Adaptive Vibration Control Using Frequency Estimation for Multiple Periodic Disturbances with Noise*

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In this study, we focus on the error of the estimated frequency of disturbance and present a new adaptive multiple frequency tracking and modification law. The relationship between the estimated frequency error and the output level after using a low-pass filter is discussed in detail. Also an algorithm for multiple frequency estimation is proposed, which is insensitive to observation noise and proves the asymptotic stability of the proposed adaptive nonlinear algorithm and the adaptive multiple frequency tracking method theoretically. The results of simulation show that when the frequency estimated by the difference equation method approaches the true value, the output error converges to zero (or equilibrium point) asymptotically. This corresponds to the asymptotic stability condition. The effectiveness of this method and the theoretical proof are verified by simulation. The experimental results show that the proposed algorithm is effective for suppressing unbalanced vibration.

Key Words: Magnetic Bearing, Adaptive Unbalance Control, Frequency Estimation, Frequency Tracking, Asymptotic Stability

1. Introduction

An adaptive disturbance rejection with frequency estimation function approach has been reported for systems subjected to single-harmonic disturbance⁽¹⁾⁻⁽⁵⁾ or multiple periodic disturbances⁽⁶⁾. It is necessary to estimate the frequency of disturbance firstly when these frequencies of disturbance are unknown. There are two methods to estimate the frequencies of disturbance. One is the cost function method proposed by Tsao and Qian⁽⁵⁾, and the other is the difference equation method proposed by Fan and Nonami⁽⁶⁾. The accuracy of estimation by these methods is high if observation noise does not exist. However, the results of simulation do not always correspond to the results of experiments, because the accuracy of frequency estimation is deteriorated by noise, causing an error in

the estimation of the Fourier coefficient⁽¹⁾. Furthermore, it is apparent that large error occurs for the estimated value of frequency when noise exists. The two methods are very sensitive to observation noise. If the estimated frequency is not equivalent to the true value, pseudo-feedforward control input signal cannot suppress the effects of the harmonic disturbance, but even excites it.

In this study, we take into account the observation noise and formulate an adaptive algorithm. Then we focus on the error of the estimated frequency of disturbance and develop an approach to solve the problems previously described. We also propose a new adaptive multiple frequency tracking and modification law. The relationship between the estimated frequency error and the output level after using a low-pass filter is discussed in detail. We also propose a multiple frequency estimation algorithm which is insensitive to observation noise and prove the asymptotic stability of the adaptive nonlinear algorithm and the adaptive frequency tracking method theoretically. The results of simulation show that when the frequency estimated by the difference equation method approaches the true value, the output

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error converges to zero asymptotically. The effectiveness of this method and the theoretical proof are verified by simulation. The experimental results show that the proposed algorithm is very effective for suppressing unbalanced vibration.

2. Frequency Estimation Using Difference Equation for Multiple Periodic Disturbance

Usually, it is necessary to estimate the frequency of the disturbance when an active adaptive technique is used to control the periodic disturbance. The difference equation method is applied to a high-speed rotating machine with unbalance when sub-harmonic or super-harmonic vibrations exist, because it can estimate several frequencies of disturbance at the same time. We will discuss the method in this section. First, we consider a steady-state single-harmonic signal as follows:

$$x[n] = \sin(\omega n). \quad (1)$$

The shortest difference equation is obtained in the following form for Eq. (1).

$$x[n] = ax[n-1] - x[n-2]. \quad (2)$$

However, the initial value is given as follows:

$$x[n_0] = \sin(\omega n_0), x[n_0-1] = \sin(\omega[n_0-1]), \\ a = 2 \cos(\omega).$$

Now we consider the signal composed of several steady-state harmonic signals (M). It is generally shown as follows:

$$x_m = c_m x_m(n-1) - x_m(n-1). \quad (3)$$

$$x[n] = \sum_{m=1}^M x_m[n], x_m[n] = \sin(\omega_m n). \quad (4)$$

In this case, we will obtain the shortest difference equation (AR model) of $2M$ dimensions as follows:

$$x[n] = \sum_{k=1}^{2M} a_k x[n-k]. \quad (5)$$

However, a_k is a function of c_m in Eq. (3).

