Rotational Vibration of a Spur Gear Pair Considering Tooth Helix Deviation* (Development of Simulator and Verification)

Yuichi OGAWA**, Shigeki MATSUMURA***,
Haruo HOUJOH***, Taichi SATO****
and Kiyohiko UMEZAWA*****

This paper proposes a simulator that can depict the rotational vibration of a spur gear having any types of tooth surface deviation including both tooth profile deviations and tooth helix deviations. The developed simulator is based on a single degree-of-freedom model consisted of equivalent mass, damping and mesh stiffness of a gear pair. The mesh stiffness is calculated by considering the effect of the tooth helix deviation which brings non-uniform load distribution along the contact line of the tooth pair. The dynamic load tests have been performed to verify the simulator changing both helix slope deviations of the driven gear and torque. The analytical result under each operating condition is in good agreement with the experimental result. Moreover, the damping ratio of the test gear system is evaluated as 0.03 with the measured vibration responses, which is less than conventional expectation value of 0.07.

Key Words: Spur Gear, Rotational Vibration, Helix Deviation, Simulation, Gear Mesh Stiffness

1. Introduction

It is widely accepted that helical gears are superior to spur gears in respect of reducing vibration and noise. However, since a helical gear causes larger axial load, it is necessary to have both strength and stiffness sufficiently for the pedestals and bearings

when it transmits heavy load. Therefore, it is difficult to reduce weight and energy consumption for geared machinery using helical gears. On the contrary, spur gear units have an advantage that there is no axial force, then pedestals can be designed compact. Thus recently, demand for spur gears becomes higher again.

The authors have developed a simulator to analyze the rotational vibration of a spur gear⁽⁵⁾, and also discussed the relationship between the type of profile error and dynamic behavior using the simulator in detail^{(4)-(6),(8)}. These works treated spur gears as having a uniform tooth profile in axial direction, and few investigation has been carried out taking into account the effect of tooth helix deviations that occurs in manufacturing process and misalignment in assembling.

A tooth helix deviation brings non-uniform load distribution along the contact line due to non-uniform tooth contact pattern. This affects the dynamic characteristics of a spur gear pair. In this study, a simulator that can depict the rotational vibration of a spur gear pair having any types of tooth helix deviations, such as helix slope deviation and intentional lead crowning, is proposed. A dynamic load test has also

* Received 27th December, 1999. Japanese original: Trans. Jpn. Soc. Mech. Eng., Vol. 65, No. 634, C (1999), p. 2486-2493 (Received 27th July, 1998)

** Department of Mechanical Engineering, Gunma National College of Technology, 580 Toriba-machi, Maebashi 371-8530, Japan. E-mail: ogawa@mech.gunma-ct.ac.jp

*** Precision and Intelligence Laboratory, Tokyo Institute of Technology, 4259 Nagatsuta-chou, Midori-ku, Yokohama 226-8503, Japan

**** Department of Intelligent Mechanical Engineering, College of Science and Engineering, Tokyo Denki University, Hatoyama-cho Ishizaka, Hiki-gun 350-0394, Japan

***** The deceased. Formerly belonged to Precision and Intelligence Laboratory, Tokyo Institute of Technology

been performed to verify the simulator changing both the amount of helix slope deviations of driven gear and torque conditions.

In this analysis, assuming that tooth load does not change its direction even though the tooth has helix deviation, the rotational vibratory motion of a spur gear pair can be treated as a single degree-of-freedom model along the line of action.

In this study, the damping coefficient of a gear pair is newly discussed through experimental investigation. The analysis is performed under sufficiently large load conditions where full-facewidth tooth contact is realized so that a gear pair produces larger vibration under heavier load condition.

