MODELLING OF A ROD BUNDLE UNDER VISCOUS AND UNCOMPRESSIBLE FLOW BY POROUS MEDIA. APPLIED TO NUCLEAR REACTOR CORE.

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Keywords : Fluid/Structure Interaction, Non Linear,

1. INTRODUCTION  
This study is about the safety of nuclear reactor core submitted to seismic loading. The purpose is to reduce the incertitude margin of the present day codes which. To reduce the incertitude margin we propose to develop a numerical model including the non linear behavior of the fluid/structure coupling. A numerical calculus with a whole core meshing would need a too large number of degree of freedom, so we establish global governing equations of the fluid and the structure by assimilating fuel assemblies to porous media.

2. METHOD  
The procedure to build equations is based on a porous media approach including an equivalent fluid model and an equivalent structure model. First, motion equation of equivalent fluid and structure are established separately. For the fluid part : fluid flow global equations through the rod bundle are obtained by spatial averaging the Navier Stokes equation with an Arbitrary Lagrangian Eulerian formulation. Equivalent fluid, of which variables are space averaged, is defined in the whole domain space. Structure effects are taken into account by a voluminal force also defined in the whole domain space. For the structure part: each fuel assembly is modeled as a porous media submitted to a voluminal force which is the opposite of the force acting on the fluid. Finally the structure and fluid motion equations are both solved.

8. CONCLUSION  
A 2D numerical model has been established with a Finites Elements Method, numerical simulations give coherent results in agreement with experimental observations.

REFERENCES  


Collard, B., Pisapia, S., Bellizzi, S., Witters, F., 2003, “PWR fuel assembly modal testing and analysis”, Symposium of Flow-Induced Vibration ASME PVP Conference, Cleveland, Ohio, USA.

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ABSTRACT

This study is about the safety of nuclear reactor core submitted to seismic loading. In order to reduce the incertitude margin of the present day codes we propose to develop a numerical code including the non linear behavior of the fluid/structure coupling.

The challenge of this work is to find out a tractable model taking the structure complexity into account. In this paper we model the nuclear reactor core mechanical behavior including the dynamics of both fuel assemblies and fluid. Each rod bundle is considered as a deformable porous media, so the velocity field of the fluid and the displacement field of the structure are defined in the whole domain space. Fluid part and structure part are in a first time considered separately, and in second time, the two parts are coupled.

The motion equations of the structure are obtained by a Lagrangian formulation, and to allow the fluid structure coupling, the motion equations of the fluid are obtained by an Arbitrary Lagrangian Eulerian formulation. The finite elements method is applied to spatially discretize the equations.

Simulations have been performed to analyze the influence of the fluid and structure characteristics, phenomena observed by the experience have been reproduced qualitatively.

1. INTRODUCTION

This study is about the safety of nuclear reactor core submitted to seismic loading. The purpose is to reduce the incertitude margin of the present day codes which. To reduce the incertitude margin we propose to develop a numerical model including the non linear behavior of the fluid/structure coupling. A numerical calculus with a whole core meshing would need a too large number of degree of freedom, so we establish global governing equations of the fluid and the structure by assimilating fuel assemblies to porous media.

2. METHOD

The procedure to build the equations of motion governing the fluid/structure medium is illustrated in Fig. 1. This procedure is based on a porous media approach including an equivalent fluid model and an equivalent structure model. First, motion equation of equivalent fluid and structure are established separately. For the fluid part : fluid flow global equations through the rod bundle are obtained by spatial averaging the Navier Stokes equation. Equivalent fluid, of which variables are space averaged, is defined in the whole domain space. Structure effects \( \vec{F}_{\text{structure/ fluid}} \) are taken into account by a
voluminal force also defined in the whole domain space. For the structure part: each fuel assembly is modeled as a porous media submitted to the force $\vec{F}_{\text{structure}}/\text{fluid}$ which is the opposite of the force $\vec{F}_{\text{fluid}}/\text{structure}$. Finally the structure and fluid motion equations are both solved.

3. PRELIMINARY: FORCES ACTING ON A ROD

In order to determine $\vec{F}_{\text{fluid}}/\text{structure}$, a model of forces acting on a rod submitted to a meanly axial flow is proposed.

As established by Païdoussis (1972) for a cylinder submitted to an axial flow, the fluid forces acting on a cylinder submit to axial and radial fluid velocity (Fig. 2) can be divided into pressure forces $\vec{F}_p$, and viscous forces (the forces are expressed by unit length).

