Inverse Dynamic Analysis of Constrained Multibody Systems Considering Friction Forces Acting on Kinematic Joints*

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A method for the inverse dynamic analysis of constrained multibody systems considering friction forces acting on kinematic joints is proposed in this paper. The stiction and the sliding which represent zero and non-zero relative motions are considered during the inverse dynamic analysis. Actuating forces to control the position or the orientation of constrained multibody systems are usually calculated in the inverse dynamic analysis. When the friction is considered, an iterative procedure need to be developed to calculate the actuating forces, which are uniquely determined during the sliding but not uniquely determined during the stiction. Numerical examples are solved to demonstrate the accuracy and the effectiveness of the proposed method.

Key Words: Inverse Dynamic Analysis, Stiction, Sliding, Kinetic Friction Coefficient, Static Friction Coefficient, Constrained Multibody System

1. Introduction

If active forces are given for a multibody system, the equations of motion of the system can be solved to obtain the motion of the system. This process is known as *forward dynamic analysis*. In some problems, a specified motion for a multibody system is sought and the objective is to determine the force that must act on the system to produce such a motion. This process is usually referred to as *inverse dynamic analysis* or *kinetostatic analysis*. While forward dynamic analysis is mainly used to obtain the motion of multibody systems, inverse dynamic analysis is

used to determine the reference input or the capacity of actuators, which are often used for multibody systems such as robotic systems.

When the friction is considered in multibody systems, several numerical difficulties such as the discontinuity of friction force, the variation of system degrees of freedom, and the coupling between normal force and friction force need to be resolved. The discontinuity of friction force involves with the stiction and the sliding which represent zero and non-zero relative motion. When two contiguous bodies slide each other, kinetic friction forces (involving the kinetic friction coefficient) are applied to the bodies. When the stiction occurs, on the other hand, static friction forces (involving the static friction coefficient) are applied to the bodies. When a transition (from the stiction to the sliding or from the sliding to the stiction) occurs, the discontinuity of friction force occurs. This discontinuity causes the discontinuity of acceleration, which often causes numerical difficulties. When the stiction occurs, constraint equations to restrict the relative motion at joint between two contiguous bodies are introduced. The introduction of the constraint equations causes

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the reduction of degrees of freedom. Thus, the degrees of freedom during the stiction become less than those during the sliding. This again causes a numerical difficulty. Friction forces are functions of normal forces, which are denoted as the Lagrange multiplier vector. Since friction forces and normal forces are coupled each other, they need to be calculated by an iterative procedure, which causes a numerical difficulty too.

Haug and his colleagues⁽¹⁾ developed a method for forward dynamic analysis of multibody systems undertaking friction forces on joints. The stiction and the sliding are considered to calculate the friction force in this method. To calculate the friction force efficiently, the concept of relative displacement and relative velocity⁽²⁾ are introduced. Thus, the friction force is represented as a linear or nonlinear spring force. While this method is convenient to model friction force, it cannot analyze the stiction properly because the friction force becomes zero with a zero relative motion. While methods for the forward dynamic analysis considering friction forces have been developed, a method for the inverse dynamic analysis considering friction forces has not been introduced yet. References (3) and (4) show typical conventional methods developed for the inverse dynamic analysis, in which augmented Lagrange equations of motion are solved to calculate the actuating forces. However, friction forces acting on kinematic joints have not been considered so far in the inverse dynamic analysis.

A method for the inverse dynamic analysis of multibody systems undertaking friction forces on kinematic joints is presented in this paper. The stiction and the sliding are considered to calculate the actuating force accurately. The variation of degrees of freedom of a system between the sliding and the stiction causes a numerical difficulty. To avoid this difficulty in this paper, equations of motion for the sliding are used to obtain the actuating force not only during the sliding but also during the stiction. To prove the numerical accuracy and the effectiveness of the proposed method, some numerical examples are solved and the results of actuating forces are compared to those used in the forward dynamic analysis to reproduce the same motion.