2. Nomenclature

- $a_c(x)$: Sneddon's contact width modulus, m
 $c(x), c(\xi)$: effect function of free edges
 D : damping coefficient, N/(m/s)
 $e(x)$: deviation along the contact line, m
 $F(t, \delta)$: excitation force due to the profile deviation, N
 G : modulus of elasticity in shear, MPa
 $K(t, \delta)$: stiffness of a gear pair, N/m
 M : equivalent mass of a gear pair along the line of action, kg
 $P(\xi)$: load distribution along the contact line, N/m
 $Q(\xi)$: load distribution due to the contact between a finite width roller and a semi-infinite plane, N/m
 t : time, s
 W : transmitting load between gears, N
 x, ξ : coordinates of measuring position along a contact line, and that of loading position, respectively, m
 δ : relative displacement of a gear pair along the line of action ($=\delta_1 - \delta_2$), m
 δ_b : deflection of a gear pair due to bending, m
 δ_{co} : displacement with contact deformation between a finite roller and a semi-infinite plane, m
 ν : Poisson's ratio
 $\Delta\xi_i$: width of divisions along the contact line, m
 Subscription
 i : division number along the contact line
 j : tooth number among meshing simultaneously

3. Procedure for Analysis of Rotational Vibration

3.1 Equation of motion

The rotational vibratory motion of a spur gear had been modeled as a single degree-of-freedom system along the line of action, i.e.; the tooth was treated as a spring, and the moment of inertia of the gear wheel as a mass as shown in Fig. 1⁽⁵⁾. The equation of

motion is expressed as

$$M\ddot{\delta} + D\dot{\delta} + K(t, \delta)\delta = W + F(t, \delta) \quad (1)$$

In order to predict dynamic behavior of a spur gear pair, even for the pair having surface deviation, it is important to obtain precise mesh stiffness of the gear pair, $K(t, \delta)$, in Eq. (1).

3.2 Mesh stiffness of a tooth pair

Elastic behavior of meshing teeth is given by the contributions from both contact deformation of tooth surface and bending deformation as shown in Fig. 2. In contrast to the former study, in which the load distribution along the contact line was treated as uniform, the mesh stiffness of a tooth pair is figured out considering the effect of non-uniform load distribution due to a "helix deviation" defined by ISO1328-1.

As shown in Fig. 2, a spring of a tooth pair is treated as series connection of the following two types of springs. One is a spring due to contact deformation of tooth surfaces, stiffness K_c . The other is a spring due to bending deformation, stiffness K_b , considering the tooth as a cantilever beam. Then stiffness of a gear pair K_t is obtained by Eq. (2).

Fig. 1 Analytical model for rotational vibration of a gear pair

Fig. 2 Model for mesh stiffness of a tooth pair with helix deviation

Fig. 3 Contact between semi-infinite plane and finite width roller having radius deviation

$$K_t = \frac{K_c K_b}{K_c + K_b} \quad (2)$$

Because tooth helix deviations have a great influence on the stiffness due to tooth contact, precise estimation of the contact stiffness is important. On the contrary, the stiffness of tooth bending is little affected by about 10 μm tooth helix deviation along the contact line.

The method of handling of K_c and K_b is indicated in the following sections.

3.2.1 Stiffness due to surface contact In the proposed simulation, contact of a tooth pair having any types of tooth surface deviation is modeled as contact between a roller of which the radius slightly changes along the axial direction and a semi-infinite plane, as shown in Fig. 3. In the figure, nominal radius of the roller is written as R . Its deviation along the axial direction is expressed as $e(x)$. And the roller approaches to the plane δ_{c0} keeping the principal axis parallel to the plane surface.

We applied Sneddon's load distribution⁽¹⁾ to estimate load distribution along the contact line, and the relationship between transmitting load and contact deformation during the tooth contact is obtained. The load distribution $Q(\xi)$ on the line of contact is given as following equation based on Sneddon's solution.

$$\delta_{c0} - e(x) = \frac{2(1-\nu)}{4\pi G} \times \sum_i \frac{c(x)c(\xi_i)c(b-x)c(b-\xi_i)Q(\xi_i)\Delta\xi_i}{\sqrt{a_c^2(\xi_i) + (x-\xi_i)^2}} \quad (3)$$

Here, $c(x)$ and $c(\xi_i)$ express the effect functions of free edges of a roller, and they make load distribution $Q(\xi_i)$ smaller near both edges than around mid area on the contact line.

Equation (3) indicates the relationship of contact between a roller and a plane. On the actual tooth

surface of a gear, the effect of the free edge on the tooth tip must be taken into account. In our previous paper⁽⁷⁾, the authors discussed the effect function of free edge of a spur gear and concluded as following: the effect of the free edge on the tooth tip of the spur gear can be neglected at an ordinary meshing position where is more than Hertzian contact width away from the tooth tip.

