Observer-Based Time-Delay Control of a Four-Cylinder Electrohydraulic Servosystem* (Stability Analysis)

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The stability test method for the observer-based time-delay control system is discussed. Because there are uncertain parameters and time-delay terms in the control system, the frequency response plot based graphical method is extended and adopted to analysis the stability of the closed-loop system. The stability of the synchronous control of a four-cylinder electrohydraulic servosystem studied in Part I is verified to be stable by using the proposed method.

Key Words: Time-Delay Control, Observer, Stability Analysis

1. Introduction

Time-delay control is a robust and efficient control strategy. It is successfully applied to robot manipulator control^{(2),(3)}, magnetic bearing control⁽⁴⁾, and electro-hydraulic servo motor control⁽⁵⁾. The stability of a time-delay control with uncertain dynamics has been discussed by Youcef-Tumi et al.⁽⁴⁾ They applied the ideas of Nyquist⁽⁶⁾ and Kharitonov⁽⁷⁾ to develop a sufficient stability condition for the time-delay control systems with uncertain dynamics, but this method is not fit to test the stability for the control system contains an observer such as the system studied in Part I⁽¹⁰⁾ of this study. Fu et al.⁽⁸⁾ investigate the robust stability problem for a class of uncertain time-delay systems with the extension of the "edge theorem"⁽¹⁾. The characteristic equation of the time-delay system is defined as a quasi-polynomial, its stability is tested by the graphical method. In this paper the graphical method concept⁽¹⁾ is extended and applied to the observer-based time-delay control of a four-cylinder electrohydraulic servosystem with un-

certain parameters.

2. System Description

Consider a n th-order linear time-invariant system with transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b}{P(s)} \quad (1)$$

where

$$P(s) = s^n + a_{n-1}s^{n-1} + \dots + a_0 \quad (2)$$

For simplification, the controller is designed based on a reduced order system model of order r , $r \leq n$. The transfer function of the reference model is defined as

$$G_m(s) = \frac{Y(s)}{R(s)} = \frac{b_m}{P_m(s)} \quad (3)$$

where

$$P_m(s) = s^r + a_{m(r-1)}s^{r-1} + \dots + a_{m0} \quad (4)$$

Convert the transfer function representation into state space in state-variable form, which yields:

For the plant,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (5)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & I^{n-1} \\ A_n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ b \end{bmatrix}, \\ A_n &= [-a_0, -a_1, -a_2, \dots, -a_{n-1}], \\ C &= [1, 0, 0, \dots, 0] \end{aligned}$$

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$\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}$ is the control input, $\mathbf{y} \in \mathbb{R}$ is the system output, $\mathbf{I}^{n-1}$ is an identity matrix and $a_0, a_1, a_2, \dots, a_{n-1}, b$ are the coefficients of the system.

For the reference model,

$$\dot{\mathbf{x}}_m = \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{r} \quad (6)$$

where

$$\mathbf{A}_m = \begin{bmatrix} \mathbf{0} & \mathbf{I}^{r-1} \\ \mathbf{A}_{mr} & \end{bmatrix}, \quad \mathbf{B}_m = \begin{bmatrix} \mathbf{0} \\ b_m \end{bmatrix},$$

$$\mathbf{A}_{mr} = [-a_{m0}, -a_{m1}, -a_{m2}, \dots, -a_{m(r-1)}]$$

$\mathbf{x}_m \in \mathbb{R}^r$ is the state vector, $\mathbf{r} \in \mathbb{R}$ is the reference command, $a_{m0}, a_{m1}, a_{m2}, \dots, a_{m(r-1)}, b_m$ are the coefficients of the reference model.