Viscous forces have an axial component $\vec{F}_V$, and a radial one $\vec{F}_N$:

$$\vec{F}_V = -\frac{1}{2} \rho D V C_T \left( \frac{\partial U_y}{\partial t} - V_y + V_z \frac{\partial U_z}{\partial x} \right) \tilde{e}_y \quad \text{Eq. 2}$$

$$\vec{F}_N = -\frac{1}{2} \rho D V C_N \left( \frac{\partial U_y}{\partial t} - V_y + V_z \frac{\partial U_z}{\partial x} \right) \tilde{e}_z \quad \text{Eq. 3}$$

where $\rho$ is the fluid density, $D$ is the rod diameter, $U_y$ is the rod displacement in the $\tilde{e}_y$ direction, $U_z$ is the rod displacement in the $\tilde{e}_z$ direction, $V_x$ is the fluid velocity in the $\tilde{e}_x$ direction, $V_y$ is the fluid velocity in the $\tilde{e}_y$ direction, $V_z$ is the fluid velocity in the $\tilde{e}_z$ direction, and $m_f$ is a virtual mass by unit length. The constant variables $C_T$ and $C_N$ have to be determined, these variables depend on the fluid viscosity, structure geometry, and confinement.

Note that if $V_y = 0$ and $V_z = 0$ then Eq. 1, 2 and 3 coincide with that proposed in Paidoussis (1972).

4. POROUS MODEL EQUATIONS

In this section we establish the motion equations of equivalent fluid and structure with the method proposed in section 2.

4.1 Hypothesis

The following theory is based on these hypothesis:

H1 The fluid is viscous incompressible and Newtonian.
H2 Gravity effects are neglected.
H3 The rod section does not deform.
H4 Distance between two rod of a Fuel Assembly stay constant.
H5 The dynamic viscosity is negligible, compared with the turbulent viscosity.
H6 Turbulent kinetic energy is negligible, compared with the gradient of pressure.
H7 A rod bundle submitted to a meanly axial flow has the same physical behavior than one rod submitted to a meanly axial flow with different physical constants.

4.2 Equivalent Fluid model

In this section we establish the equation of an equivalent fluid. First the fluid motion equations are space averaged, then turbulence of smaller scale are modeled with a turbulent viscosity, and finally the structure/fluid efforts are assimilated to forces acting on a rod as established in preliminary (section 3).

4.2.1 Arbitrary Lagrangian Eulerian

Usually fluid equations are written with an Eulerian formulation, the fluid is observed trough a fixed window, and structure equations with Lagrangian formulation, structure is followed in its motion. To allow the coupling of fluid and structure equations we have to describe both fluid and structure from the same point of view, so we use an Arbitrary Lagrangian Eulerian formulation (ALE) to write the fluid equations. The fluid is observed trough a moving window which follow the structure motion. The Navier Stokes equations written with ALE formulation for an incompressible fluid gives:

\[ \rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \left( \frac{\partial \vec{V}}{\partial t} + \left( \vec{V} - \frac{\partial \vec{U}}{\partial t} \right) \nabla \vec{V} \right) d\Omega = \text{div} \sigma \quad \text{Eq. 4} \]

\[ \text{div}\vec{V} = 0 \quad \text{Eq. 5} \]

where \( \vec{U} \) is the structure displacement, and \( \vec{V} \) the fluid velocity.

4.2.2 Space averaging

Let’s consider a control volume \( \Omega_j(x,y,z) \) (Fig. 3). \( A(x,y,z) \) is the center of the control volume, \( \Omega_j(x,y,z) \) is the fluid domain and \( V_\Omega_j(x,y,z) \) is its volume, \( \Omega_j(x,y,z) \) is the structure domain and \( V_\Omega_j(x,y,z) \) is its volume, and \( \Omega_j(x,y,z) \) is the whole domain of the control volume and \( V_\Omega_j(x,y,z) \) is its volume.

