Decentralized Variable Structure Control of Vehicle Active Suspensions*

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A design method for active suspension systems of vehicles is proposed based on the decentralized variable structure controller (DVSC) and a 7-degree-of-freedom full-car model. Its nominal stability and stability robustness to parameter variations are assured through stability analysis. The performance of the DVSC is compared with that of a LQR controller in computer simulations. From the simulations, it is found that active suspensions with the DVSC reduce the acceleration of the sprung mass in the heaving, rolling, and pitching directions when the car is driven on a normal road or through an asymmetric bump. The suspension stroke and road holding capability are also compared under the assumption of equal power consumption.

Key Words: Active Suspensions, Full-Car Model, Pitch, Roll, Sliding-Mode Control

1. Introduction

Over the past few years, modern vehicles equipped with active suspension systems have appeared and demonstrated their potentials in improving driver's ride quality, meeting structural constraints, and maintaining good road-holding capability. Most studies on active suspensions^{(1),(2)}, however, have been devoted to rather simplified 2-degree-of-freedom quarter-car models, with only a few studies addressing the overall motion control based on "full" vehicle models.

Lately, some researches⁽³⁾⁻⁽⁵⁾ have been done to control rolling and pitching as well as vertical acceleration of the chassis, but they lacked the robustness to system parameter changes.

Sliding mode controllers have the advantage of overcoming system parameter variations in the system to be controlled⁽⁶⁾⁻⁽⁸⁾. However, it is difficult to design a satisfactory sliding-mode controller for a full-car model using conventional methods, due to the

fact that the procedure to derive its discontinuity gains is complexly coupled, resulting in a very complex sliding-mode control law.

However, it is possible to solve for simple discontinuity gains if the sliding mode control called the "decentralized variable structure control (DVSC)", based on a regular state-space form of the plant dynamics, is used^{(6),(7)}.

This paper proposes a design procedure for the DVSC for active suspensions which is robust to system parameter variations, as well as extending the vehicle model to a more complete one, based on a rigid sprung mass and 4 wheel-axle assemblies.

Section 2 describes a full-car model based on 7 degrees of freedom, on which the DVSC will be based. Section 3 includes explanations on the DVSC and its design procedure as well as a brief review on a typical LQG design. The performance of the DVSC and the LQG designs is compared in computer simulations in Section 4 under the assumption that they consume almost equal amount of actuator power. Section 4 concludes.

2. Vehicle Modeling

The vehicle model considered in this paper comprises five parts: the chassis (sprung mass) and 4 wheel-axle assemblies (unsprung masses) as shown in

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Fig. 1 Schematic diagram of the full-car model

Fig. 1. The sprung mass is assumed to be a rigid body and has freedoms of motion in the vertical, pitch, and roll directions. Each of the unsprung masses has a freedom of motion in the vertical direction. Thus, the full-car model has 7 degrees of freedom. Each suspension unit consists of a linear spring, a damper, and an actuator to generate pushing force between the chassis and the axle. The dynamics of the actuators are assumed to be negligible compared to the response of suspensions.

The governing equations of motion for the full-car model are:

$$M_s \ddot{q} = G B_s (\dot{z}_u - \dot{z}_s) + G k_{sr} (z_u - z_s) + G u \quad (1)$$

$$M_u \ddot{z}_u = B_s (\dot{z}_s - \dot{z}_u) + K_{ss} (z_s - z_u) + K_t (z_r - z_u) - u \quad (2)$$

where

$$\begin{aligned} q &= [z_c \quad \theta \quad \phi]^T \in \mathcal{R}^3, \\ z_u &= [z_{1u} \quad z_{2u} \quad z_{3u} \quad z_{4u}]^T \in \mathcal{R}^4, \\ z_s &= [z_{1s} \quad z_{2s} \quad z_{3s} \quad z_{4s}]^T \in \mathcal{R}^4, \\ z_r &= [z_{1r} \quad z_{2r} \quad z_{3r} \quad z_{4r}]^T \in \mathcal{R}^4, \\ u &= [u_1 \quad u_2 \quad u_3 \quad u_4]^T \in \mathcal{R}^4, \end{aligned}$$

where z_c , θ , and ϕ denote the vertical displacement at the center of gravity, the roll angle, and the pitch angle, respectively, of the sprung mass; and z_{iu} , z_{is} , z_{ir} , and u_i ($i=1, \dots, 4$) denote the vertical displacement of the unsprung mass, the vertical displacement of the sprung mass, the vertical displacement of the road, and the pushing force generated by the actuator at suspension unit i , respectively. The matrices that appear in Eqs. (1) and (2) are expressed as

$$\begin{aligned} M_s &= \text{diag}(m_s, I_\theta, I_\phi), \\ M_u &= \text{diag}(m_f, m_f, m_r, m_r), \\ B_s &= \text{diag}(b_f, b_f, b_r, b_r), \\ K_{ss} &= \text{diag}(k_f, k_f, k_r, k_r), \\ K_t &= \text{diag}(k_{ft}, k_{ft}, k_{rt}, k_{rt}), \end{aligned}$$

