Two-dimensional joints analysis under shear load using an enriched finite element

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1. Introduction

Conventional finite element method has difficulty in predicting accurate stresses around dissimilar-material junctions where has singularities created by discontinuities in material properties across an interface. For this reason, there are some special elements or methods developed to obtain accurate stress field around a singular point. An enriched element method developed by Benzley(1) is one method that can be used to obtain stress intensity factor around singular point.

In this study, we tried to apply the enriched finite element method on two-dimensional joints under shear load. In additional, the changing element size results have been investigated for finding suitable element size.

2. Analytical formula

2-D enriched elements equations were developed by Benzley(1) to solve for stress and displacement for two-dimensional singular point. The 4-node quadrilateral element with a singular point at node ‘o’ is shown in Fig.1(b)(type A element). The displacement field is of form

\[ u_i = \alpha_{ia} + \alpha_{ib}a + \alpha_{ic}b + \alpha_{id}ab + KQ_i(r, \theta) \]  (1)

In eq.(1), \( u_i \) and \( u_j \) represent the displacements of a point within the element in the x and y direction, respectively, a and b are natural co-ordinates of the element, \( \alpha_{ij} \) are the unknown coefficients, \( Q_i(r, \theta) \) are the intensities (unknown coefficients) of singular terms, and \( K \) represent the stress intensity factors.

Solving eq.(1) for the unknown coefficients \( \alpha_{ij} \)’s in terms of the nodal displacements, \( u_{k} \) where \( k = 1,2,3,4 \) the displacement assumption may be written as

\[ u_i = \sum_{k=1}^{4} f_i u_k + KQ_i(r, \theta) \]  (2)

In equation (2), \( Q_i \) is the values of \( Q_i(r, \theta) \) evaluated at node \( k \) and \( f_i = 0.25(1-a_k)(1-b_k) \).

For transition elements (type B element), they are needed to join enriched elements to standard elements in a finite element model. The displacement field of a transition element is given by the relation

\[ u_i = \sum_{k=1}^{4} f_i u_k + R(a,b)(KQ_i(r, \theta) \sum_{k=1}^{4} f_i \bar{Q}_k) \]  (3)

where \( R(a,b) \) is set a ‘zeroing’ function, equals 1 along ‘enrich’ boundaries and equals 0 along ‘standard’ boundaries.

For bi-material plate with plain strain condition, the stress becomes singular at ‘o’ point in Fig.1 due to mismatch in young’s modulus of the two materials. By using eigen analysis method, the order of the stress singularity \( \lambda \) and the angular variation of the displacement fields can be determined. Then, the function \( Q_i \) may be written as

\[ Q_1 = r^{-\lambda} (u_1(\theta) \cos(\theta) - u_0(\theta) \sin(\theta))/E \]
\[ Q_2 = r^{-\lambda} (u_0(\theta) \sin(\theta) + u_1(\theta) \cos(\theta))/E \]  (4)

In case of the bi-material plate applied a uniform shear stress, \( \tau \), \( u_1 \) and \( u_0 \) can be normalized followed the equation (5) by assume \( \tau_{1\theta}(\theta = 0) = K, r^{-1}\)

\[ u_1(\theta) = S_{1,0} + S_{1,1} \sin((P+1)\theta) + S_{1,2} \cos((P+1)\theta) \]
\[ + S_{1,3} \sin((P-1)\theta) + S_{1,4} \cos((P-1)\theta) \]
\[ u_0(\theta) = S_{0,0} + S_{1,0} \sin((P+1)\theta) + S_{2,0} \cos((P+1)\theta) \]
\[ + S_{3,0} \sin((P-1)\theta) + S_{4,0} \cos((P-1)\theta) \]  (5)

while \( P = \lambda + 1 \)

3. Numerical process

Model for analysis is shown in Fig.1. It is the bi-material plate fixed on the bottom side and applied shear stress on the top. Material properties are shown in Table 1.

![Table 1 Material Properties](image)

<table>
<thead>
<tr>
<th></th>
<th>Young’s modulus(GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material 1 (M1)</td>
<td>160.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Material 2 (M2)</td>
<td>4.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

![Figure 1](image)
The order and angular variation of the displacements associated with the singular point are determined by eigen analysis. The finite element model for eigen analysis is shown in Fig. 2(a).

Figure 2(b) shows the order of stress singularity (λ) and the angular variation of the displacement fields that lead to a singular stress state on this case.

The previous study’s results (Pageau(2)) showed that the size of the enriched region is one importance factor for enriched analysis. On this time, the model is divided element size into 4 cases; enriched element size 50x50, 40x40, 30x30 and 25x25 nm² and every model consists of 2 enriched elements for 1 singular point.

In addition, calculated stress intensity factor by using FEM is also done for a comparison with the enriched element method. The results are shown in Fig. 4.

4. Results

The model in Fig. 1 is shown that there are 2 singular points in this case. Figure 3 is shown the results of stress intensity factor from both singular points. K1 and K2 represent the stress intensity factor of left and right side, respectively. Naturally, K1 equals −K2 on a symmetric model but the results from numerical process have some numerical errors. Differences of K1 and K2 decrease when the size of element decreases.

Figure 4 Stress intensity factor result from FEM

Figure 5 Stress distributions near singular point by FEM

Stress intensity factor from using FEM is determined by the equation

\[
\frac{\tau}{\sigma} = k \left(\frac{r}{L}\right)^{-\lambda} + c
\]

while \( k = K/\epsilon L^3 \)

The results from dividing elements near singular points to be very small element (\( r/L = 10^{-3} \)) is shown in Fig. 4. We use relation between shear stress and distance from singular point to study its singularity. And then, changing near the interface has been considered for finding the stress intensity factor. The stress intensity factor from this method is −5.2141.

From Fig. 5, we found that even if the model is applied with shear load, singularity in normal stress still must be considered.

5. Summary

The result from applied the enriched finite element on shear load model is agreed with using FEM. However, the results from changing the size of enriched element effects the stress intensity factors varied from FEM results. This is an important point for continue study. In additional, on this time we have considered only shear stress loading. The next step is to combine both shear and normal stress singularities on the model under shear load.

6. References