\( \Omega_j(x,y,z) = \Omega_j(x,y,z) \cup \Omega_j(x,y,z) \)

\( V_\Omega_j(x,y,z) = V_\Omega_j(x,y,z) + V_\Omega_j(x,y,z) \)

Dimension of the control volume is \( a.a.d \), \( a \) is the distance between two rod center (and is supposed to be constant), so the control volume contains exactly the section of one rod, and the fluid presence fraction does not depend on the control volume position. We can write:

\( V_j(x,y,z) = V_j \quad \text{Eq. 6} \)

\( V_j(x,y,z) = V_j \quad \text{Eq. 7} \)

\( V_j(x,y,z) = V_j \quad \text{Eq. 8} \)

Integration of the Navier Stokes equations written with ALE formulation is over control volume for an incompressible Newtonian fluid gives:

\[ \rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \left( \frac{\partial \vec{V}}{\partial t} + \left( \vec{V} - \frac{\partial \vec{U}}{\partial t} \right) \nabla \vec{V} \right) d\Omega = \iint_{\Omega_j(x,y,z)} \text{div} \sigma d\Omega \quad \text{Eq. 9} \]

\[ \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \text{div} \vec{V} d\Omega = 0 \quad \text{Eq. 10} \]

\( \sigma = -P I + \mu \left( \nabla \vec{V} + \nabla \vec{V}^T \right) \quad \text{Eq. 11} \)

where \( P \) is the pressure, \( \mu \) is the viscosity and \( \sigma \) is the constraint tensor.

According to the Leibniz and Gauss theorem, integrals over control volume can be transformed:

\[ \rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \left( \frac{\partial \vec{V}}{\partial t} + \left( \vec{V} - \frac{\partial \vec{U}}{\partial t} \right) \nabla \vec{V} \right) d\Omega = \rho \frac{1}{V_\Omega} \frac{\partial}{\partial t} \iint_{\Omega_j(x,y,z)} \vec{V} d\Omega \]

\[ -\rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \vec{V} \nabla \vec{V} d\Omega \]

\[ -\rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \vec{V} \nabla \vec{V} d\Omega \]

\[ -\rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \frac{\partial \vec{U}}{\partial t} \nabla \vec{V} d\Omega \]

\[ + \rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \text{div} \left( \vec{V} \nabla \vec{V} \right) d\Omega \]

\[ -\rho \frac{1}{V_\Omega} \iint_{\Omega_j(x,y,z)} \vec{V} \nabla d\Omega \quad \text{Eq. 12} \]
\[
\frac{1}{V_{\Omega}} \iiint \nabla \cdot \sigma d\Omega = \frac{1}{V_{\Omega}} \nabla \cdot \iiint \sigma \partial \nabla \cdot d\Omega + \frac{1}{V_{\Omega}} \nabla \nabla \cdot \sigma d\Omega \quad \text{Eq. 13}
\]

where \( A_t(x,y,z) \) is the control volume surface, \( V_t \) is the velocity of this surface, \( A_{f}(x,y,z) \) is the fluid structure frontier surface, and \( V_A \) is the velocity of this surface (Fig. 3).

Exploiting the H1, H3 and H4 hypothesis, and substituting Eq. 12 and Eq. 13 in Eq. 9 and Eq. 11 gives:

\[
\frac{1}{V_{\Omega}} \frac{\partial}{\partial t} \left( \iiint \sigma d\Omega \right) + \frac{1}{V_{\Omega}} \nabla \cdot \iiint \sigma \partial \nabla \cdot d\Omega = 2\rho \frac{1}{V_{\Omega}} \frac{\partial}{\partial t} V \iiint \sigma d\Omega \quad \text{Eq. 14}
\]

\[
- \rho \frac{1}{V_{\Omega}} \nabla \cdot \iiint \sigma d\Omega + \frac{1}{V_{\Omega}} \nabla \cdot \iiint \sigma dS \quad \text{Eq. 15}
\]

\[
\frac{1}{V_{\Omega}} \nabla \cdot \iiint \bar{V} d\Omega = 0 \quad \text{Eq. 15}
\]

By noticing that, the average of the fluctuating part is equal to zero, we can substitute the velocity in the convective term:

\[
\frac{1}{V_{\Omega}} \rho \nabla \cdot \iiint \bar{V} \bar{V} d\Omega = 0
\]

\[
\frac{1}{V_{\Omega}} \rho \nabla \cdot \iiint \bar{V} \bar{V} d\Omega = \frac{1}{V_{\Omega}} \rho \nabla \cdot \iiint \bar{V} \bar{V} d\Omega \quad \text{Eq. 17}
\]

\[
+ \frac{1}{V_{\Omega}} \rho \nabla \cdot \iiint \bar{V} \bar{V} d\Omega \quad \text{Eq. 17}
\]

We recognize the Reynolds tensor:

\[
\sigma_{re} = \rho V_{\Omega} \iiint \bar{V} \bar{V} d\Omega \quad \text{Eq. 18}
\]

which according to Smagorinsky can be expressed by:

\[
\sigma_{re} = - \frac{2}{3} \rho k T \quad \text{Eq. 19}
\]

where \( \mu_f \) is a turbulent viscosity, and \( k \) is the turbulent kinetic energy (equal to the kinetic energy of the fluctuating velocity). There are several models to express \( \mu_f \), for simplicity we choose a \( \mu_f \) which does not depend on time and space.

