}_1 \), and \( \cos \dot{\theta}_1 = 1 \) about the point of equilibrium, and obtain the following linearized equations by linearizing Eq. (1) and (2).

\[
\begin{align*}
(m_1 l^2 + J_1 + n^2 J_m) \ddot{\theta}_1 + (m_1 rl - n^2 J_m) \ddot{\theta}_2 \\
+ f_s (\dot{\theta}_1 - \dot{\theta}_2) - m_1 gl \sin \theta_1 = -nKu, \\
(m_1 rl - n^2 J_m) \dot{\theta}_1 + ((m_1 + m_2) r^2 + J_2 + n^2 J_m) \dot{\theta}_2 \\
- f_s (\dot{\theta}_1 - \dot{\theta}_2) = nKu + Fr 
\end{align*}
\]

(3)

(4)

Here, we regard the unknown external force applied to the wheeled inverted pendulum \( F_e \) as a disturbance of the system and try to estimate it by an observer regarding it as one of the states of the system. We assume that the dynamics of the applied external force is such that

\[ F_e = 0 \tag{5} \]

From Eq. (3) and (4), the state equation and the output equation of the wheeled inverted pendulum can
be represented as a fifth-order system as follows:
\[ \dot{x} = Ax + bu, \]  
\[ y = Cx, \] 

where \[
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
F_1 \\
F_2
\end{bmatrix},
\begin{bmatrix}
a_1 & 0 & a_3 & a_5 & 0 \\
0 & 0 & 0 & 0 & 0 \\
a_3 & 0 & -a_1 & a_5 & a_5 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
F_1 \\
F_2
\end{bmatrix},
\begin{bmatrix}
a_1 & a_3 & a_3 & a_5 & a_5 \\
0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
\end{bmatrix}
\]
\[
a_1 = m_1 g \alpha_{2a} \Delta \quad a_2 = -m_1 g \alpha_{a2} \Delta \\
a_3 = -(a_{2a} + a_{12})/\Delta \\
a_4 = (a_{11} + a_{12}) f_2/\Delta \\
a_5 = -a_{12} r/\Delta \\
a_6 = a_{12} r/\Delta \\
b_1 = -n K (a_{11} + a_{12})/\Delta \\
b_2 = n K (a_{11} + a_{12})/\Delta \\
\Delta = a_{11} a_{2a} - a_{12}^2 \\
a_{11} = m_1 I_1 + J_1 + n^2 J_m \\
a_{12} = m_1 rl + n^2 J_m \\
a_{2a} = (m_1 + m_2) r^2 + J_2 + n^2 J_m.
\]

Here, \( q_1, q_2, q_3 \) in the output vector, \( y \), are the inclination angular velocity of the body and the relative angle and angular velocity between the body and the wheels, respectively. \( q_2 \) and \( q_3 \) can be obtained from the sensors, and \( q_1 \) can be calculated by differentiating \( q_2 \). Since the system \((C, A)\) is observable, we can construct an observer that estimates the external force, \( F_e \), and the inclination angle of the body, \( \theta_b \), which are the unknown states of the system.

Equation (5) represents that the external force \( F_e \) is always constant or changes its value stepwise. Even in the case that the external force changes continuously, we can well estimate the external force if the sampling rate of the observer is small enough to permit the changes regarded as stepwise changes. The observer estimates all the states of the wheeled inverted pendulum using the value of the control input to the wheeled inverted pendulum and the output values from it when that input is given. This can be represented as an equation as follows:
\[ \dot{x}^* = (A - KC)x^* + Ky + bu, \] 
where \[
\begin{bmatrix}
\theta_1^* \\
\theta_2^* \\
\theta_3^* \\
F_1^* \\
F_2^*
\end{bmatrix}.
\]
\( \theta_1^* \) to \( F_2^* \) are the states estimated by the observer. We can specify the poles of \( A - KC \) by modifying the feedback gain matrix of the observer \( K \) and thereby change the characteristics of the state estimates of the system to follow the true state values. The observer is an identity observer whose order is the same as the order of the system and estimates not only the unknown values, the external force, \( F_e \), and the inclination angle of the body, \( \theta_b \), but also all the other states of the system. We use the estimated states, rather than the states measured by sensors, for the state feedback control, and the wheeled inverted pendulum will be stabilized and driven by the state feedback control.