Table 1 Parameters of the vehicle model used

Parameters (Symbols)	Values
sprung mass (m_s)	1,460 kg
front unsprung mass (m_f)	40 kg
rear unsprung mass (m_r)	35.5 kg
roll moment of inertia of the sprung mass (I_θ)	460 kg·m ²
pitch moment of inertia of the sprung mass (I_ϕ)	2,460 kg·m ²
front suspension damping rate (b_f)	1,290 N·s/m
rear suspension damping rate (b_r)	1,620 N·s/m
front suspension stiffness (k_f)	19,960 N/m
rear suspension stiffness (k_r)	17,500 N/m
front antiroll-bar stiffness (r_f)	19,200 N·rad/m
rear antiroll-bar stiffness (r_r)	9,600 N·rad/m
front tire stiffness (k_{ft})	175,500 N/m
rear tire stiffness (k_{rt})	175,500 N/m
front thread (t_f)	1.522 m
rear thread (t_r)	1.510 m
distance between the c.g. and the front axle (l_f)	1.011 m
distance between the c.g. and the rear axle (l_r)	1.803 m

$$K_{sr} = \begin{bmatrix} k_f + r_f/2 & -r_f/2 & 0 & 0 \\ -r_f/2 & k_f + r_f/2 & 0 & 0 \\ 0 & 0 & k_r + r_r/2 & -r_r/2 \\ 0 & 0 & -r_r/2 & k_r + r_r/2 \end{bmatrix},$$

and

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -t_f/2 & t_f/2 & -t_r/2 & t_r/2 \\ -l_f & -l_f & l_r & l_r \end{bmatrix}.$$

All the above parameters are explained in Table 1 along with their numerical values used in the later simulations.

And z_s and q have a kinematic relationship of

$$z_s = G^T q. \quad (3)$$

Substituting Eq. (3) into Eqs. (1) and (2),

$$M \ddot{z} + B_z \dot{z} + K_a z - K_b z_r = G_a u \quad (4)$$

where

$$z = \begin{bmatrix} q \\ z_u \end{bmatrix} \in \mathcal{R}^7$$

and all the matrices are of appropriate dimensions:

$$\begin{aligned} M &= \begin{bmatrix} M_s & 0 \\ 0 & M_u \end{bmatrix}, \quad B_z = \begin{bmatrix} G B_s G^T & -G B_s \\ -B_s G^T & B_s \end{bmatrix} \\ K_a &= \begin{bmatrix} G K_{sr} G^T & -G K_{sr} \\ -K_{ss} G^T & K_t + K_{ss} \end{bmatrix}, \quad K_b = \begin{bmatrix} 0 \\ K_t \end{bmatrix}, \quad \text{and} \\ G_a &= \begin{bmatrix} G \\ -I \end{bmatrix}. \end{aligned}$$

Equation (4) can be represented in the state-space form of

$$\dot{x} = A x + B u + F z_r \quad (5)$$

where

$$x = [z^T \quad \dot{z}^T]^T \in \mathcal{R}^{14}$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K_a & -M^{-1}B_z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}G_a \end{bmatrix}, \quad \text{and}$$

$$F = \begin{bmatrix} 0 \\ M^{-1}K_b \end{bmatrix}.$$

3. Controller Design

3.1 Decentralized variable structure controller (DVSC) design

In order to apply the sliding mode control to the suspension system, it is required to transform the original plant model of Eq. (5) to a regular form^{(6),(7)} of

$$\begin{aligned} \dot{\gamma}_1 &= f_1(\gamma) \\ \dot{\gamma}_2 &= f_2(\gamma) + u \end{aligned} \quad (6)$$

where u is the control input, $\gamma = [\gamma_1^T \quad \gamma_2^T]^T \in \mathcal{R}^{14}$, $\gamma_1 \in \mathcal{R}^{10}$, and $\gamma_2 \in \mathcal{R}^4$, with a linear transformation $T \in \mathcal{R}^{14 \times 14}$, i.e.,

$$\gamma = Tx. \quad (7)$$

This requirement is expressed as

$$TB = [0_{4,10} \quad I_4]^T, \quad (8)$$

where I_4 denotes a 4-by-4 identity matrix.