As we suppose in H5 and H6, we can write:

\[
\sigma_{re} + \frac{1}{V_{\Omega}} \iiint \sigma d\Omega = - \frac{1}{V_{\Omega}} \iiint P d\Omega + \mu_f \left[ \nabla \cdot \iiint \bar{V} \bar{V} d\Omega + \left( \nabla \cdot \iiint \bar{V} \bar{V} d\Omega \right)^T \right] \quad \text{Eq. 20}
\]

\[
4.2.3 \text{ Turbulence modelling}
\]

\[\text{The fluid velocity is decomposed in an averaged part and a fluctuating part } \bar{V}' : \]

\[\bar{V} = \frac{1}{V_{\Omega}} \iiint \bar{V} d\Omega + \bar{V}' \quad \text{Eq. 16}\]

Fig. 3 Control volume for space averaging

4.2.4 Structure efforts acting on the fluid

As the term \( \frac{1}{V_{\Omega}} \iiint \sigma dS \) in Eq. 14 is the structure efforts on the fluid, we can write:

\[\frac{1}{V_{\Omega}} \iiint \sigma dS = F_{\text{structure} \rightarrow \text{fluid}} \quad \text{Eq. 21}\]

The control volume contain one rod (Fig. 4), as we supposed in H7 we use the modified Païdoussis expression (Eq. 1, 2 and 3) to write the structure/fluid efforts:
\[ \tilde{F}_{\text{structure} \rightarrow \text{fluid}} = \frac{1}{S} (\tilde{F}_I + \tilde{F}_N + \tilde{F}_L) \]  
Eq.22

where \( S = a^2 \).

The constant variables \( C_T \) and \( C_N \) will be determined with tests on a rod bundle.

\[ \rho_{eq} \frac{\partial \tilde{V}_{eq}}{\partial t} + \rho_{eq} \text{div} \tilde{V}_{eq} \otimes \tilde{V}_{eq} = -\nabla P_{eq} + \mu_{eq} \Delta \tilde{V}_{eq} \]

\[ + 2\rho_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \nabla \tilde{V}_{eq} \]  
Eq.28

\[ - \rho_{eq} \nabla \left( \frac{\partial \tilde{U}_{eq}}{\partial t} \tilde{V}_{eq} \right) \]

\[ + \tilde{F}_{\text{structure} \rightarrow \text{fluid}} \]

\( \text{div} \tilde{V}_{eq} = 0 \)  
Eq.29

with:

\[ \tilde{F}_{\text{structure} \rightarrow \text{fluid}} = \frac{1}{S} \left[ \frac{m_j}{V_{\Omega_j}} \left( \frac{\partial}{\partial t} \left( \tilde{V}_{eq} - V_{eq} \right) + \tilde{V}_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \right) \right. \]

\[ + \frac{\rho_{eq}}{2} V_{eq} C_N \left( \frac{\partial \tilde{U}_{eq}}{\partial t} - V_{eq} + \tilde{V}_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \right) \]

\[ + \frac{1}{S} \left[ \frac{m_j}{V_{\Omega_j}} \left( \frac{\partial}{\partial t} \left( \tilde{V}_{eq} - V_{eq} \right) + \tilde{V}_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \right) \right. \]

\[ + \frac{\rho_{eq}}{2} V_{eq} C_N \left( \frac{\partial \tilde{U}_{eq}}{\partial t} - V_{eq} + \tilde{V}_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \right) \]

\[ \left. \left. + \frac{\rho_{eq}}{2} V_{eq} C_N \left( \frac{\partial \tilde{U}_{eq}}{\partial t} - V_{eq} + \tilde{V}_{eq} \frac{\partial \tilde{U}_{eq}}{\partial t} \right) \right) \right] \]

\[ \text{Fig. 4 Coupling terms identification} \]

**4.2.5 Equivalent fluid**

Let's consider equivalent variables:

\[ \beta = \frac{V_{\Omega_j}}{V_{\Omega_j}} \]  
Eq.23

\[ \rho_{eq} = \beta \rho \]  
Eq.24

\[ \tilde{V}_{eq} = \frac{1}{V_{\Omega_j}} \iiint_{\Omega_j} \tilde{V} d\Omega \]  
Eq.25

\[ P_{eq} = \beta \frac{1}{V_{\Omega_j}} \iiint_{\Omega_j} P d\Omega \]  
Eq.26

\[ \mu_{eq} = \beta \mu_T \]  
Eq.27

where \( \beta \) is the porosity.