2. Equations of Motion

Let n generalized coordinates of a multibody system be denoted as

$$\mathbf{q} = [q_1 \ q_2 \ \cdots \ q_n]^T \quad (1)$$

Let constraint equations which are used to restrict relative motions of contiguous bodies be expressed as follows:

$$\Phi(\mathbf{q}, t) = [\phi_1 \ \phi_2 \ \cdots \ \phi_m] = 0 \quad (2)$$

where m denotes the number of independent constraint equations. Then augmented Lagrange equations of motion for a constrained multibody system are written as

$$\mathbf{M}\ddot{\mathbf{q}} + \Phi_q^T \lambda = \mathbf{Q} \quad (3)$$

Where $\mathbf{M}$ (dimension $n \times n$) is the mass matrix that depends on the generalized coordinates, Φ_q (dimension $m \times n$) is the Jacobian matrix which is the partial derivative of the constraint equations with respect to the generalized coordinates, λ (dimension m) is the Lagrange multiplier vector, and $\mathbf{Q}$ (dimension n) is the general force vector. Double dots over a symbol (shown in Eq.(3)) denote the double time differentiation of the symbol. The general force vector $\mathbf{Q}$ is composed of three components as follows:

$$\mathbf{Q} = \mathbf{Q}^A + \mathbf{Q}^F + \mathbf{Q}^E \quad (4)$$

where $\mathbf{Q}^A$, $\mathbf{Q}^F$, and $\mathbf{Q}^E$ denote actuating forces, friction forces on kinematic joints, and the other external forces, respectively.

With the Jacobian matrix, a full row and column pivoting is performed to automatically partition the generalized coordinates $\mathbf{q}$ into dependent coordinates $\mathbf{q}_d$ of dimension m and independent coordinates $\mathbf{q}_i$ of dimension $n - m$. So, the generalized coordinates can be rewritten as

$$\mathbf{q} = [\mathbf{q}_d^T \ \mathbf{q}_i^T]^T \quad (5)$$

To reduce the dimension of the equations of motion, a matrix which is the null space of the Jacobian matrix is introduced. The matrix is called hereinafter the partial velocity matrix and expressed as follows:

$$\mathbf{B} = \begin{bmatrix} -\Phi_{q_d}^{-1} \Phi_{q_i} \\ \mathbf{I} \end{bmatrix} \quad (6)$$

In the above equation, Φ_{q_d} and Φ_{q_i} denote the Jacobian matrices of constraint equations with respect to $\mathbf{q}_d$ and $\mathbf{q}_i$; and $\mathbf{I}$ denotes the identity matrix. The dimensions of Φ_{q_d} , Φ_{q_i} , and $\mathbf{I}$ are $m \times m$, $m \times (n - m)$, and $(n - m) \times (n - m)$ respectively. By pre-multiplying Eq.(3) by the transpose of the partial velocity matrix, the following equations of motion can be obtained.

$$\mathbf{B}^T \mathbf{M} \ddot{\mathbf{q}} = \mathbf{B}^T \mathbf{Q} \quad (7)$$

The formulation of Eq.(7) constitutes minimum number of equations of motion for a constrained multibody system. Thus, the formulation is computationally efficient.

3. Inverse Dynamics

In this study, the number of actuating forces is assumed to be same as the number of degrees of freedom of the system of concern during the sliding. Substitution Eq.(4) into Eq.(7), the following equation can be obtained.

Fig. 1 Two bodies connected with a kinematic joint

$$B^T Q^A - \kappa = 0 \quad (8)$$

where

$$\kappa \equiv B^T (M\ddot{q} - Q^F - Q^E) \quad (9)$$

In Eqs. (8) and (9), all terms except Q^A and Q^F are determined in the inverse dynamic analysis since Q^E , q , $\dot{q}$, and $\ddot{q}$ are given.

Figure 1 (a) shows body i and body j connected with a kinematic joint. In the figure, $x_k - y_k - z_k$ ($k = i, j$) is a reference frame fixed to body k , and s_k is the position vector from the origin of the reference frame k to the kinematic joint. Figure 1(b) shows an actuating force and an actuating torque acting on the two bodies. In the figure, F_{Aij} and T_{Aij} denotes the magnitude of the force and the torque exerted by an actuator; and $\hat{g}_{ij}$ and $\hat{h}_{ij}$ are the unit vectors along the axis of relative translation and rotation. Therefore actuating force F_{Aij}^i and torque T_{Aij}^i applied to the body i can be written as

$$F_{Aij}^i = -F_{Aij} \hat{g}_{ij} \quad (10)$$

$$T_{Aij}^i = -T_{Aij} \hat{h}_{ij} \quad (11)$$

Actuating force F_{Aij}^j and torque T_{Aij}^j applied to the body j have the same magnitude (but opposite direction) as F_{Aij}^i and T_{Aij}^i . So, F_{Aij}^j and T_{Aij}^j are written as

$$F_{Aij}^j = F_{Aij} \hat{g}_{ij} \quad (12)$$

$$T_{Aij}^j = T_{Aij} \hat{h}_{ij} \quad (13)$$

In case that the velocity of the origin of a body-fixed reference frame and the angular velocity of the reference frame are selected as generalized velocities, the general force vectors Q_k^A acting on body k ($k = i, j$) due to the actuating force and torque can be obtained as

$$Q_j^A = \begin{bmatrix} -F_{Aij} \hat{g}_{ij} \\ -F_{Aij} \hat{s}_i' A_i^T \hat{g}_{ij} - T_{Aij} A_i^T \hat{h}_{ij} \end{bmatrix} \quad (14)$$

