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Transient Analysis based on Non-Fourier Heat Conduction Law

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1. Introduction

In the classical Fourier heat conduction theory, the velocity of temperature propagation is assumed to be infinite, and so thermal disturbance is transmitted to every other point in the domain instantaneously. It has been pointed out that this is contradictory to the reality, although this classical model is compatible with the second law of thermodynamics. In order to overcome such unreality, investigations⁽¹⁻⁵⁾ have been carried out from the viewpoint that the actual heat conduction must depend on the propagation velocity of temperature wave, which should evidently be finite. Particularly in the field of nano-technology such as pulse laser heating, the non-Fourier effect is supposed to be dominant since both the time scale and space scale are so small that the coupled phenomenon between temperature wave and stress wave should not be negligible. This paper discusses the effect of non-Fourier heat conduction on the thermo-mechanical behavior of materials.

2. Governing Equations

Cattaneo and Vernotte^(3,4) proposed a non-Fourier law based on the idea of relaxing the heat flux q for heat conduction thereby leading to

$$q = -k\nabla T - \tau \frac{\partial q}{\partial t}, \quad (1)$$

where τ is the relaxation time and k is the thermal conductivity. Hence the heat conduction equation involving non-Fourier effect is shown as

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \alpha \nabla^2 T. \quad (2)$$

Since eq. (2) is a hyperbolic type differential equation, this modification may violate the second law of thermodynamics. It should also be noted the relation between the velocity of heat propagation c_h and the relaxation time τ is given by

$$c_h = \sqrt{\frac{k}{\tau \rho C_v}} = \sqrt{\frac{\alpha}{\tau}}, \quad (3)$$

where ρ and C_v represent the density and the specific heat, respectively.

The mechanical balance law reads

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \sigma + \rho b, \quad (4)$$

where b is the body force and σ is the stress. The stress is given by the constitutive law which relates the stress to strain ϵ as

$$\sigma = E\epsilon - \beta T I. \quad (5)$$

Here we employ the conventional thermo-elasticity model. E stands for the elasticity tensor and β for the thermo-elasticity coefficient. We need to refine the constitutive law, say a thermo-elastic-plastic model, for further development in the

practical application. Since the temperature is involved in eq. (5), the coupled equations (2) and (4) have to be solved. A finite element code is developed, based on the Newmark- β method.

3. Problem Statement

Let us calculate the temperature distribution field in a two-dimensional square block whose configuration is schematically shown in Fig. 1. A pulse heat flux is imposed at the left-top of the objective domain, while the temperature along the bottom line of the domain is kept constant. Along the other boundaries, the adiabatic boundary condition is imposed. The shape of the heat flux is sinusoidal so that the initial velocity and the acceleration are to be given. The material parameters are assumed to be constant. Nondimensionalized equations are used for convenience, introducing the following dimensionless variables, as

$$\begin{aligned} \theta &= \frac{T(x, y, t) - T_0}{T_w - T_0}, & \xi &= \frac{c_h^2 t}{2\alpha}, \\ \eta &= \frac{c_h x}{2\alpha}, & \delta &= \frac{c_h y}{2\alpha}. \end{aligned} \quad (6)$$

The reference temperatures T_0 and T_w are set zero and unity, respectively. The nondimensionalized representation of the governing equations for the hyperbolic heat conduction model is expressible as

$$\frac{\partial^2 \theta}{\partial \xi^2} + 2 \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\partial^2 \theta}{\partial \delta^2}. \quad (7)$$

The domain is subdivided into a finite element mesh by 1600 triangle elements with 841 nodes.

4. Results and Discussion

Figure 2 shows the comparison of dimensionless temperature curves of the nodes located at the left side of the objective domain $\eta=0$ with respect to various time step. Figure 2(a) corresponds to the result by the non-Fourier analysis and Fig. 2(b) to variation of that by the classical Fourier heat conduction model. It can be found from this figure that the temperature history are similar to each other during the heating process until the 30 step. The temperature rises gradually and reaches the maximal value at the 30 step. After that step, when the heat flux begins to reduce, the temperature also begins to decrease. As for the non-Fourier heat conduction, a clear temperature wave appears and travels forward at the finite speed in the domain. In contrast, such a wave is not observed in the classical Fourier heat conduction model.

Figure 3 demonstrates the temperature distribution at the 120 step, in which the results by the non-Fourier and the classical Fourier analyses are shown in Figs. (a) and (b), respectively. It can be found that the maximal temperature level travels through the domain at the finite velocity as shown in Fig. 3 (a), and the

temperature propagation is vividly illustrated. In contrast a gradual increase in temperature is observed in Fig. 3(b). This implies that the heat travels in the infinite speed and that the temperature distribution is just subject to the conventional law. Further, we point out that the maximal temperature by the non-Fourier analysis is higher than that by the classical Fourier analysis.

5. Conclusion

In order to clarify the effect of non-Fourier heat conduction effect, finite element analysis is carried out. As a result a large difference between the non-Fourier and Fourier heat conduction is found in the temperature distribution. In the non-Fourier heat conduction, temperature wave travels and the level is sharper and higher than results by the classical Fourier heat conduction model. Even the reflection of temperature wave occurs, and such particular temperature distribution may affect the thermal stress. For the further development on the pico-second or femto-second pulsed laser heating, we expect that the non-Fourier heat conduction has a significant influence on the temperature distribution and also to the stresses and material structures.

References

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Fig.1 Schematic view of the objective domain.

Fig.2 Dimensionless temperature curves for the non-Fourier and Fourier models.

Fig.3 Dimensionless temperature distribution for the non-Fourier and Fourier models at the 120 step.