Eq. 12 became:

**4.3 Equivalent Structure model**

A fuel assembly is assimilated to a porous media (Fig. 5) with a linear behavior, so as we did for the fluid unknown, the structure displacement field is now defined on the whole space domain. Because of the dimensions of a fuel assembly we can use a beam model, but the shear stiffness of a fuel assembly is very weak so the Euler Bernoulli approximation isn’t verified, that’s why we will use a Timoshenko beam model.

\[ \text{Fig. 5 Porous media modelling} \]
4.3.1 Cinematic of a fuel assembly
The cinematic of a Timoshenko beam is defined by the displacement of the mean line \( \bar{u} \) and by the rotation of the cross-section \( \bar{\theta} \) (Fig. 6).

![Fig. 6 Cinematic of a Timoshenko beam](image)

We suppose that the fuel assembly is not submitted to axial displacement, and to torsion rotation, so we can write:
\[
\bar{u} = u_y \hat{e}_y + u_z \hat{e}_z \quad \text{Eq. 31}
\]
\[
\bar{\theta} = \theta_y \hat{e}_y + \theta_z \hat{e}_z \quad \text{Eq. 32}
\]
It is possible to express the displacement \( \bar{U}(x,y,z) \) of a fuel assembly point which is not on the mean line:
\[
\bar{U}(x,y,z) = \bar{u}(x) + \bar{\theta}(x) \wedge (y \hat{e}_y + z \hat{e}_z) \quad \text{Eq. 33}
\]
The displacement depend only on the \( x \) direction for one fuel assembly. It is important to notice that each fuel assembly has its own displacement, so the displacement field defined on the whole space is continue by part, discontinuity are located on the frontier surface delimiting fuel assemblies domains.

4.3.2 Fluid efforts acting on the structure
Fluid efforts acting on a fuel assembly are the opposite of the coupling effort of the fluid integrated over the fuel assembly cross-section:
\[
\vec{F}_{\text{fluid} \rightarrow \text{structure}} = -\vec{F}_{\text{structure} \rightarrow \text{fluid}} \quad \text{Eq. 34}
\]
\[
\vec{F}_{\text{fluid} \rightarrow \text{FA}} = \int \vec{F}_{\text{fluid} \rightarrow \text{structure}} dS \quad \text{Eq. 35}
\]
\[
\bar{M}_{\text{fluid} \rightarrow \text{FA}} = \int \left( y \hat{e}_y + z \hat{e}_z \right) \wedge \vec{F}_{\text{fluid} \rightarrow \text{structure}} dS \quad \text{Eq. 36}
\]
where \( S_{\text{ac}} \) is the cross-section surface of a fuel assembly.