The division width $\Delta\xi_i$ of contact line is determined from the contact width modulus as

$$\Delta\xi_i = \begin{cases} 1.53a_c(\xi_i) & (1.53a_c(\xi_i) > 0.1) \\ 0.1 & (1.53a_c(\xi_i) \leq 0.1) \end{cases} \quad (4)$$

$a_c(\xi_i)$ is the contact width modulus which is determined in terms of the Hertzian contact width modulus $a_H(\xi_i)$ at a loading point as

$$a_c(\xi_i) = a_H(\xi_i)/2 \quad (5)$$

Summing up the load distribution $Q(\xi_i)$ along the contact line, transmitting load W is obtained as

$$W = \sum_i Q(\xi_i) \cdot \Delta\xi_i \quad (6)$$

As mentioned above, the relationship between the transmitting load and the load distribution for a tooth contact is obtained taking any types of tooth helix deviation into account.

Therefore, the stiffness K_c due to tooth contact can be evaluated from the ratio of ΔW to $\Delta\delta_{c0}$, Eq. (7), as a function of δ_{c0} .

$$K_c(\delta_{c0}) = \frac{\Delta W}{\Delta\delta_{c0}} \quad (7)$$

3.2.2 Stiffness due to tooth bending We show an estimation method of stiffness due to tooth bending next. Under the condition that transmitting load between gears is sufficiently heavy and then the meshing teeth have full-facewidth contact, the stiffness K_b due to the bending deformation is calculated by using Ishikawa's equation⁽⁶⁾ in which a gear tooth is approximated as a cantilever formed three trapezoids, Fig. 2, taking into account the effect of shear and the inclination of tooth root. The influence of tooth helix deviation on the bending deflection gently spreads over the whole tooth surface in comparison with that on contact deformation.

Therefore in this paper, even though gear tooth surfaces have helix deviations, the mesh stiffness due to bending, K_b , is treated approximately as same as the gear tooth without helix deviation under sufficiently heavy load condition which brings full-facewidth contact along the contact line.

$$\delta_b = W/K_b \quad (8)$$

The validity of this approximation on bending deflection, δ_b , is verified in section 3.2.3.

3.2.3 Elastic behavior of a tooth pair Stiffness of a tooth pair K_t is obtained from Eq. (2) using K_b and K_c . To reduce computational complexity,

Table 1 Specifications of test gear

Module (mm)	2.5
Number of teeth	48
Pitch circle diameter (mm)	120.0
Facewidth (mm)	15.0
Profile shift coefficient	0
Pressure angle (deg.)	20
Addendum (×module) (Tooth tip chamfer (mm))	1.0 (0.37)
Contact ratio	1.51*

* Substantial contact ratio due to tooth tip chamfer

Fig. 4 Relative displacement of a tooth pair vs. tooth load (16 μm helix slope deviation)

relative displacement between a tooth pair is obtained approximately as sum of a bending deflection δ_b and a contact deformation δ_{c0} ;

$$\delta = \delta_b + \delta_{c0} \quad (9)$$

In order to verify the validity of the approximation on bending deflection described in section 3.2.2, the calculated relationship between tooth load (torque) and relative displacement is compared with the analysis method presented by Umezawa et al.⁽⁹⁾, which takes into account the effect of the distribution of bending deflection. On a gear pair (Table 1) used for the experiment described in chapter 4, the comparison was made by changing the helix slope deviation from 0 μm to 16 μm . Thus, it has become clear that the difference between these results is about 11% maximum, under the torque condition from 49 Nm to 196 Nm.

Figure 4 is an example of the comparison with 16 μm tooth helix slope deviation. In this case, the difference is within about 6%. The relative displacement of the tooth pair is obtained with sufficient

Fig. 5 Mesh stiffness vs. relative displacement on the pitch point

accuracy by using approximated bending deflection as shown in Fig. 4. Because the exact calculation without approximation on bending deflection is complicated, we approximate the bending spring as a single spring, which is the same with a tooth pair that has no helix deviation on the surfaces.