### 3.1.2 Stabilization control using compensation for the external force

The inclination angular velocity of the body, \( \dot{\theta}_b \), the inclination angular acceleration of the body, \( \ddot{\theta}_b \), the rotational angular velocity of the wheel, \( \dot{\theta}_w \), and the rotational angular acceleration of the wheel, \( \ddot{\theta}_w \), are zero at the state of equilibrium, and the following equations can be obtained from the equations of motion (1) and (2):
\[
m_1 gl \sin \theta_b = n K u_{off},
\]
\[
F_{ex} = -n K u_{off},
\]
where \( u_{off} \) is the current which maintains the wheeled inverted pendulum at the states of equilibrium and \( \theta_b \) is the inclination angle of the body at the state of equilibrium. The wheeled inverted pendulum stands up vertically at the states of equilibrium when no external force is applied to it. That is, \( \theta_b = 0 \). The wheeled inverted pendulum inclines by \( \theta_b \) from the vertical position and stands up at the states of equilibrium when the external force is applied to it. From Eq. (9) and (10), the relation between the inclination angle of the body, \( \theta_b \), and the applied external force, \( F_e \), can be represented as follows:
\[
m_1 gl \sin \theta_b = -F_{ex}.
\]
As shown in Fig. 4, the relation between the external force applied to the wheeled inverted pendulum at the axle height and the inclination angle of the body in Eq. (11)

![Balance of moment about the contact point](image-url)
means that the wheeled inverted pendulum inclines so that the moment caused by the external force balances with the moment caused by the weight of the body about the contacting point on the ground. From Eq. (9) and (10), the following equations can be obtained.

\[ T_{off} = u_{off} = nK_n u_{off} \]
\[ = m_g l \sin \theta_b \]
\[ = -F \dot{r} \],

(12)

where \( T_{off} \) is the torque to maintain the wheeled inverted pendulum at the state of equilibrium. That is to say, the body inclines because of the applied external force, and then a torque is produced about the axle because of the inclination of the body. From the relation of Eq. (12), the current which maintains the torque can be obtained as follows:

\[ u_{off} = -\frac{F \dot{r}}{nK_n} \].

(13)

When the external force is applied, this current \( u_{off} \), which maintains the inclination of the body, has to be compensated to let the wheeled inverted pendulum stand stably around the inclination of equilibrium \( \theta_b \).

In the actual control phase, \( u_{off} \) can be obtained by substituting the estimated force \( F^e \) into Eq. (13). Therefore, the control system which includes the external force can be obtained by adding, as feedforward term, the compensation current \( u_{off} \), which maintains the inclination of the body caused by the applied external force term, to the system which does not include the external force. The control law is represented as follows:

\[ u = f \begin{bmatrix} \dot{\theta}_b^* - \dot{\theta}_b^* \\ \theta_{ref} - \theta_b^* \\ \dot{\theta}_{ref} - \dot{\theta}_b^* \\ \omega_{ref} - \omega_b^* \end{bmatrix} + u_{off} \],

(14)

where

\[ \theta_b^* = -\frac{r}{m_g l} F^e \]
\[ u_{off} = \frac{-r}{nK_n} F^e \]
\[ f = [f_1 \ f_2 \ f_3 \ f_4] \]

and \( \theta_{ref} \) is the target rotational angle of the wheel, \( \omega_{ref} \) is the target rotational angular velocity of the wheel, and \( f \) is the state feedback gain vector, which stabilizes the fourth-order system excluding the state \( F^e \) when there is no external force applied. By rewriting Eq. (14), we can obtain the following equation.

\[ u = f \begin{bmatrix} \dot{\theta}_b^* \\ \theta_{ref} - \theta_b^* \\ \dot{\theta}_{ref} - \dot{\theta}_b^* \\ \omega_{ref} - \omega_b^* \end{bmatrix} - \left( \frac{f_3}{m_g l} + \frac{1}{nK_n} \right) r F^e. \]