Transformation matrices that satisfy both Eqs. (7) and (8) can be expressed as

$$T = B_\gamma B^+ + Q^T \quad (9)$$

where $B_\gamma = [0_{4,10} \quad I_4]^T$ and B^+ is the Moore-Penrose inverse of matrix B . The columns of Q spans the null space of B^T .

By substituting Eq. (9) into Eq. (5), a regular form is obtained as

$$\dot{\gamma} = A_\gamma \gamma + B_\gamma u + F_\gamma z_r \quad (10.a)$$

where

$$A_\gamma = TAT^{-1}, \quad B_\gamma = TB, \quad F_\gamma = TF$$

or

$$\begin{bmatrix} \dot{\gamma}_1 \\ \dot{\gamma}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} 0_{10,4} \\ I_4 \end{bmatrix} u + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} z_r \quad (10.b)$$

We begin the design of the sliding mode controller by defining a sliding surface as

$$\sigma = S\gamma = [S_1 \quad S_2] \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = S_1 \gamma_1 + S_2 \gamma_2 \quad (11)$$

where $\sigma \in \mathcal{R}^4$, $S_1 \in \mathcal{R}^{4 \times 10}$, and nonsingular $S_2 \in \mathcal{R}^{4 \times 4}$.

If the system is on the sliding surface,

$$\sigma = 0, \quad (12)$$

and, thus

$$\gamma_2 = -S_2^{-1}S_1\gamma_1 = -F_b\gamma_1, \quad (13)$$

where $F_b = S_2^{-1}S_1$.

Substituting Eq. (13) into Eq. (10.b),

$$\begin{aligned} \dot{\gamma}_1 &= A_{11}\gamma_1 + A_{12}\gamma_2 + F_1 z_r \\ &= (A_{11} - A_{12}F_b)\gamma_1 + F_1 z_r \end{aligned} \quad (14)$$

The reduced-order dynamics of Eq. (14) on the sliding surface is independent of control input u and

exhibits a state-feedback structure where F_b and A_{12} represent a "state feedback matrix" and an "input" matrix, respectively.

If system (A_{11}, A_{12}) is stabilizable, it is possible to find the optimal F_b , a "feedback control gain", that minimizes performance index

$$J = \int_0^\infty (\gamma_1^T Q \gamma_1 + \gamma_2^T R \gamma_2 + \gamma_1^T N \gamma_2) dt \quad (15)$$

where the lower limit of the integration refers to the initiation of sliding; $Q \geq 0$, $R > 0$. This optimal F_b maximizes index J and asymptotically stabilizes γ_1 .

Note that it would be difficult to find appropriate matrices in Eq. (15) for desirable performance of the system since γ 's are not directly associated with physical state variables. Therefore, it is desirable to find perform indices expressed in terms of physical state variables.

By using the relation of

$$x = T^{-1}\gamma = \begin{bmatrix} W_1 & W_2 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = W_1 \gamma_1 + W_2 \gamma_2 \quad (16)$$

a cost index which depends on the physical state variables can be expressed in terms of state variables of the regular form. Suppose a cost function is defined as an integral of a quadratic function of x . Then, it can be expressed in the form of Eq. (15) as

$$\begin{aligned} J &= \int_0^\infty x^T Q_s x dt \\ &= \int_0^\infty (\gamma_1^T W_1^T Q_s W_1 \gamma_1 + 2\gamma_1^T W_1^T Q_s W_2 \gamma_2 \\ &\quad + \gamma_2^T W_2^T Q_s W_2 \gamma_2) dt \end{aligned} \quad (17)$$

where $Q_s > 0$.

For the simplicity, S_2 is selected as

$$S_2 = I_4,$$

resulting in simplified switching matrix

$$S = [F_b \quad I_4]. \quad (18)$$

From Eqs. (10.b), (11), and (18),

$$\begin{aligned} \dot{\sigma} &= F_b \dot{\gamma}_1 + \dot{\gamma}_2 \\ &= F_b(A_{11}\gamma_1 + A_{12}\gamma_2 + F_1 z_r) + A_{21}\gamma_1 \\ &\quad + A_{22}\gamma_2 + u + F_2 z_r. \end{aligned} \quad (19)$$

Thus, the equivalent control, which satisfies $\dot{\sigma} = 0$, becomes

$$\begin{aligned} u_{eq} &= -F_b(A_{11}\gamma_1 + A_{12}\gamma_2 + F_1 z_r) - A_{21}\gamma_1 \\ &\quad - A_{22}\gamma_2 - F_2 z_r \end{aligned} \quad (20)$$

