Analysis for singular stress field in 2D-dissimilar material joints using an enriched finite element method (Discussion on enriched area and mesh types)

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1. Introduction

Dissimilar material joints have singularities created by discontinuities in material properties across an interface. For this reason, it is difficult to predict accurate stress by using conventional FEM. There are some special elements or methods developed for solving this problem. An enriched finite element method developed by Benzley\(^\text{(1)}\) is one method that can directly compute the intensity of singularity and do not need to divide meshes around the singular point to be very small meshes. However, this method is not applied widely because the accuracy of the results depends on the enriched element’s size and shape.

In the present study, the singular stress field at bi-material joints is analyzed by an enriched finite element method. An eigenvalue and eigenvector analysis is applied to calculate the order of stress singularities and the asymptotic displacement fields on the enriched elements. This study is focused on the effect of changing enriched area and mesh types on the accuracy of the results. Both 4-node and 3-node elements are used to study its convergence and fine suitable element for each case.

2. Analytical formula

2-D enriched element equations were developed by Benzley\(^\text{(1)}\) to solve an intensity in stress and displacement for two-dimensional singular point. In this method, three different elements; enriched, transition and standard elements, are used. (See Fig.1)

![Element Models for Enriched Analysis](image)

<table>
<thead>
<tr>
<th>C C C</th>
<th>B B C</th>
<th>A B C</th>
<th>B B C</th>
<th>C C C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>A = ‘Enriched Element’</td>
<td>B = ‘Transition Element’</td>
<td>C = ‘Standard Element’</td>
<td>O = ‘Singular Point’</td>
</tr>
</tbody>
</table>

Stress distribution around the singular point with one real singularity can be described as

\[
\sigma_i(r,\theta) = K_i r^{-\lambda} f_i(\theta)
\]

where \(r\) is the radial distance from the singular point, \(\lambda\), \(K_i\) and \(f_i(\theta)\) are the orders of the stress singularity, the intensity of singularity and angular functions, respectively.

The displacement field is of form

\[
u_i = \alpha_i + \alpha_i a + \alpha_i b + \alpha_i a b + K_{m0} Q_i(r,\theta)
\]

In eq.(2), \(u_i\) and \(u_j\) represent the displacements of a point within the element in the x and y direction, respectively, \(a\) and \(b\) are natural co-ordinates of the element, \(\alpha_i\) are the unknown coefficients, and \(Q_i(r,\theta)\) is the intensity of singularities coefficients of singular terms.

Solving eq.(2) for the unknown coefficients \(\alpha_i\) in terms of the nodal displacements, \(u_i\), the displacement assumption may be written as

\[
u_i = \sum_{m=1}^{\infty} g_m u_m + K_{m0}(Q_i - \sum_{m=1}^{n} g_m Q_m)
\]

where \(R(a,b)\) is set a ‘zeroing’ function, equals 1 along ‘enrich’ boundaries and equals 0 along ‘standard’ boundaries.

For finding the asymptotic displacement fields on enriched element, eigen value analysis by FEM is separately conducted. By using eigen analysis method, the order of the stress singularity \(\lambda\) and the angular variation of the displacement fields can be determined in polar coordinates \((u_x, u_y)\). Then, the function \(Q_i\) \((i=1,2)\) converted from polar coordinates to Cartesian coordinates may be written as

\[
Q_i = r^{n-\lambda}(u_x(\theta) \cos(\theta) - u_y(\theta) \sin(\theta)) / E
\]

where \(E\) is Young’s modulus of material 1.

3. Numerical analysis

3-1 The model for analysis. The model for analysis is shown in Fig.2(a). The bottom side in a bi-material model is fixed and a shear load is applied on the top. A plane strain condition is employed in this analysis. Material properties are shown in Table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material 1</td>
<td>160.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Material 2</td>
<td>4.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>
3-2 Eigenvalue analysis
Eigen analysis is used to find displacement fields in the enriched element. The finite element model for eigen analysis is shown in Fig. 2(b). The eigen equation can be expressed as:

$$ (p^2[A] + p[B] + [C])\{u\} = 0 $$

where \([A], [B]\) and \([C]\) are matrices composing of Young’s modulus and Poisson’s ratio, \(p = 1 - \lambda\) and \(\{u\}\) is the eigenvector of displacement.

The order of stress singularity, \(\lambda\), from eigenvalue analysis is 0.2626915. The angular variation of displacement is analyzed by eigenvector analysis. Figure 3 shows the displacement profile and angular function that is used in additional terms on the enriched elements.

3-3 Enriched FEM
As shown in Fig. 4, 3 types of element model are investigated to study the effect of element shape on the accuracy of results. Model 1 uses 4-node rectangular elements, while models 2 and 3 use 3-node triangular element with different number of element. In addition, the enriched element size, \(x\), is varied from 0.1 to 0.8 mm to study the influence of enriched area on the result convergence.

The results of the intensity of singularity, \(K_{\text{int}}\), with various enriched area sizes are shown in Fig. 5. As can be seen on this figure, the results of all models agree with the results from the conventional FEM. The intensity of singularity is calculated by fitting a curve of stress distribution at the interface, \(K_{\text{int}} = 0.5275\).

Comparing within these 3 models, model 3 has the best result at \(x = 0.2-0.4\) mm and the rate of divergence is lower than other 2 models. The advantage of 3-node element type is able to adjust the area to match easily with the singular region.

Figure 6 shows the distribution of stress of all models by fixed \(x = 0.3\) mm. It shows that the stress distributions from enriched FEM are in good agreement with conventional FEM result.

![Fig. 2](image)

(a) Analytical model and boundary condition (b) FEM model for eigen analysis

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![Fig. 5](image)

The intensity of singularity against the enriched area of 3 models compared with conventional FEM result.

![Fig. 6](image)

Distribution of stress at the interface by fixed the enriched area (x) at 0.3 mm. compared with conventional FEM results

4. Summary
The results obtained applying the enriched FEM are agreed with those using conventional FEM. The enriched area and mesh types are important factors for modeling selection. A relationship between enriched mesh model and singular stress field is an influence on the accuracy of the result.

5. References