(15)

Using the following expression,

\[ f_3 = \left( \frac{f_1}{m_g l} + \frac{1}{nK_n} \right) r, \]

(16)

the control law can be represented as follows:

\[ u = f' (x_{ref} - x^*), \]

(17)

where

\[ f' = [f_1 \ f_2 \ f_3 \ f_4 \ f_5], \]
\[ x_{ref} = [0 \ \theta_{ref} \ \dot{\theta}_{ref} \ \omega_{ref} \ \dot{\omega}_{ref}]^T. \]

The body of the wheeled inverted pendulum inclines when the external force is applied. This motion of the wheeled inverted pendulum means that the wheeled inverted pendulum is compensating for the external force by changing its attitude, and this control enables the wheeled inverted pendulum to maintain its attitude stably against the force from the transported object when it transports the object cooperatively.

3.2 Control of the force applied to the object

The control system, which stabilizes the wheeled inverted pendulum against the external force, built in 3.1.2 is a position control system. On the other hand, it is also apparent from Fig. 2 that the wheeled inverted pendulum has to apply some force to the transported object held by two wheeled inverted pendulums in the cooperative transportation. This force is specified as a target force. We will establish a force control system which enables the wheeled inverted pendulum to exert a specified force. We assume that the wheeled inverted pendulum exerts the specified force at the axle height for simplicity as is the case of the applied external force.

Here, we also assume that the object to which the wheeled inverted pendulum applies a force has a positive compliance. In other words, if the wheeled inverted pendulum moves against the external force, we suppose that the force should increase, and if the wheeled inverted pendulum moves to follow the force from the object, the force should decrease. The wheeled inverted pendulum sustains, as the external force, the reaction force from the object which corresponds to the force it applies to the object. We build a control system to drive the wheeled inverted pendulum towards the transported object if the estimated force is smaller than the command force and away from the object if the estimated force is larger than the command force.

There is a relation between the external force applied to the wheeled inverted pendulum and the motor current, which produces the motor torque corresponding to it, such as Eq. (13) at the state of equilibrium. The target current, \( u_{cmd} \), which produces the motor torque corresponding to the target force, \( F_{cmd} \), is obtained from Eq. (13) as follows:

\[ u_{cmd} = -\frac{F_{cmd}}{nK_n}. \]

(18)
The specified force can be exerted by changing the target rotational angle of the wheel so that the current
to the wheeled inverted pendulum $u$ coincides with the target current $u_{cmd}$. That is, the target current, which produces the motor torque corresponding to the target force, is first calculated, and next the output force is controlled by changing the target position of the wheeled inverted pendulum so that the output current of the wheeled inverted pendulum coincides with the target current. The target rotational angle of the wheel of the pendulum $\theta_{cmd}$ for the force control is obtained as the integration of the difference between the present current, $u$, and the target current, $u_{cmd}$, multiplied by a gain $G$ as follows:

$$\theta_{cmd} = \int G(u_{cmd} - u) dt.$$  (19)

For this force control, the new target vector $x_{ref}'$ is introduced by adding the target rotational angle of the wheel to the target vector $x_{ref}$ as follows:

$$x_{ref}' = [0 \quad \theta_{ref} + \theta_{cmd} \quad 0 \quad \omega_{ref} \quad 0].$$  (20)

The control law in this case is as follows:

$$u = f'(x_{ref}' - x^*).$$  (21)

The following equation is also obtained by rewriting Eq. (21).

$$u = f'(x_{ref}' - x^*) + f_2 \theta_{cmd}$$
$$= f'(x_{ref}' - x^*) + f_2 \int G(u_{cmd} - u) dt.\quad (22)$$

It can be recognized from Eq. (22) that the control system for the force control can be obtained by adding an integration of the difference between the present current and the target current multiplied by some gain to the position control system which enables the wheeled inverted pendulum to stand stably against the external force. The value of the gain $G$ is determined experimentally within the velocity limit the wheeled inverted pendulum can follow. In this force control system, by letting the gain $G$ equal to zero, we can also get a position control system. That is to say, the force control system includes a position control system which stabilizes the wheeled inverted pendulum against the external force. This control system can let the wheeled inverted pendulum follow the movement of the transported object and also exert the specified force. In short, this control system provides the wheeled inverted pendulum with an appropriate cooperative behavior.