In order to take care of uncertainty of the system, the actual control input is modified as

$$u = \hat{u}_{eq} - H \cdot \text{sgn}(\sigma) \quad (21)$$

where

$$H = \text{diag}(h_1, h_2, h_3, h_4),$$

$$\text{sgn}(\sigma) = [\text{sgn}(\sigma_1) \quad \text{sgn}(\sigma_2) \quad \text{sgn}(\sigma_3) \quad \text{sgn}(\sigma_4)]^T,$$

and

$$\begin{aligned} \hat{u}_{eq} &= -F_b(\hat{A}_{11}\gamma_1 + \hat{A}_{12}\gamma_2 + \hat{F}_1 z_r) \\ &\quad - \hat{A}_{21}\gamma_1 - \hat{A}_{22}\gamma_2 - \hat{F}_2 z_r \end{aligned} \quad (22)$$

where $\hat{u}_{eq}$ is the equivalent control which is based on estimates of system parameters rather than actual

values. Term $H \cdot \text{sgn}(\sigma)$ represents discontinuous control input and makes the sliding mode controller robust to plant parameter variations.

In order to prove the stability of Eq.(21), a Lyapunov function candidate is selected as

$$V = \frac{1}{2} \sigma^T \sigma. \quad (23)$$

By differentiating Eq.(23), and using Eqs.(10.b), (11), and (18),

$$\begin{aligned} \dot{V} &= \sigma^T (F_b \dot{\gamma}_1 + \dot{\gamma}_2) \\ &= \sigma^T \{ F_b (\tilde{A}_{11} \gamma_1 + \tilde{A}_{12} \gamma_2 + \tilde{F}_1 z_r) \\ &\quad + \tilde{A}_{21} \gamma_1 + \tilde{A}_{22} \gamma_2 + \tilde{F}_2 z_r \} - \sum_{i=1}^4 h_i |\sigma_i| \end{aligned} \quad (24)$$

where $\tilde{A}_{ij} = A_{ij} - \hat{A}_{ij}$ for $i, j = 1, 2$.

If the components of diagonal matrix H are selected such that

$$\begin{aligned} h_i &\geq \eta_i + \|F_b \tilde{A}_{11} + \tilde{A}_{21}\| |\gamma_1| + \|F_b \tilde{A}_{12} + \tilde{A}_{22}\| |\gamma_2| \\ &\quad + \|F_b \tilde{F}_1 + \tilde{F}_2\| |z_r|, \end{aligned} \quad (25)$$

Eq.(24) results in

$$\dot{V} \leq - \sum_{i=1}^4 \eta_i |\sigma_i|. \quad (26)$$

Typically, the signum function in Eq.(21) can be replaced by the saturation function in order to eliminate the chattering effect, and thus the following control input is used for the later simulations.

$$\begin{aligned} u &= -F_b (\hat{A}_{11} \gamma_1 + \hat{A}_{12} \gamma_2 + \hat{F}_1 z_r) - \hat{A}_{21} \gamma_1 \\ &\quad - \hat{A}_{22} \gamma_2 - \hat{F}_2 z_r - H \cdot \text{sat}(\sigma/\Phi) \end{aligned} \quad (27)$$

where Φ is the thickness of the boundary layer and

$$\text{sat}(x) = \begin{cases} 1 & \text{if } x \geq 1, \\ x & \text{if } -1 < x < 1, \\ -1 & \text{if } x \leq -1. \end{cases}$$

3.2 LQR design

The road/terrain irregularities can be modeled as the output of a linear shaping filter with an input of white noise. Here, the filter is modeled as a first-order system (1) as

$$\dot{z}_r + \alpha z_r = w \quad (28)$$

where w is the white noise and α is the product of the road type coefficient and the vehicle speed. If the state vector is defined as $x_s = [x^T \ z_r^T]^T$, the augmented state-space representation of the plant becomes, from Eqs.(5) and (28),

$$\dot{x}_s = A_s x_s + B_s u + F_s w \quad (29)$$

where

$$A_s = \begin{bmatrix} A & F \\ 0 & -\alpha I \end{bmatrix}, \quad B_s = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \text{and} \quad F_s = \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

Then, the LQR controller is designed to minimize a cost function of chassis acceleration, suspension deflection and tire deflection, represented by

$$\begin{aligned} J &= \lim_{t \rightarrow \infty} \frac{1}{t} E \left\{ \int_0^t \left(\rho_1 \ddot{z}^2 + \rho_2 \dot{\theta}^2 + \rho_3 \dot{\phi}^2 + \rho_4 \sum_{i=1}^4 (z_{si} - z_{ui})^2 \right. \right. \\ &\quad \left. \left. + \rho_5 \sum_{i=1}^4 (z_{ui} - z_{ri})^2 + \rho_6 \sum_{i=1}^4 u_i^2 \right) dt \right\}. \end{aligned} \quad (30)$$

The values of weighting factors are shown on Table 2.