(a) System configuration

(b) Free body diagram of body 1

Fig. 2 Simple example of translationally moving contiguous bodies

$$Q_j^A = \begin{bmatrix} F_{Aij} \hat{g}_{ij} \\ F_{Aij} \hat{s}_i' A_i^T \hat{g}_{ij} + T_{Aij} A_i^T \hat{h}_{ij} \end{bmatrix} \quad (15)$$

where A_k ($k = i, j$) is the orientation matrix of the reference frame fixed to body k , s_k' is $A_k^T s_k$, and $\hat{s}_k'$ is a skew symmetric matrix of s_k' . All terms except F_{Aij} and T_{Aij} in Eqs. (14) and (15) are determined since generalized coordinates are prescribed during the inverse dynamic analysis. From Eqs. (14) and (15), it is obvious that Q^A is a linear function of F_{Aij} and T_{Aij} . So, Eq. (8) can be rewritten as

$$B^T (Q^A)_{F_A} F_A = \kappa \quad (16)$$

where F_A is the vector that has actuating forces and torques as its elements and $(Q^A)_{F_A}$ is the Jacobian matrix of Q^A with respect to F_A . To obtain the solution of the above equation, it is necessary to find Q^F (the general force vector due to friction forces) to calculate κ . However, Q^F cannot be calculated directly since it is a function of the normal forces at joints. The normal forces can be obtained from the following augmented Lagrange equations of motion.

$$\Phi_q^T \lambda = Q - M\ddot{q} \quad (17)$$

Since Q in Eq. (17) includes Q^F , Eqs. (16) and (17) are coupled. So they must be solved simultaneously by using an iteration method. In this work, the fixed point iteration method (See Ref. (5)) is employed.

A simple example that consists of two translationally moving contiguous bodies (as shown in Fig. 2) is given to explain how the actuating forces can be calculated when the sliding and the stiction occur during the inverse dynamic analysis. In the figure, m denotes the mass of body 1 and g denotes the gravitational acceleration. v_1 and a_1 denote the velocity and the acceleration of body 1; and v_2 and a_2 denote those of body 2. F , F_s , and N denote the actuating force, the

Fig. 3 Two bodies connected with a revolute joint

friction force, and the normal force acting on body 1. From the free body diagram of body 1 (shown in Fig. 2(b)), the equation of motion can be derived as follows:

$$F - F_s = ma_1 \quad (18)$$

From the above equation, the actuating force can be determined as follows:

$$F = ma_1 + F_s \quad (19)$$

Now in case of sliding, i.e. when v_1 is not equal to v_2 , F_s can be determined uniquely as follows:

$$F_s = \mu_k N \frac{(v_1 - v_2)}{|v_1 - v_2|} \quad (20)$$

where μ_k denotes the kinetic friction coefficient. In case of stiction, i.e. when v_1 is equal to v_2 , F_s cannot be determined uniquely. Instead, a bounded range for F_s is given as follows:

$$-\mu_s N < F_s < \mu_s N \quad (21)$$

where μ_s denotes the static friction coefficient. Since F_s is bounded during the stiction, the actuating force F is also bounded. Thus,

$$ma_1 - \mu_s N < F < ma_1 + \mu_s N \quad (22)$$

When the stiction occurs between two contiguous bodies, the degrees of freedom and the equations of motion to be solved may be reduced. If the equations of motion are reduced, however, the friction forces between the two contiguous bodies cannot be determined properly. Therefore, actuating forces cannot be determined properly from the inverse dynamic analysis. In the present work, the original equations of motion are consistently employed for the inverse dynamic analysis. If the sliding occurs, F_s is given as a unique value to calculate the actuating forces. If the stiction occurs, F_s is given with bounded values and the actuating force is also given with bounded values.

Figure 3 shows the case of two rotationally moving contiguous bodies. Two bodies are connected with a revolute joint. ω_{ij} denotes relative angular

Fig. 4 A simple bar pendulum

velocity of body j with respect to body i . R_{ij} and d_{ij} denote the radius and the length of the joint respectively. During the sliding, the actuating torque can be obtained uniquely; and during the stiction, bounded values for actuating torques can be obtained by using the same method as explained in the previous example.