The reduced order observer is described as

$$\dot{\mathbf{z}} = \hat{\mathbf{A}}\mathbf{z} + \hat{\mathbf{B}}\mathbf{u} + \mathbf{L}(\mathbf{y} - \hat{\mathbf{y}}) \quad (7)$$

$$\hat{\mathbf{y}} = \mathbf{C}\mathbf{z}$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{I}^{(r-1) \times (r-1)} \\ \hat{\mathbf{A}}_r & \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \hat{b} \end{bmatrix},$$

$$\hat{\mathbf{A}}_r = [-\hat{a}_0, -\hat{a}_1, -\hat{a}_2, \dots, -\hat{a}_{r-1}]$$

$\mathbf{z} \in \mathbb{R}^r$ is the observer state vector, $\hat{a}_0, \hat{a}_1, \hat{a}_2, \dots, \hat{a}_{r-1}, \hat{b}$ are the coefficients of the observer.

The time-delay control law via observer state signal feedback, derived in Part I⁽¹⁰⁾, is expressed as

$$\mathbf{u} = \hat{\mathbf{B}}^+ [-\dot{\mathbf{z}}(t-\tau) + \hat{\mathbf{B}}\mathbf{u}(t-\tau) + \mathbf{A}_m \mathbf{z} + \mathbf{B}_m \mathbf{r}] \quad (8)$$

where

$$\hat{\mathbf{B}}^+ = (\hat{\mathbf{B}}^T \hat{\mathbf{B}})^{-1} \hat{\mathbf{B}}^T = \begin{bmatrix} 0, \dots, 0, \frac{1}{\hat{b}} \end{bmatrix}$$

and

$$\hat{\mathbf{B}}^+ \hat{\mathbf{B}} = \mathbf{I}$$

The simplified block diagram of the closed-loop control system is shown in Fig. 1.

Laplace Eq. (7) yields

$$\mathbf{Z}(s) = (s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})^{-1} (\hat{\mathbf{B}}\mathbf{U}(s) + \mathbf{L}\mathbf{Y}(s)) \quad (9)$$

Laplace Eq. (8) and combined with Eq. (9), then

$$\begin{aligned} \mathbf{U}(s) &= (1 - e^{-\tau s})^{-1} \hat{\mathbf{B}}^+ [(-e^{-\tau s} s\mathbf{I} + \mathbf{A}_m) \mathbf{Z}(s) \\ &\quad + \mathbf{B}_m \mathbf{R}(s)] \\ &= (1 - e^{-\tau s})^{-1} \hat{\mathbf{B}}^+ [(-e^{-\tau s} s\mathbf{I} + \mathbf{A}_m)(s\mathbf{I} - \hat{\mathbf{A}} \\ &\quad - \mathbf{L}\mathbf{C})^{-1} (\hat{\mathbf{B}}\mathbf{U}(s) + \mathbf{L}\mathbf{Y}(s)) + \mathbf{B}_m \mathbf{R}(s)] \end{aligned} \quad (10)$$

The inverse of the matrix $(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})$ is

$$(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})^{-1} = \frac{\text{adj}(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})}{\det(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})} \quad (11)$$

For a SISO system, defining

Fig. 1 Simplified block diagram of observer-based time-delay control system

$$N_1(s) = -s\hat{\mathbf{B}}^+ \text{adj}(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})\hat{\mathbf{B}} \quad (12)$$

$$N_2(s) = \hat{\mathbf{B}} + \mathbf{A}_m \text{adj}(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})\hat{\mathbf{B}} \quad (13)$$

$$M_1(s) = -s\hat{\mathbf{B}}^+ \text{adj}(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})\mathbf{L} \quad (14)$$

$$M_2(s) = \hat{\mathbf{B}} + \mathbf{A}_m \text{adj}(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C})\mathbf{L} \quad (15)$$

$$D(s) = \det(s\mathbf{I} - \hat{\mathbf{A}} + \mathbf{L}\mathbf{C}) \quad (16)$$

and substituting Eqs. (12)–(16) into Eq. (10) to get

$$\begin{aligned} U &= \frac{1}{1 - e^{-\tau s}} \left[\frac{(N_1 e^{-\tau s} + N_2)U + (M_1 e^{-\tau s} + M_2)Y}{D} \right. \\ &\quad \left. + \frac{b_m R}{\hat{b}} \right] \end{aligned} \quad (17)$$