In this study, we assume that the signal is composed of a maximum of four fixed harmonic signals for simplicity. We estimate the coefficient c_m of Eq. (3) using sampled data $x[n], x[n-1], x[n-2], \dots$, which is dependent on the least-squares method. Then, using those coefficients, we compute c_m ($m=1 \dots M$) and obtain the estimated value of frequency ω_m ($m=1 \dots M$).

3. Multiple Adaptive Disturbance Rejection and Adaptive Multiple Frequency Tracking Method

3.1 Adaptive rejection approach of multiple periodic disturbances with observation noise

In this section, we develop an adaptive algorithm for the control of periodic disturbance. The major feature of this method is that it does not need a model for the control object. The block diagram of an adaptive rejection using this method is shown in Fig. 1. Furthermore, the details of the adaptive algorithm are shown in Fig. 2.

In Fig. 2, the left block shows the calculation of the harmonic components as sine and cosine functions, the middle block shows the modification of Fourier coefficients of the input signal to plant adaptability, and the right block shows the control input generated for the plant from the Fourier coefficient. The purpose of this algorithm is such that the generated pseudo-feedforward control input signal to the plant in Fig. 2 convolutes in the presence of a disturbance as shown in Fig. 1 and cancels out the error; thus, the error signal becomes zero.

Observation noise (with color and white noise) exists in practice and can be defined as follows:

$$N_d = \sum_{i=1}^n \{ \alpha_i \sin(\omega_i t) + \beta_i \cos(\omega_i t) \} + \zeta \times \text{rand}(t), \quad (6)$$

where ω_i is the frequency of color noise i and α_i, β_i are amplitudes of color noise i . The white noise is defined as $\text{rand}(t)$. In this case, the system

Fig. 1 Block diagram of the control system rejecting periodic disturbance

Fig. 2 Block diagram of adaptive algorithm for internally generating reference signal

Fig. 3 Details of the block SEC of Fig. 2

disturbance is expressed in Eq. (7) which has been mixing noise such as Eq. (6).

$$d = \sum_{j=1}^M \{ \alpha_{dj} \sin(\omega_{0j}t) + \beta_{dj} \cos(\omega_{0j}t) \} + N_d \quad (7)$$

where ω_{0i} is the frequency of disturbance and α_{dj}, β_{dj} are the amplitudes of disturbance. However, $\alpha_i, \beta_i \ll \alpha_{dj}, \beta_{dj}$ ($i=1 \dots M$). $\omega_{0j} \ll \omega_j$ (when $i=j$). We assume that the transfer function of the plant is $G(s) = Ae^{j\theta}$. The control input r for the plant is defined here as follows:

$$r = \sum_{j=1}^M \{ \alpha_j \sin(\omega_j t) + \beta_j \cos(\omega_j t) \} \quad (8)$$

where ω_j is the frequency estimated by the frequency estimation algorithm proposed in section 2. α_j, β_j are the modified Fourier coefficients determined by the adaptive control algorithm. In this case, the steady-state output y of the plant is shown as follows:

$$y = rG(j\omega) = \sum_{j=1}^M A_j [\alpha_j \sin(\omega_j t) + \beta_j \cos(\omega_j t)] \quad (9)$$

However, A_j and θ_j are the gain and the phase of the plant at frequency ω_j . Then, the steady-state error e of the system becomes

$$e = y + d = rG(j\omega) = \sum_{j=1}^M [A_j \{ \alpha_j \sin(\omega_j t) + \beta_j \cos(\omega_j t) \} + \alpha_{dj} \sin(\omega_{0j}t) + \beta_{dj} \cos(\omega_{0j}t)] + N_d \quad (10)$$

We consider the error signal e in the signal processing, as shown in Fig. 3. We define $n'_1(t)$ and $n'_2(t)$ before the two filters as follows:

$$\begin{cases} n'_1(t) = \sum_{j=1}^M n'_{1j}(t) = e(t) \sum_{j=1}^M \sin(\omega_j t) \\ n'_2(t) = \sum_{j=1}^M n'_{2j}(t) = e(t) \sum_{j=1}^M \cos(\omega_j t) \end{cases} \quad (11)$$