Figure 5 shows mesh stiffness K_t between a tooth pair having helix slope deviation obtained with the proposed method. It shows the relationship between the mesh stiffness of the tooth pair K_t and the relative displacement δ under the condition that the pair has 7 μm tooth helix slope deviation and is mating on the pitch point. The mesh stiffness varies significantly when the displacement δ is small. On the contrary, when the relative displacement δ is large enough to realize full-facewidth contact condition (more than 116 N/mm transmitting load indicated in the figure), the mesh stiffness is a linear function of δ .

In this study, we mainly analyze such case under sufficient heavy transmitting load.

3.3 Stiffness of multi-pair tooth mesh

As mentioned above, the analysis for mesh stiffness of a tooth pair is complicated because of its non-linear elastic behavior of contact stiffness. In order to develop practical vibration analysis of a spur gear pair having any types of tooth surface deviations, mesh stiffness of a gear pair $K(t, \delta)$ is obtained taking into account of multi-pair tooth mesh by following procedure.

Figure 6 illustrates the case that two pairs are meshing simultaneously. The meshing positions A and B are positioned with one normal pitch spacing along the line of action. At each meshing position, the relationship between mesh stiffness K_t and relative displacement δ is obtained with the proposed analysis.

Fig. 6 Instantaneous stiffness of a gear pair at multi-pair tooth mesh

Fig. 7 Effect of difference of load torque on mesh stiffness of a gear pair

When tooth pairs are in mesh simultaneously, the relative displacements of the tooth pairs δ are equal. Equilibrium relative displacement δ_0 can be obtained with the condition that the total load is equal to static transmitting load between a gear pair, W . Thus as shown in Fig. 6, δ_0 is determined by an iterated calculation. Then the mesh stiffness of each tooth pair $K_v(\delta_0)$ is obtained.

According to our previous paper⁽⁵⁾, the stiffness function of tooth tip mesh is approximated as linearly chamfered shape in Hertzian contact width from the tooth tip along the line of action, and it is zero at the tooth tip. At any mesh positions along the line of action, the mesh stiffness of the gear pair, $K(t, \delta_0)$, is obtained by summing up each mesh stiffness of tooth pairs mating at the instant.

Fig. 8 Effect of difference of amount of helix slope deviation on mesh stiffness of a gear pair

Figure 7 shows mesh stiffness variation of a gear pair that has $7 \mu\text{m}$ helix slope deviation along the line of action under torque condition from 49 Nm to 196 Nm. The rate of increase in mesh stiffness of the gear pair becomes smaller as torque increases, and the mesh stiffness curve converges to a specific shape under sufficient heavy torque condition. Observing more detail, though the value of mesh stiffness is small during one pair mesh that is indicated as "One pair" in Fig. 7, the difference of mesh stiffness due to load condition becomes smaller because the surface load is relatively large. On the other hand, during two-pair mesh (indicated as "Two pairs"), although the mesh stiffness is large, the difference of mesh stiffness corresponding to load condition becomes larger because surface load becomes lower depending on load sharing between two tooth pairs. It causes non-linear behavior of mesh stiffness even though the load is large. The mesh stiffness under 49 Nm torque is also indicated in Fig. 7, where full-facewidth contact is not realized, which would be referred as a rough estimation.

Figure 8 shows the mesh stiffness of a gear pair changing the amount of helix slope deviation under 147 Nm torque condition. During two-pair mesh, the mesh stiffness becomes small, as the amount of helix deviation increases. In the region of one pair mesh, the value of mesh stiffness of a gear pair is not affected with helix deviation from $0 \mu\text{m}$ to $10 \mu\text{m}$. However, the value of the mesh stiffness is small with $16 \mu\text{m}$ helix deviation. The influence of helix deviation on the mesh stiffness is remarkable during two-pair mesh.

Therefore, it is important to obtain precise mesh stiffness of a gear pair taking tooth helix deviation

Fig. 9 Measured rotational acceleration response and half power bandwidth of a resonance

Fig. 10 Value of damping ratio obtained from measured each resonant peak

into account.

3.4 Damping coefficient

The damping effect of a gear system is assumed as viscous damping between gear teeth. In the previous study⁽⁵⁾, the damping ratio was set empirically as 0.07. With more exact viewpoint, however, a gear unit should have its own specific damping ratio.

Therefore, in this study, the damping ratio ζ is determined by resonant frequency f_n and its half power bandwidth Δf , which were measured by our experiment, as shown in Fig. 9. In the experiment, remarkable peaks indicated as f_n/m ($m=2, 3, 4, 5, \dots$) in Fig. 9 are m -th harmonic resonance by tooth mesh frequency f_z at mesh natural frequency f_n .