Rearranging the above equation yields

$$U = \frac{(M_1 e^{-\tau s} + M_2)Y + \frac{b_m}{\hat{b}} DR}{(1 - e^{-\tau s})D - e^{-\tau s} N_1 - N_2} \quad (18)$$

Substituting Eq. (18) into Eq. (1) then the closed-loop transfer function can be obtained as

$$\frac{Y}{R} = \frac{b \frac{b_m}{\hat{b}} D}{(1 - e^{-\tau s})PD - PN_1 e^{-\tau s} - PN_2 - b(M_1 e^{-\tau s} + M_2)} \quad (19)$$

The closed-loop characteristic equation is

$$(PD - PN_2 - bM_2) + e^{-\tau s}(-PD - PN_1 - bM_1) = 0 \quad (20)$$

Defining a quasi-polynomial as

$$\delta(s) = (PD - PN_2 - bM_2) + e^{-\tau s}(-PD - PN_1 - bM_1) \quad (21)$$

3. Stability Criterion

Fu, M. et al. put forward a stability test method⁽⁸⁾ for a class of uncertain time-delay systems. The closed-loop characteristic equation involves a polytope of quasi-polynomials.

Consider a quasi-polynomial family of

$$\begin{aligned} \delta(s, A) &= a_n s^n + a_{n-1} e^{-sT_{n-1}} p_{n-1}(s) \\ &\quad + a_{n-2} e^{-sT_{n-2}} p_{n-2}(s) + \dots + a_0 e^{-sT_0} p_0(s) \end{aligned} \quad (22)$$

in which each polynomial $p_i(s)$, may be real or complex, has degree order less than n ,

$$0 < T_0 < T_1 < \dots < T_{n-1}$$

and $a_i \in [a_i^-, a_i^+]$ are real parameters varying in the box

$$A = \{a: a_i^- \leq a_i \leq a_i^+, i=0, 1, \dots, n\} \quad (23)$$

Under the assumption that the “degree” of the family remains invariant (a_n does not pass through zero), the boundary crossing theorem⁽¹⁾ applied to this family and conclude from image set argument that the stability of the exposed edges implies stability of the entire polytope.

Consider a n th-order (real or complex) quasi-polynomial $\delta(s)$. $\delta_0, \delta_1, \delta_2, \dots, \delta_l$ denote the vertices of $\delta(s)$; $\delta_{E_1}, \delta_{E_2}, \dots, \delta_{E_l}$ denote the edges of $\delta(s)$. The k th-edge of the quasi-polynomial is defined as

$$\delta_{E_k} = \{\delta_{k\lambda}(s) = (1 - \lambda)\delta_{k0}(s) + \lambda\delta_{k1}(s): \lambda \in (0, 1]\} \quad (24)$$

where $\delta_{k0}(s)$ and $\delta_{k1}(s)$ denote the vertices of the edge

δ_{E_k} . Then, $\delta(s)$ is stable if and only if the following two conditions hold for every δ_{E_k} , $1 \leq k \leq t$.

(a) The frequency response plot of $\delta_{k0}(j\omega)/(j\omega+1)^n$ does not encircle the origin.

(b) The frequency response plot of $\delta_{k1}(j\omega)/\delta_{k0}(j\omega)$ does not cross $(-\infty, 0]$ (the nonpositive part of the real axis).

Based on the principle of the argument⁽¹⁾, the number of zeros of the δ_{k0} in the closed right half plane is equal to the encirclements of $\delta_{k0}(j\omega)/(j\omega+1)^n$ around the origin. Since $(s+1)^n$ is stable, therefore, δ_{k0} is stable if condition (a) hold.