Firstly, let us consider $n_{1j}(t)$ as follows:

$$\begin{aligned} n'_1(t) &= \sum_{j=1}^M [A_j \{ \alpha_j \sin(\omega_j t) + \beta_j \cos(\omega_j t) \} \sin(\omega_j t) \\ &\quad + \{ \alpha_{dj} \sin(\omega_{0j}t) + \beta_{dj} \cos(\omega_{0j}t) \} \sin(\omega_j t) \\ &\quad + N_d \sin(\omega_j t)] \\ &= \sum_{j=1}^M [0.5A_j \{ \alpha_j(t) (\cos(\omega_j t + \theta_j) - \cos \theta_j) \\ &\quad + \beta_j (\sin(\omega_j t + \theta_j) - \sin \theta_j) \} \\ &\quad + 0.5 \{ \alpha_{dj}(t) (\cos(\omega_{0j} - \omega_j)t - \cos(\omega_{0j} + \omega_j)t) \\ &\quad + \beta_{dj} (\sin(\omega_{0j} - \omega_j)t - \sin(\omega_{0j} + \omega_j)t) \} \\ &\quad + N_d \sin \omega_j t] \end{aligned} \quad (12)$$

Here, we define the cutoff frequency ω_{Bj} of the low-pass filter as $\omega_{Bj} \ll 2\omega_{0j}$ and assume the color noise frequency of Eq. (6) as $\omega_{Bj} \ll \omega_j$. As a result, we can neglect the relatively small gain at high-frequency components to determine the true value of the disturbance. We define the error of the disturbance as follows:

$$\Delta\omega_j = \omega_{0j} - \omega_j. \quad (13)$$

We can obtain the output of the low-pass filter at the steady state as follows:

$$n_1(t) = \sum_{j=1}^M [0.5A_j \{ \alpha_j(t) \cos \theta_j - \beta_j(t) \sin \theta_j \} + 0.5A_{dj} \{ \sin(\Delta\omega_j t - \theta_{dj}) \}] \quad (14)$$

However, $A_{dj} = \sqrt{\alpha_{dj}^2 + \beta_{dj}^2}$; $\theta_{dj} = \tan^{-1} \left(\frac{\alpha_{dj}}{\beta_{dj}} \right)$. We can also obtain output $n_{2j}(t)$ as follows:

$$n_2(t) = \sum_{j=1}^M [0.5A_j \{ \beta_j(t) \cos \theta_j - \alpha_j(t) \sin \theta_j \} + 0.5A_{dj} \{ \cos(\Delta\omega_j t - \theta_{dj}) \}] \quad (15)$$

Firstly, let us consider the case in which $\Delta\omega_j = 0$. In this case, an adaptive law does not depend on phase θ_j and is defined as follows:

$$\begin{cases} \alpha_j(k+1) = \alpha_j(k) - \mu_{1j}(k+1)n_{1j}(k) \\ \beta_j(k+1) = \beta_j(k) - \mu_{2j}(k+1)n_{2j}(k) \end{cases} \quad (16)$$

Here, μ_{1j}, μ_{2j} are the step sizes. Furthermore,

$$n_{1j}(k+1) = n_{1j}(k) + 0.5A_j \{ \mu_{2j}(k+1)n_{2j}(k) \sin \theta_j - \mu_{1j}(k+1)n_{1j}(k) \cos \theta_j \} \quad (17)$$

$$n_{2j}(k+1) = n_{2j}(k) - 0.5A_j \{ \mu_{2j}(k+1)n_{2j}(k) \cos \theta_j - \mu_{1j}(k+1)n_{1j}(k) \sin \theta_j \} \quad (18)$$

where

$$\begin{cases} \mu_{1j}(k+1) = \mu_{1j}(k) \operatorname{sgn}(n_{1j}(k-1)^2 - n_{1j}(k)^2) \\ \mu_{2j}(k+1) = \mu_{2j}(k) \operatorname{sgn}(n_{2j}(k-1)^2 - n_{2j}(k)^2) \end{cases} \quad (19)$$