Thus, ζ is obtained from Eq.(10) with these peaks.

$$\zeta = \Delta f / (2f_n) = \Delta N / (2N_0) \quad (10)$$

The measured values of ζ with helix slope deviations from 0 μm to 16 μm are shown in Fig. 10. The

Fig. 11 Procedure for numerical calculation of the developed simulator

values of ζ are seen in the range from 0.015 to 0.045. However, the dependence on deviation can not be observed. Under these experimental conditions, the value of ζ is decided to be 0.03.

The damping force between a gear pair is collected every tooth pair in mesh simultaneously. The damping coefficient D , in Eq. (1), is obtained by Eq. (11).

$$D = \sum_j D_j = \sum_j 2\zeta \sqrt{M \cdot K_{tj}} \quad (11)$$

3.5 Procedure for numerical analysis of rotational acceleration

Solving Eq.(1) with $K(t, \delta)$ and D in section 3.3 and 3.4 using Runge-Kutta-Gill method, rotational acceleration of a spur gear pair is obtained. A flow chart of the proposed analysis is shown in Fig. 11.

4. Experimental Verification for Analytical Results

4.1 Test apparatus and tested gears

Figure 12 shows a schematic of the test gear unit. It is a power-absorbing type dynamic load test machine. To avoid the influence of stiffness of shafts and pedestals on gear vibration and to examine only the effect of tooth helix deviation, the gear shafts and pedestals are designed to have enough stiffness. Both driving and driven gear shafts are 40 mm diameter at gear mount positions, and there are 10 mm diameter through holes at their centers for electrical wiring of accelerometers. The material of the shafts is SKS2, and their surfaces are finished by grinding. Both shafts are mounted on roller bearings (NUP207), which are considered to be stiffer than ball bearings,

Fig. 12 Apparatus for dynamic load test of a spur gear pair

Fig. 13 Measured helix deviation of test gears

and the axial distance between two bearings is 164 mm. The center distance is 120.1 mm.

During the test, the gear pair is lubricated by supplying about 0.45 l/min flow rate of additive free turbine oil (JIS No.3) to tooth mesh part. In this study, the specification of the test gears is shown in Table 1. Although the geometrical contact ratio for a full depth tooth pair is 1.75, the substantial contact ratio is calculated to be 1.51 due to 0.37 mm chamfer in the tooth tips. The material of the gears is SCM3, and the teeth have H_{RC} 55 hardness, after induction hardening and tempering, and are finished by tooth surface grinding.

Figure 13 shows the tooth helix deviations of the test gears measured by using a coordinate measuring machine. They are manufactured to have different amount of helix slope deviation shown in this figure. Every 48 tooth surface is finished carefully, and both of the shape and the amount of helix deviation are within 1 μm variation.

4.2 Measurement system

The rotational acceleration of the gear wheel is measured using two piezoelectric type accelerometers

Fig. 14 Influence of torque on rotational vibration without helix deviation

(303A, PCB) placed axisymmetrically on both driving and driven gear. Acceleration responses are measured keeping the torque constant and the rotational speed is swept down from 3 500 rpm to 600 rpm at the rate of 14 rpm/s. The waveforms and the spectrum data are obtained with an FFT analyzer.

4.3 Verification results

In order to verify the accuracy of the developed simulator, the influence of helix slope deviation on rotational vibration of a spur gear pair was examined. The driving gear has no helix deviation, and the gears having three different helix slope deviations, 0 μm, 10 μm and 16 μm, were prepared for the driven gears, shown in Fig. 13.

The over-all value (rms) of rotational acceleration of the analytical and the experimental results are shown in Figs. 14, 15 and 16. Each figure indicates three dynamic responses under 49 Nm, 98 Nm and 147 Nm load torque conditions in column from the bottom, respectively. Because the proposed analysis method is based on a single degree-of-freedom model, the calculated results depict only the peaks correlated to f_n . For example, the peaks at 2 000 rpm and near 3 100 rpm are only observed in the experimental

Fig. 15 Influence of torque on rotational vibration with 10 μm helix deviation

results. The peaks of $f_n/2$ to $f_n/5$ in the calculated results are in good agreement with those in the experimental results.