4. Experiments

4.1 Some conditions for the experiments

The parameter values of the wheeled inverted pendulum used in the experiments (Fig. 1) are shown in Table 1. The actuator is a DC servo motor with a nominal power of 11 W. A personal computer with an 80386 CPU, 20 MHz, is used as the controller. The continuous form control law mentioned in the previous section is simply discretized, and the experiments are performed using digital control with the computer. The control cycle period is 2 ms, which is determined from the calculation time of the computer.

The state feedback gain vector of the wheeled inverted pendulum $f$ and the feedback gain matrix are determined experimentally based on the pole assignment method. Since the wheeled inverted pendulum has nonlinear elements such as Coulomb friction and insensitivity of the motor, we compensate for them and then add a dither signal of rectangular wave with a frequency of 250 Hz and an amplitude of 244 mV to the output current. Figure 5 shows the feedback block diagram which includes compensation for nonlinearities.

4.2 Stable standing control using the estimation of the external force and the compensation for the external force

Using the position feedback control system of Eq. (17), we conducted an experiment on the estimation of the external force applied to the wheeled inverted pendulum and stabilization control of the wheeled inverted pendulum using the estimated external force.

As shown in Fig. 6(a), a weight is suspended from the body of the wheeled inverted pendulum at the axle height via a wire and pulley. Therefore statically, a constant external force is applied by the weight to the body at the axle height. Then, the applied external force is changed with time by a human pulling the wire or loosening it. The tension of the wire is measured by a force sensor attached to the wire. Figure 6(b) shows an experimental result.

![Feedback block diagram.](image)

Fig. 5 Feedback block diagram.
The solid line is the external force estimated by the observer, the dotted line is the tension of the wire, and the broken line is the weight (1.065 N). A constant external force by the weight is applied to the wheeled inverted pendulum during the periods of about 0 s through 6 s, 13 s through 20 s, and 33 s through 36 s. No external force is applied during the period of about 6 s through 13 s by lifting up the weight by a human. A force by a human is added to the force of the weight and is then applied during the period of about 20 s through 33 s.

Looking at the parts where the external force is constant, we can see that, even though the estimated value of the external force is vibrating a little, the applied external force is estimated fairly well. Also in the case that the external force is not constant, it can be confirmed that the observer can produce an estimate well following the change of the external force. This vibration of the estimated value is considered to be caused by the existence of backlash of the gear, the nonlinear friction, etc. which are hard to model in the real wheeled inverted pendulum. It was confirmed that the wheeled inverted pendulum can stand stably against the external force by the standing control based on the estimated external force.

4.3 Exertion of a specified force
We conducted an experiment on exertion of a specified force by the wheeled inverted pendulum using the feedback control system for force control of Eq. (21). As shown in Fig. 7(a), one end of a string is fixed and the other end is connected to the wheeled inverted pendulum at the axle height. Therefore, if the wheeled inverted pendulum pulls the string, it will sustain the same magnitude force at the axle height. In the experiment, the command force was first set to zero, and then it was increased stepwise about every 8 seconds.

Figure 7(b) shows an experimental result. The solid line in the graph is the external force estimated by the observer and the broken line is the command force. The estimated value by the observer is vibrating. Since the previous experiment has confirmed that the estimated external force coincides with the applied external force very well, this vibration of the estimated external force means that the applied external force itself is actually vibrating. This vibration of the estimated external force is caused by the vibration of the wheeled inverted pendulum. The vibration gets harder as the command force increases. The reasons for this seems to be that the friction between the body and the axle increases as the applied external force increases. Although the estimated external force
Table 2  Time change of the external force