The above equation becomes a nonlinear adaptive law. The results of simulation show that the stability of a nonlinear system can be easily guaranteed if the first value of μ_j satisfies some conditions for different phases θ_j . Next, we consider the case in which $\Delta\omega_j \neq 0$. We can obtain the form of $n_{1j}(t)$ in the discrete system from Eq. (14) as follows:

$$\begin{aligned} n_1(k+1) &= \sum_{j=1}^M n_{1j}(k+1) \\ &= \sum_{j=1}^M [0.5A_j \{ \alpha_j(k+1) \cos \theta_j - \beta_j(k+1) \sin \theta_j \} \\ &\quad + 0.5A_{dj} \sin(\Delta\omega_j(k+1) + \theta_{dj})] \\ &= \sum_{j=1}^M [0.5A_j \{ (\alpha_j(k) - \mu_{1j}(k+1)n_{1j}(k)) \cos \theta_j \\ &\quad - (\beta_j(k) - \mu_{2j}(k+1)n_{2j}(k)) \sin \theta_j \} \\ &\quad + 0.5A_{dj} \sin(\Delta\omega_j k + \theta_{dj})] \end{aligned} \quad (20)$$

Generally, $\Delta\omega_j \ll \omega_j$ and we assume that $\cos \Delta\omega_j \approx 1$ and $\sin \Delta\omega_j \approx \Delta\omega_j$ here. Then Eq. (20) becomes:

$$\begin{aligned} n_1(k+1) &= n_1(k) + \sum_{j=1}^M [0.5A_j \{ \mu_{2j}(k+1)n_{2j}(k) \sin \theta_j \\ &\quad - \mu_{1j}(k+1)n_{1j}(k) \cos \theta_j \} + 0.5A_{dj} \\ &\quad \times \{ \sin(\Delta\omega_j k + \theta_{dj}) + \Delta\omega_j \cos(\Delta\omega_j k + \theta_{dj}) \}] \end{aligned} \quad (21)$$

$n_{2j}(t)$ can also be shown as follows:

$$\begin{aligned} n_{2j}(k+1) &= n_{2j}(k) \\ &- \sum_{j=1}^M [0.5A_j\{\mu_{2j}(k+1)n_{2j}(k)\cos\theta_j \\ &- \mu_{1j}(k+1)n_{1j}(k)\sin\theta_j\} + 0.5A_{dj} \\ &\times \{\cos(\Delta\omega_j k + \theta_{dj}) + \Delta\omega_j \sin(\Delta\omega_j k + \theta_{dj})\}]. \end{aligned} \quad (22)$$

It is known that $n_{1j}(k+1)$ and $n_{2j}(k+1)$ depend not only on $n_{1j}(k)$ and $n_{2j}(k)$ but also on the estimated error of frequency $\Delta\omega_j$. In this case, the results of simulation show that the stability of a nonlinear system cannot be guaranteed and unstable based on Eqs. (21) and (22). The control error always diverges even if the first values of μ_{1j}, μ_{2j} ($j=1\cdots M$) are chosen. The reason for the instability is because the error $\Delta\omega$ of the estimated frequency. It is necessary to modify estimated frequency and make it approach the true value. We propose a new algorithm to realize $\Delta\omega_j=0$ in the next section and develop an adaptive system for asymptotic stability.

3.2 Adaptive multiple frequency tracking method

In this study, the algorithm for reducing the frequency error $\Delta\omega$ of the disturbance to zero is called an adaptive frequency tracking method. We can obtain the frequency adaptive law from Eqs. (14) and (15).

$$\begin{aligned} N_j(t) &= n_{1j}(t) + n_{2j}(t) \\ &= 0.5A_j\{\alpha_j(t)(\cos\theta_j + \sin\theta_j) \\ &+ \beta_j(t)(\cos\theta_j - \sin\theta_j)\} + 0.5A_{dj} \\ &\times \{\sin(\Delta\omega_j + \theta_{dj}) + \sin(\Delta\omega_j + \theta_{dj})\}. \end{aligned} \quad (23)$$