Table 2 Weighting factors of optimal cost function

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
1×10^6	3×10^5	3×10^5	1×10^8	1×10^8	1

Fig. 2 Power spectral densities of the heaving acceleration

4. Simulations

For simulations of active suspension control, two types of road inputs are considered to measure the responses of the vehicle over a specific band of frequencies⁽⁹⁾ and the peak values of the responses in time domain. For all simulations, the inputs to the rear wheels are assumed to be the time-delayed ones to the front wheels.

In the first simulation, the car is driven at 80 km/hr on a normal road. The power spectral densities (PSDs) of the heaving acceleration, the pitching angular acceleration, the suspension deflection of the front left wheel, and the tire deflection of the rear right wheel are shown in Figs. 2, 3, 4, and 5, respectively. Figures 2 and 3 show that the DVSC exhibits a smaller level of heaving acceleration and pitching angular acceleration than that of the LQR design for the most of the frequency range. As far as the rattle space is concerned, the DVSC also shows a better performance than the LQR above 0.5 Hz. At the low frequency range, the DVSC also shows remarkably small level in tire deflection. The power consumption level of the DVSC suspension is almost identical to that of the LQR suspension.

In the second simulation, the car is driven at 35 km/hr on an asymmetric bump whose profile is shown in Fig. 6. The heaving acceleration, the rolling angular velocity, the pitching angular velocity of the chassis, and the deflection of the rear right suspension are shown in Figs. 7, 8, 9, and 10, respectively. The

Fig. 3 Power spectral densities of the pitching angular acceleration

Fig. 4 Power spectral densities of the rattle space of the front left wheel

Fig. 5 Power spectral densities of the tire deflection of the rear right wheel

Fig. 6 Bump profile

Fig. 7 Heaving acceleration of the chassis

Fig. 8 Rolling rate of the chassis

DVSC and LQR exhibit almost identical peak levels in the heaving acceleration and the rolling rate. But the DVSC has a smaller settling time than the LQR. The pitching rate of the chassis and the rattle space are considerably smaller for the DVSC.

In the third simulation, the response and the stability of the vehicle is measured when the plant parameter variations occur. It is assumed that the

Fig. 9 Pitching rate of the chassis

Fig. 12 Rolling rate of the chassis as its mass changes

Fig. 10 Deflection of the rear right suspension

Fig. 13 Pitching rate of the chassis as its mass changes

Fig. 11 Heaving acceleration of the chassis as its mass changes

mass of the chassis changes $\pm 15\%$ from its nominal value and that other parameters such as unsprung masses, moment of inertia of the sprung mass, suspension damping rate, suspension spring stiffness, and anti-roll bar rate also vary from their nominal values. Some of the simulation results are shown from Fig. 11 to Fig. 16. Figures 11, 12, and 13 show the heaving acceleration, the rolling rate, and the pitching rate of the chassis, respectively, in the bump test when its mass changes $\pm 15\%$ from its nominal value. Figures 14, 15, and 16 show the heaving acceleration, the rolling rate, and the pitching rate of the chassis, respectively, in the bump test when the stiffness of the suspension springs changes $\pm 15\%$ from its nominal value. From the simulations, it is concluded that the responses of the vehicle are almost identical to those of the normal plant in the normal road test and the bump test.

5. Conclusions

A decentralized variable structure controller

Fig. 14 Heaving acceleration of the chassis as the stiffness of the suspensions changes

Fig. 16 Pitching rate of the chassis as the stiffness of the suspensions changes

Fig. 15 Rolling rate of the chassis as the stiffness of the suspensions changes

(DVSC) is designed for a full-car model using a plant transformation to a regular form. In designing the sliding surface, a new method minimizing the performance index, which is a function of the original state, is used. In order to evaluate the performance of the suspension, two simulation tests are performed: driving on a regular road and driving on a bump. Performance evaluations are made in terms of the chassis acceleration, the rattle space and the tire deflection as well as the power consumed. For the nominal plant, the suggested DVSC shows a better performance than that of the LQR. The stability robustness to the system parameter variations is also assured.

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