The numerical algorithm of the proposed method can be summarized as follows:

Step 1: In the beginning (when no preceding time step exists), the normal forces obtained from Eq. (17) without considering friction are used as the initial estimate of N . Otherwise, the solution of normal force vector N at the preceding time step is used as the initial estimate of N for the current time step.

Step 2: Calculate all the matrices determined in the inverse dynamic analysis such as M , B , Q^E and $(Q^A)_{FA}$.

Step 3: If the sliding occurs, calculate Q^F due to sliding, otherwise (if the stiction occurs) calculate Q^F which is the bounded values of static friction force. Then, calculate κ .

Step 4: Obtain the actuating force vector F_A by solving Eq. (16). If F_A is calculated for the first time, go to Step 5. Otherwise, calculate the actuating force vector variance $(F_A^{new} - F_A^{old})$. If it is less than the specified error tolerance, then go to Step 6. Otherwise, go to Step 5.

Step 5: Calculate a Lagrange multiplier vector λ by solving Eq. (17). Then calculate new normal forces N on the joints by using the Lagrange multiplier vector λ and the Jacobian matrix Φ_q . Go to Step 3.

Step 6: If the current time reaches or exceeds the final time, stop. Otherwise, increase the time and go to Step 1.

4. Numerical Examples

Figure 4 shows a simple bar pendulum undertaking friction force on its joint. The pendulum and the ground are connected with a revolute joint. In the

Table 1 Inertia properties and initial conditions of a simple bar pendulum

Inertia properties	
m, J	$1.00[\text{kg}], 0.400[\text{kg} \cdot \text{m}^2]$
Initial conditions	
$\mathbf{r}$	$[0.217 \ 0.125]^T [\text{m}]$
θ	$0.524[\text{rad}]$
$\dot{\mathbf{r}}$	$[-0.125 \ 0.217]^T [\text{m/sec}]$
$\dot{\theta}$	$1.00[\text{rad/sec}]$

Fig. 5 Orientation angle, angular velocity, and angular acceleration of a simple pendulum

figure, $x'-y'$ is a reference frame fixed to the pendulum and R is the radius of the joint. Gravitational force and an actuating torque are applied to the pendulum. Table 1 presents the inertia properties and initial conditions for the forward and the inverse dynamic analysis. In the table, m and J denote the mass and the mass moment of inertia of the pendulum respectively. θ and $\dot{\theta}$ denote the orientation angle and the angular velocity of the pendulum respectively. $\mathbf{r}$ and $\dot{\mathbf{r}}$ denote the position and the velocity of the mass center of the pendulum respectively. In performing forward dynamic analysis, an actuating torque T_A which has the value of $2 \sin(2t)[\text{N} \cdot \text{m}]$ is applied. The kinetic and the static friction coefficients are 0.3 and 0.35; and the radius of the joint is 0.1 [m]. Figure 5 shows the orientation angle, the angular velocity, and the angular acceleration vs. time. As shown in Fig. 5, there exist discontinuities in the angular acceleration, and the stiction occurs between 1.07 [sec] and 1.21 [sec]. To perform the inverse dynamic analysis, the position, the velocity, and the acceleration obtained from the forward dynamic analysis are used as given motions. Figure 6 shows the comparison of the actuating torque obtained from the inverse dynamic analysis and that used for forward dynamic analysis. The analysis results show that the actuating torque

Fig. 6 Comparison of the actuating torque obtained by the inverse dynamic analysis and that used for the forward dynamic analysis

Fig. 7 A spatial multi-link mechanism connected by revolute joints

obtained from the inverse dynamic analysis is identical to that used for the forward dynamic analysis except the results in the region of the stiction. In the region of the stiction, the actuating torque used in the forward dynamic analysis exists within the bounded region obtained from the inverse dynamic analysis. From the above analysis results, it is found that the proposed method in this paper has the accuracy of the numerical solution.