The expression of $N_j(t)$ of a discrete system is shown as follows:

$$\begin{aligned} N_j(k+1) &= N_j(k) \\ &+ 0.5A_j\{\mu_{1j}(k+1)n_{1j}(k)(\cos\theta_j + \sin\theta_j) \\ &+ \mu_{2j}(k+1)n_{2j}(k)(\cos\theta_j - \sin\theta_j)\} \\ &+ 0.5A_{dj}\Delta\omega_j\{\cos(\Delta\omega_j(k+1) + \theta_{dj}) \\ &- \sin(\Delta\omega_j(k+1) + \theta_{dj})\} \\ &= 0.5N_j(k) + 0.5A_{dj}\Delta\omega_j\{\cos(\Delta\omega_j k + \theta_j) \\ &- \sin(\Delta\omega_j k + \theta_j)\}. \end{aligned} \quad (24)$$

It is necessary to let $\Delta\omega_j \rightarrow 0$ when $k \rightarrow \infty$ from Eq. (24). We define the modifications law as follows:

$$\omega_j(k+1) = \omega_j(k) - \mu_{3j}(k+1)N_j(k), \quad (25)$$

where

$$\begin{cases} \mu_{3j}(k+1) = \mu_{3j}(k) \operatorname{sgn}\{N_j(k-1)^2 - N_j(k)^2\} \\ \mu_{3j}(0) > 0. \end{cases} \quad (26)$$

Namely, we use output $N_j(t)$ to modify the estimated frequency. In this case, $\mu_{3j}(0)$ is the number of circles and the conditions are obtained in the following section. The estimated frequency gradually approaches the true value, and the output of the disturbance is controllable and its asymptotic stability is guaranteed.

3.3 Proof of asymptotic stability of adaptive rejection algorithm for disturbance

As mentioned above, Eq. (16) is an adaptive law

which does not depend on the phase θ_j about α_j and β_j . We assume that $N_j(k+1) = n_{1j}(k+1) + n_{2j}(k+1)$ and Eqs. (17) and (18) can be obtained as follows:

$$\begin{aligned} N_j(k+1) &= N_j(k) - 0.5A_j\{(\mu_{2j}(k+1)n_{2j}(k) \\ &+ \mu_{1j}(k+1)n_{1j}(k))(\cos\theta_j - \sin\theta_j)\}. \end{aligned} \quad (27)$$

Also $\mu_j(k+1)$ is shown as follows:

$$\begin{aligned} \mu_j(k+1) &= \{\mu_{2j}(k+1)n_{2j}(k) \\ &+ \mu_{1j}(k+1)n_{1j}(k)\}/N_j(k). \end{aligned} \quad (28)$$

Furthermore, we assume $|\mu_{2j}(k+1)| \leq |\mu_{1j}(k+1)| = |\mu_j(0)|$ and Eq. (28) can be written as follows:

$$\begin{aligned} |\mu_j(k+1)| &= \{|\mu_{2j}(k+1)n_{2j}(k) \\ &+ \mu_{1j}(k+1)n_{1j}(k)|\}/|N_j(k)| \\ &\leq \{|\mu_{2j}(k+1)n_{2j}(k)| + |\mu_{1j}(k+1)n_{1j}(k)|\} \\ &/|N_j(k)| \leq \{|\mu_j(0)|(|n_{2j}(k)| + |n_{1j}(k)|)\} \\ &/|N_j(k)| \leq |\mu_j(0)|. \end{aligned} \quad (29)$$

Thus, Eq. (27) can be rewritten as follows:

$$\begin{aligned} N_j(k+1) &= N_j(k) - 0.5A_j\{(\mu_j(k+1)N_j(k)) \\ &\times (\cos\theta_j - \sin\theta_j)\}. \end{aligned} \quad (30)$$

Here, we assume $V(k) = N(k)^2 = \left\{\sum_{j=1}^M N_j(k)\right\}^2$ is a candidacy of the Lyapunov function and define the form as $\Delta N_j(k+1) = N_j(k+1) - N_j(k)$.