If δ_{k0} is stable, then condition (b) is the necessary and sufficient condition for the stability of all $\delta_{k\lambda}$. $\delta_{k\lambda}$ is stable which implies that

$$\delta_{k\lambda}(s) = (1-\lambda)\delta_{k0}(s) + \lambda\delta_{k1}(s) \neq 0 \quad (25)$$

Fig. 2 Frequency response plot of $(j\omega+10)^4/(j\omega+1)^4$

Fig. 3 Frequency response plot of $(j\omega+10)^4/(j\omega+9)^4$

for $\lambda \in (0, 1]$ and for all $\omega \in \mathfrak{N}$. This is equivalent to

$$\frac{1-\lambda}{\lambda} + \frac{\delta_{k1}(j\omega)}{\delta_{k0}(j\omega)} \neq 0 \quad (26)$$

Since the value of $(1-\lambda)/\lambda$ varies in $[0, \infty)$ as λ varies in $(0, 1]$, it is necessary that $\delta_{k1}(j\omega)/\delta_{k0}(j\omega)$ does not cross $(-\infty, 0]$.

However, there exists an example which contradicts the condition (a). A stable polynomial for this example is $\delta(s) = (s+10)^4$, the frequency response plot of $(j\omega+10)^4/(j\omega+1)^4$ as shown in Fig. 2 encircles the origin though it is a stable system. Figure 3 shows the frequency response plot of $(j\omega+10)^4/(j\omega+9)^4$, it does not encircle the origin. Therefore the condition (a) of this stability test theorem is extended here as: The frequency response plot of $\delta_{k0}(j\omega)/(j\omega+\omega^*)^n$ where $0 < \omega^* \in \mathfrak{N}$, in place of $\delta_{k0}(j\omega)/(j\omega+1)^n$, does not encircle the origin. ω^* can be obtained by try and error approach.

4. Stability Analysis for the Observer-based Time-delay Synchronizing Control of a Four-cylinder Electrohydraulic Servosystem

In this section the stability of the observer-based closed-loop time-delay control system with uncertain parameters mentioned in Part I⁽¹⁰⁾ will be tested by the graphical method.

The individual servo hydraulic cylinder subsystem can be described as the following third-order linear system.

$$Y_i(s) = \frac{Kv_i\omega_{ni}^2 U_i(s) + D_i'(s)}{s(s^2 + 2\zeta_i\omega_{ni}s + \omega_{ni}^2)} \quad (27)$$

Neglecting the external force, the open-loop transfer function can be written as

$$\frac{Y_i(s)}{U_i(s)} = \frac{b_i}{s(s^2 + a_{2,i}s + a_{1,i})} \quad (28)$$

where $a_{1,i} = \omega_{ni}^2$, $a_{2,i} = 2\zeta_i\omega_{ni}$ and $b_i = Kv_i\omega_{ni}$.

The first order reference model which generates the desired trajectory is expressed as

$$\dot{x}_m = -\lambda_m x_m + \lambda_m r, \quad (29)$$

its pole is located at $s = -\lambda_m$.

The reduced order observer is expressed as

$$\dot{z}_i = \hat{K}_{vi}u_i + l(y_i - \tilde{y}_i) \quad (30)$$

$$\tilde{y}_i = z_i$$

where $\hat{K}_{vi}$ is the steady-state speed gain, and the pole of the observer is located at $s = -l$.

The control law with the observed state variables is expressed as

$$u_i = \frac{1}{\hat{K}_{vi}} [-\dot{z}_i(t-\tau) + \hat{K}_{vi}u_i(t-\tau) - \lambda_m z + \lambda_m r] \quad (31)$$

For this control system,

$$P(s) = s^3 + a_{2,i}s^2 + a_{1,i}s, \quad D(s) = s + l$$

$$P_m(s) = s + \lambda_m, \quad b_m = \lambda_m$$

$$b = b_i,$$

$$\hat{b} = \hat{K}_{vi}$$

$$N_1(s) = -s, \quad N_2(s) = -\lambda_m$$

$$M_1(s) = -\frac{l}{b_i}s, \quad M_2(s) = -\frac{l}{b_i}\lambda_m$$

Equation (21) leads to

$$\delta(s, a_1, a_2, b) = (\hat{b}s^4 + \hat{b}(\lambda_m + l + a_2)s^3 + (a_2\lambda_m + a_2l + a_1)s^2 + (a_1\lambda_m + a_1l)s + b\lambda_m l) + e^{-\tau s}(-ls^3 - a_2ls^2 + (-a_1l + bl)s) \quad (32)$$