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>External Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ~ 8</td>
<td>$F_e = 0$</td>
</tr>
<tr>
<td>8 ~ 16</td>
<td>$F_e = F_{cmd}$</td>
</tr>
<tr>
<td>16 ~ 24</td>
<td>$F_e &gt; F_{cmd}$</td>
</tr>
<tr>
<td>24 ~ 32</td>
<td>$F_e = F_{cmd}$</td>
</tr>
<tr>
<td>32 ~ 40</td>
<td>$F_e &lt; F_{cmd}$</td>
</tr>
<tr>
<td>40 ~ 48</td>
<td>$F_e &gt; F_{cmd}$</td>
</tr>
<tr>
<td>48 ~ 56</td>
<td>$F_e = 0$</td>
</tr>
</tbody>
</table>

Fig. 8 Cooperative transportation with a human.

vibrates, the mean of the vibrating estimated external force coincides well with the command force. Therefore, it was confirmed that the wheeled inverted pendulum can exert a specified force fairly accurately using the established force control system.

4.4 Cooperative transportation with a human

We conducted an experiment on cooperative transportation of an object with a human. An object is placed between a human operator’s hand and the wheeled inverted pendulum, and the wheeled inverted pendulum is controlled to exert a specified force. A human operator takes the initiative in moving the object. That is, by moving the hand towards the wheeled inverted pendulum or away from the wheeled inverted pendulum or stopping the movement, the operator can let the wheeled inverted pendulum cooperatively transport the object. The force applied to the wheeled inverted pendulum via the object by the human operator is changed every 8 seconds according to Table 2. The command force of the wheeled inverted pendulum is kept constant (0.785 N).

Figure 8 shows an experimental result of the cooperative transportation. The solid line in the upper graph of Fig. 8 is the rotational angle of the wheel, and the broken line is the target rotational angle of the wheel which is derived from target force. The solid line in the lower graph of Fig. 8 is the external force estimated by the observer, and the broken line is the command force. The wheeled inverted pendulum followed the movement of the operator’s hand well and did not drop the object, which was placed between the operator’s hand and the wheeled inverted pendulum, as long as the speed of transportation movement was low.

In Fig. 8, we can see that the inverted pendulum moves as if it is pushed by the hand, if the force applied by the hand is larger than the command force, and that it moves as if it pushes the hand, if the applied force is smaller than the command force. The wheeled inverted pendulum stays standing if the applied force is equal to the command force. Consequently, the wheeled inverted pendulum moves according to the difference between the applied external force and the command force, and we can confirm that the wheeled inverted pendulum can cooperatively transport an object with a human with this kind of movement (Fig. 9).

5. Conclusions

In this paper, we consider a control method for a mobile robot which enables the robot to transport an object cooperatively even under the situation that the stability of the robot is influenced by the force from the object. In such cooperative transportation, the robot has to maintain its attitude stably while transporting the object cooperatively. We pointed out
that the robot has to have three functions: the function to maintain the attitude of the robot stably against the external force, the function to apply an appropriate force to the transported object, and the function to follow the movement of the object, and they constitute “cooperative behavior” of the robot for the realization of cooperative transportation.

We chose a wheeled inverted pendulum as an instance of a mechanically unstable robot. As the control system which realizes the above-mentioned three functions in an integrated manner, we built a position control system to estimate the external force applied to the wheeled inverted pendulum using an observer and maintain standing stably using the estimated external force, and a force control system to produce a specified force at the axle height of the wheeled inverted pendulum. The wheeled inverted pendulum exerts the force by inclining its body according to the specified force and stabilizing itself in that position. From this viewpoint, a mechanically unstable robot differs from a mechanically stable robot such as a four-wheeled robot. It was confirmed from the experimental results that the external force applied to the wheeled inverted pendulum can be estimated well and the wheeled inverted pendulum can stand stably against the external force based on the proposed control law. It was also confirmed that the wheeled inverted pendulum can exert a specified force and thus the proposed methods work effectively.

Using the force control system, we also conducted an experiment on cooperative transportation with a human who takes the initiative. The wheeled inverted pendulum can cooperatively transport an object with a human, though slowly, and object transportation is realized by the cooperative behavior of the wheeled inverted pendulum using the established control system.

References