The asymptotic condition of the system is $V_j(k+1) - V_j(k) < 0$ according to the Lyapunov stability theorem. Squaring the two sides of Eq. (30) gives

$$\begin{aligned} V(k+1) - V(k) &= N(k+1)^2 - N(k)^2 \\ &= \left\{\sum_{j=1}^M N_j(k+1)\right\}^2 - \left\{\sum_{j=1}^M N_j(k)\right\}^2 \\ &= \sum_{j=1}^M [-2 \times 0.5A_j\{\mu_j(k+1)N_j(k)^2 \\ &\times (\cos\theta_j - \sin\theta_j)\} + \{0.5A_j\mu_j(k+1) \\ &\times N_j(k)(\cos\theta_j - \sin\theta_j)\}^2] \\ &= \sum_{j=1}^M [(-\sqrt{2}A_j\mu_j(k+1) \\ &+ (0.5\sqrt{2}A_j\mu_j(k+1))^2)N_j(k)^2] \end{aligned} \quad (31)$$

Here, we assume $\eta_j = 0.5\sqrt{2}A_j\mu_j(k+1)$. The necessary and sufficient condition is $0 < \eta_j < 2$. Therefore, we obtain the condition $|\mu_j(k+1)| < 2\sqrt{2}/A_j$ from Eq. (31). The asymptotic stability of the adaptive algorithm can be guaranteed if the initial value is as follows:

$$|\mu_j(0)| < 2\sqrt{2}A_j. \quad (32)$$

The results of simulation show that the asymptotic stability of the adaptive rejection system can be guaranteed if the initial value is set such that $|\mu_j(0)| < 2.83/A_j$. The results correspond to Eq. (32).

3.4 Proof of asymptotic stability of the adaptive multiple frequency tracking method

Here, Eq. (14) is given by:

$$\begin{aligned} n_{1j}(k+1) &= 0.5A_j\{\alpha_j(k+1)\cos\theta_j - \beta_j(k+1)\sin\theta_j\} \\ &+ 0.5A_{dj}\{\sin(\Delta\omega_j(k+1) + \theta_{dj})\} \\ &= 0.5n_{1j}(k) + 0.5A_{dj}\Delta\omega_j\cos(\Delta\omega_j k + \theta_{dj}). \end{aligned} \quad (33)$$

We can also obtain the following equation from Eq.

Fig. 4 Results of disturbance attenuation

(15).

$$n_{2j}(k+1) = 0.5n_{2j}(k) - 0.5A_{dj}\Delta\omega_j \sin(\Delta\omega_j k + \theta_{dj}). \quad (34)$$

In this case, from Eqs. (33) and (34), $N_j(k+1) = n_{1j}(k+1) + n_{2j}(k+1)$ become:

$$N_j(k+1) = 0.5N_j(k) + 0.5A_{dj}\Delta\omega_j \times \{\cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj})\}. \quad (35)$$

If $N_j(k+1) \neq 0$, then Eq. (35) is a nonlinear system.

We also assume the candidacy of the Lyapunov function and define $\Delta N_j(k+1) = N_j(k+1) - N_j(k)$. Then,

$$V_j(k+1) - V_j(k) = N_j(k+1)^2 - N_j(k)^2 = 2\Delta N_j(k+1)N_j(k) + \Delta N_j(k+1)^2. \quad (36)$$

The control system becomes stable when the right-hand side is smaller than 0 and Eq. (36) is expanded as follows:

$$\begin{aligned} V_j(k+1) - V_j(k) &= 2[-0.5N_j(k) + 0.5A_{dj}\Delta\omega_j\{\cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj})\}]N_j(k) \\ &\quad + [-0.5N_j(k) + 0.5A_{dj}\Delta\omega_j\{\cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj})\}]^2 \\ &= -0.75N_j(k)^2 \\ &\quad + 0.5N_j(k)\Delta\omega_j A_{dj}\{\cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj})\} + 0.25A_{dj}^2\Delta\omega_j^2 \\ &\quad \times \{\cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj})\}^2. \end{aligned} \quad (37)$$