After substituting $\bar{K}v_i=1$, $\lambda_m=10$, $l=15$, $\tau=0.02$, the same as the ones used in Part I⁽¹⁰⁾, into Eq.(32), then the characteristic polynomial becomes

$$\delta(s, a_1, a_2, b) = (s^4 + (25 + a_2)s^3 + (25a_2 + a_1)s^2 + (25a_1)s + 150b + e^{-0.02s}((-15s^3 - 15a_2)s^2 + 15(b - a_1)s)) \quad (33)$$

The parameters a_1 , a_2 and b are bounded by $a_1 \in [a_1^-, a_1^+]$, $a_2 \in [a_2^-, a_2^+]$ and $b \in [b^-, b^+]$ we have the following eight vertex quasi-polynomials

$$\begin{aligned} \delta_0(s) &= \delta(s, a_1^-, a_2^-, b^-) \\ \delta_1(s) &= \delta(s, a_1^-, a_2^-, b^+) \\ \delta_2(s) &= \delta(s, a_1^-, a_2^+, b^-) \\ \delta_3(s) &= \delta(s, a_1^-, a_2^+, b^+) \\ \delta_4(s) &= \delta(s, a_1^+, a_2^-, b^-) \\ \delta_5(s) &= \delta(s, a_1^+, a_2^-, b^+) \\ \delta_6(s) &= \delta(s, a_1^+, a_2^+, b^-) \\ \delta_7(s) &= \delta(s, a_1^+, a_2^+, b^+) \end{aligned}$$

Fig. 4 Frequency response plot of $\delta_0(j\omega)/(j\omega + 20)^4$

Fig. 5 All frequency response plots of $\delta_{k1}(j\omega)/\delta_{k0}(j\omega)$

$$\delta_7(s) = \delta(s, a_1^+, a_2^+, b^+)$$

and twelve edges as follows:

$$\begin{aligned} \delta_{E_1} &\in [\delta_0, \delta_1], & \delta_{E_2} &\in [\delta_0, \delta_2], & \delta_{E_3} &\in [\delta_0, \delta_4], \\ \delta_{E_4} &\in [\delta_1, \delta_3], & \delta_{E_5} &\in [\delta_1, \delta_5], & \delta_{E_6} &\in [\delta_2, \delta_3], \\ \delta_{E_7} &\in [\delta_2, \delta_6], & \delta_{E_8} &\in [\delta_3, \delta_7], & \delta_{E_9} &\in [\delta_4, \delta_5], \\ \delta_{E_{10}} &\in [\delta_4, \delta_6], & \delta_{E_{11}} &\in [\delta_5, \delta_7], & \delta_{E_{12}} &\in [\delta_6, \delta_7] \end{aligned}$$

Because different working area on both sides of the single-rod hydraulic cylinder and the effect of load mass, the system coefficients are identified for upstroking and downstroking individually via the step responses of the piston velocity. For the system pressure between 60 kg/cm² and 80 kg/cm², the coefficients variation ranges found by using the Matlab identification tool box⁽⁹⁾ are

$$\begin{aligned} a_1 &\in [46903, 19602], & a_2 &\in [285, 75], \\ b &\in [50466, 13296] \end{aligned}$$

The frequency response plot of $\delta_0(j\omega)/(j\omega + 20)^4$ is shown in Fig. 4. It can be seen that this plot does not encircle the origin, it means that the vertex δ_0 is stable

Figure 5 shows that all frequency response plots of $\delta_{k1}(j\omega)/\delta_{k0}(j\omega)$ do not intersect the nonpositive part of the real axis. It can be concluded that the system is robustly stable.

5. Conclusion

A frequency response plot based graphical test for checking the stability of time-delay systems was discussed in this paper and an example that the graphical test theorem does not hold has been demonstrated. After extension, the graphical approach can be used to analysis the observer-based time-delay control system with uncertain dynamics. The observer-based time-delay synchronizing control of a four-cylinder electrohydraulic servosystem studied in Part I⁽¹⁰⁾ has been proved to be stable by this generalized graphical test.

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