A three link mechanism which often appears in robotic systems is shown in Fig. 7. The ground and link 1, link 1 and link 2, and link 2 and link 3 are connected with revolute joints. The friction force is assumed to act only on the joint between link 1 and link 2. In the figure, $X-Y-Z$ and $x'_k-y'_k-z'_k$ ($k=1, 2, 3$) denote reference frames fixed to the ground and the link k respectively, and $\hat{\mathbf{h}}_{l(l+1)}$ ($l=0, 1, 2$) denotes a unit vector fixed to the axis of rotation of the joint between link l and link $l+1$. In the magnified drawing, R_{12} and d_{12} are the length and the radius of the joint between link 1 and link 2 and have the values of 0.08 [m] and 0.1 [m] respectively. Table 2 shows the

Table 2 Inertia properties and initial conditions of a multi-link mechanism

Inertia properties	
m_1	4.00[kg]
$J'_{1xx}, J'_{1yy}, J'_{1zz}$	0.500, 0.200, 0.500[kg·m ²]
m_2	3.00[kg]
$J'_{2xx}, J'_{2yy}, J'_{2zz}$	0.400, 0.400, 0.150[kg·m ²]
m_3	1.00[kg]
$J'_{3xx}, J'_{3yy}, J'_{3zz}$	0.100, 0.200, 0.200[kg·m ²]
Initial conditions	
$\mathbf{r}_1$	[0.000 0.200 0.000] ^T [m]
$\mathbf{e}_1$	[0.000 0.000 0.000 1.00] ^T
$\dot{\mathbf{r}}_1$	[-0.200 0.000 0.000] ^T [m/sec]
ω_1	[0.000 0.000 1.00] ^T [rad/sec]
$\mathbf{r}_2$	[0.000 0.400 -0.250] ^T [m]
$\mathbf{e}_2$	[0.000 0.000 0.000 1.00] ^T
$\dot{\mathbf{r}}_2$	[-0.400 -0.250 0.000] ^T [m/sec]
ω_2	[-1.00 0.000 1.00] ^T [rad/sec]
$\mathbf{r}_3$	[0.150 0.400 -0.500] ^T [m]
$\mathbf{e}_3$	[0.000 0.000 0.000 1.00] ^T
$\dot{\mathbf{r}}_3$	[-0.400 -0.350 -0.150] ^T [m/sec]
ω_3	[-1.00 1.00 1.00] ^T [rad/sec]

Fig. 8 Relative angular velocity of link 2 with respect to link 1

inertia properties and initial conditions of the three links. $\mathbf{e}_i$'s are the Euler parameters which represent the orientation of the links. Gravitational force is applied to the links in the opposite direction of Y . For actuating torques T_{A01} , T_{A12} , and T_{A23} shown in the Fig. 7, the values of $5 \sin(t)$, $5 \sin(2t)$, and $5 \sin(3t)$ [N·m] are used respectively for the forward dynamic analysis. The actuating torque $-T_{A(l+1)}\hat{\mathbf{h}}_{l(l+1)}$ ($l=0, 1, 2$) is applied to the link l and $T_{A(l+1)}\hat{\mathbf{h}}_{l(l+1)}$ is applied to the link $l+1$. Figure 8 shows the relative angular velocity of the link 2 with respect to the link 1 along the $\hat{\mathbf{h}}_{12}$ axis. It is shown that the stiction occurs between the time 0.521 [sec]

Fig. 9 Comparison of actuating torques obtained by the inverse dynamic analysis and the forward dynamic analysis

Fig. 10 Comparison of actuating torques obtained by the inverse dynamics analyses with friction force and without friction force

and 0.558 [sec]. Figure 9 shows the comparison of actuating torques obtained from the inverse dynamic analysis and those used in the forward dynamic analysis. From the analysis results shown in the figure, the actuating torques obtained from the inverse dynamic analysis are almost identical to those used for the forward dynamic analysis except the region of the stiction. In the region of the stiction, the actuating torque T_{A12} used for the forward dynamic analysis exists within the bounded values obtained from the inverse dynamic analysis. Figure 10 shows the comparison of the numerical results of the system with and without the friction force on the joint between the link 1 and link 2. From the analysis results shown in Fig. 10, it is clearly exhibited that the friction results in the significant difference.

5. Conclusions

A method for the inverse dynamic analysis considering friction forces on the kinematic joints is proposed in this work. The sliding and the stiction are considered in the method. Numerical examples are

given to demonstrate the accuracy of the proposed method. It is shown that while actuating forces are uniquely determined during the sliding, they are not uniquely determined during the stiction. To avoid numerical difficulty due to the variation of degrees of freedom, equations of motion for the sliding are used to obtain the actuating force not only during the sliding but also during the stiction. Numerical study also shows that actuating forces are heavily influenced by friction forces on the joints. Thus, in designing multibody systems such as robotic systems, the proposed method can be used effectively to calculate the actuating force which is significantly influenced by the friction force. The capacity of actuators and the feed forward reference input can be determined accurately by using the proposed method.

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