We can obtain $\Delta\omega_j = \omega_j(k+1) - \omega_j(k) = -\mu_{3j}(k+1)N_j(k)$ from Eq. (25) and substitute it into Eq. (37). Move over, we can obtain the following equation using $-\sqrt{2} \leq \cos(\Delta\omega_j k + \theta_{dj}) - \sin(\Delta\omega_j k + \theta_{dj}) \leq \sqrt{2}$:

$$\begin{aligned} V(k+1) - V(k) &= \sum_{j=1}^M \{V_j(k+1) - V_j(k)\} \\ &\leq \sum_{j=1}^M \{-N_j(k)^2 + 0.25N_j(k)^2(\xi+1)^2\} \end{aligned} \quad (38)$$

where $\xi = \sqrt{2} A_{dj} |\mu_{3j}(k+1)|$.

Therefore, $V(k+1) - V(k) < 0$ can be guaranteed if the first value of μ_{3j} satisfies the following condition:

$$0 < \mu_{3j}(0) < 1/(\sqrt{2} A_{dj}). \quad (39)$$

It is shown that the control system is asymptotically stable if an appropriate initial value of μ_{3j} is chosen and frequency approaches the true value.

Fig. 5 Results obtained using the adaptive multiple frequency tracking method

4. Simulations

In this section, we will verify the validity of the above method by simulation. Here, we assume two frequencies as $\omega_{d1} = 107$ Hz and $\omega_{d2} = 302$ Hz for periodic disturbances. At the same time, we let white noise and same color noise join it. Firstly, we estimate the frequencies of disturbances from a signal composed of white noise and same color noise using the multiple frequency estimation method based on the difference equation. The estimated frequencies are $\omega_{d1} = 106.6113$ Hz and $\omega_{d2} = 311.6651$ Hz. Figure 4 (b) shows the results of simulation using the adaptive algorithm described in section 3.1. The amplitude of the steady-state error e is reduced to 49% compared with that without control. The application of the adaptive multiple frequency tracking method, which was proposed in section 3.2, is shown in Fig. 5. It can be shown that the two frequencies approach $\omega_{d1} = 106.9981$ Hz, $\omega_{d2} = 312.0104$ Hz using the adaptive multiple frequency tracking method. The results of simulation show that the adaptive multiple frequency tracking method can modify frequencies exactly and promptly even for multiple periodic disturbances.

The rejection results of this case are shown in Fig. 4 (c). The steady-state error e is suppressed by 7.8% compared to that without control. In this study,

Fig. 6 Test apparatus

Fig. 7 Block diagram for control system

Fig. 8 Trajectories of geometric center positions when rotating at 6024 rpm

we also obtained good results of an adaptive rejection well for periodic disturbance with three and four frequencies (data not shown).

5. Experiments on Magnetic Bearing

In order to confirm the results of simulation, we carried out experiments with consideration of two harmonic disturbances. The magnetic bearing test apparatus is shown in Fig. 6.

We also used adaptive control against the vibration of a rotor with unbalance. The block diagram of the control system is shown in Fig. 7 and the results

Fig. 9 Stationary response of geometric center positions when rotating at 6024 rpm

(a) Without application of the proposed algorithm

(b) With application of proposed algorithm

Fig. 10 Spectra of stationary response of geometric center positions when rotating at 6024 rpm

are shown in Figs. 8 - 10.

6. Conclusions

In this study, we estimated the frequencies of multiple periodic disturbances using a difference equation and proposed the adaptive multiple frequency tracking method to modify the frequency of disturbance. We also proved the asymptotic stability of the

adaptive multiple frequency tracking method and the adaptive rejection algorithm for disturbance. The conclusions are summarized as follows:

(1) The asymptotic stability and the stability conditions of the proposed algorithms, namely, the coefficient modification laws of the adaptive rejection method and the adaptive multiple frequency tracking method, are derived using Lyapunov's stability theory.

(2) The results of experiments show that the periodic disturbance signal can be reduced by 20 db even when observation noise exists.

(3) The result of this study proves that the proposed method can be applied to MIMO systems such as a magnetic bearing with strong nonlinearity.

For future research, we plan to study the control of nonstationary periodic disturbance.

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