Without helix slope deviation in Fig. 14, in particular, the calculated and the experimental results are in good agreement on the point that the peak of $f_n/2$ is the highest under 147 Nm torque condition, while the peak of $f_n/3$ is the highest under 98 Nm and 49 Nm torque condition.

Likewise, in Fig. 15 for 10 μm helix slope deviation, the peak of $f_n/2$ is the highest under 147 Nm torque condition, and the peak of $f_n/2$ is almost same amplitude as that of $f_n/3$ under 98 Nm torque condition. For 10 μm helix slope deviation under 49 Nm torque condition, the calculated amplitude becomes higher, and the resonant speed becomes lower. A premise for the proposed method which realize full-facewidth contact, is not satisfied under this light load condition. However, the feature that the peak amplitude of $f_n/2$ becomes very low is depicted even under this condition.

For 16 μm helix slope deviation, in Fig. 16, since full-facewidth contact condition is not satisfied under

Fig. 16 Influence of torque on rotational vibration with 16 μm helix deviation

98 Nm and 49 Nm torque, the difference between the calculated and the experimental results are observed under these conditions. However, similar tendencies are shown in the variation of peak amplitudes and resonant speeds changing torque conditions. Moreover, under 147 Nm torque, full-facewidth contact condition, it is recognized that the calculated results of the peak amplitudes and the resonant speeds are in good agreement with the experimental results.

From above, the proposed simulator depicts the effect of tooth helix deviation well under full-facewidth contact condition, and therefore the validity of the simulator was confirmed. Moreover, it is possible to predict dynamic behavior of a spur gear pair even under "almost" full-facewidth contact condition, i.e. partial-facewidth contact, if some estimation error is allowed.

5. Conclusion

A simulator for dynamic behavior of a spur gear pair having tooth helix deviation is proposed for low vibration design, and the conclusions are given as

follows.

(1) Tooth helix deviations considerably affect vibration response of a spur gear pair. The applicability of the developed simulator is verified by experiment.

(2) From the experimental result, damping ratio of a spur gear pair was evaluated as 0.03, and its value was shown to be smaller than that used for conventional one, as 0.07.

References

- (1) Sneddon, I., Fourier Transforms, (1951), p. 473, McGraw-Hill, New York.
- (2) Umezawa, K., The Meshing Test on Hermal Gears under Load Transmission; 3rd Report, The Static Behaviors of Driven Gears, Bull. JSME, Vol. 17, No. 112 (1974), p. 1348.
- (3) Fujii, K., Ishikawa, J. and Umezawa, K., On Deflection of a Spur Gear Tooth, (in Japanese), Proc. JSME, 780-2, (1978), p. 175-180.
- (4) Sato, T., Umezawa, K. and Kono, K., Performance of Rotational Vibration and Error on Spur Gear for Power Transmission; 1st Report, Influence of Pressure Angle Error and Normal Pitch Error, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 49, No. 443, C(1983), p. 1217-1225.
- (5) Umezawa, K., Sato, T. and Ishikawa, J., Simulation on Rotational Vibration of Spur Gears, Bull. JSME, Vol. 27, No. 223 (1984), p. 102-109.
- (6) Umezawa, K. and Sato, T., Performance of Rotational Vibration and Error on Spur Gear for Power Transmission; 2nd Report, Influence of Waved Profile Error, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 461, C(1985), p. 148-151.
- (7) Sato, T. and Umezawa, K., Mesh Stiffness of Spur Gear during Tooth Tip Mesh, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51, No. 463, C (1985), p. 558-596.
- (8) Umezawa, K. and Sato, T., Performance of Rotational Vibration and Error on Spur Gear for Power Transmission; 3rd Report, Accumulative Pitch Error, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 51 No. 464, C (1985), p. 831-838.
- (9) Umezawa, K., Matsumura, S., Houjoh, H. and Ichikawa, N., Rotational Vibration of a Hermal Gear Pair Having Tooth Surface Deviation during Transmission of Light Load; 1st Report, An Analysis of Approach between Gear Teeth with Surface Deviation, Trans. Jpn. Soc. Mech. Eng., (in Japanese), Vol. 60, No. 575, C (1994), p. 2429-2435.