4.4 Coupled model
Variational formulation of the coupled fluid structure movement equations gives:
\[
\text{find } \vec{V}_{\text{eq}} \in \mathbf{V}_{\text{CA}}, \rho \vec{u} \in \mathbf{U}_{\text{CA}}, \bar{\theta} \in \mathbf{\Theta}_{\text{CA}} \text{, } \forall \delta \vec{V} \in \mathbf{V}_{\text{CA}}, \forall \delta \rho \vec{u} \in \mathbf{U}_{\text{CA}}, \forall \delta \bar{\theta} \in \mathbf{\Theta}_{\text{CA}}
\]
\[
\rho_a \int_{\Omega} \frac{\partial \vec{V}}{\partial t} \cdot \delta \vec{V} + \rho_a \int_{\Omega} \text{div} \vec{V}_{\text{eq}} \otimes \delta \vec{V} = -\int_{\Omega} \rho_a \rho \rho \vec{u} \cdot \delta \vec{V}
\]
\[
- \mu_a \int_{\Omega} \delta \vec{V}_{\text{eq}} \cdot \delta \vec{V} \quad \text{Eq. 37}
\]
\[
+ 2 \rho_a \int_{\Omega} \left( \frac{\partial \vec{u}}{\partial t} \cdot \delta \vec{V}_{\text{eq}} + \delta \vec{u} \cdot \frac{\partial \vec{V}_{\text{eq}}}{\partial t} \right)
\]
\[
- \rho_a \int_{\Omega} \delta \vec{V}_{\text{eq}} \cdot \delta \vec{V}
\]
\[
+ \int_{\Omega} \vec{F}_{\text{structure} \rightarrow \text{fluid}} \left( \delta \vec{v}_{\text{eq}} \cdot \delta \vec{u} \right)
\]
\[
\int \nabla \vec{V}_{\text{eq}} d\Omega = 0 \quad \text{Eq. 38}
\]
\[
\sum_{\text{FuelAssemblies}} \left\{ \frac{m_{\text{f}}}{2} \int_{L_{\text{fa}}} \frac{\partial^2 \bar{u}}{\partial t^2} \cdot \delta \bar{u} + \frac{1}{2} I_{\mu} \int_{L_{\text{fa}}} \frac{\partial^2 \bar{\theta}}{\partial t^2} \cdot \delta \bar{\theta} \right\} = \left\{ \begin{array}{l}
\frac{1}{2} \int_{L_{\text{fa}}} T_{\mu} \left( \frac{\partial \bar{\theta}}{\partial x} - \delta \theta \right) \\
\frac{1}{2} \int_{L_{\text{fa}}} T_{\mu} \left( \frac{\partial \bar{\theta}}{\partial x} + \delta \theta \right)
\end{array} \right\}
\]
\[
+ \int_{L_{\text{fa}}} M_{\text{fluid} \rightarrow \text{FA}} \left( \delta \bar{v}_{\text{eq}} \cdot \delta \bar{u} \right)
\]
\[
+ \int_{L_{\text{fa}}} \bar{F}_{\text{structure} \rightarrow \text{fluid}} \left( \delta \vec{v}_{\text{eq}} \cdot \delta \vec{u} \right)
\]
where \( M \) is the bending moment, \( \bar{T} \) is the shear effort, \( \mathbf{V}_{\text{CA}} \) is the space of vector field that satisfy fluid velocity boundary conditions, \( \mathbf{U}_{\text{CA}} \) is the space of vector field that satisfy structure displacement boundary conditions, \( \mathbf{\Theta}_{\text{CA}} \) is the space of vector field that satisfy structure rotation boundary conditions, \( \Omega \) is the whole space domain, \( L_{\text{fa}} \) is the length of a fuel assembly, \( m_{\text{f}} \) is the mass per unit length of a fuel assembly, and \( I_{\mu} \) is the inertial moment per unit length of a fuel assembly.

5. NUMERICAL MODEL
5.1 Spatial discretisation
The variational formulation is spatially discretised by a 2D Finite Elements Method, fluid velocity field is discretised with a 9 nodes mesh, pressure field is discretised with a 4 nodes mesh, and displacement and rotation structure are discretised with a 3 nodes beam mesh.

5.1 Temporal discretisation
Two different classics schemes are chosen to discretise the fluid and structure equations, fluid equations are temporally discretised with an Uzawa scheme, and structure equation are discretised with a Newmark scheme.

6. MULTI SCALES MODEL
The model proposed in this paper presents several scales (Fig. 8), the core reactor is made of fuel assemblies, and fuel assemblies are made of rods which correspond to the control volume scale. It is important to notice that the mesh scale must be smaller than the fuel assembly scale in order to have significant results, but wider than the control volume scale, otherwise the physical notion of space averaging would be lost. The smaller scale is the turbulence scale which is not explicitly represented but modeled by a turbulent viscosity term at the control volume scale.

7. RESULTS
2D dynamic simulations of two fuel assemblies submitted to a meanly axial flow have been performed. Initial displacement of the left fuel assembly is imposed, the right fuel assembly is in its equilibrium position (Fig. 10). At $t = 0$ the system is set free of external forces, so free oscillations are observed. The simulation is performed with three different fluid velocity. Result are presented in Fig. 10 and 11.

As it has be observed by Collard and al (2001, 2003) in experimental tests, the damping increases with the fluid velocity, and the coupling between the two fuel assemblies increases with fluid velocity.

8. CONCLUSION
In this article we proposed a global model of a nuclear reactor core taking into account non linear behavior fluid and structure, and fluid/structure
coupling. A 2D numerical model has been established with a Finite Elements Method, numerical simulations give coherent results in agreement with experimental observations. A 3D numerical model will be developed, and next an experimental validation of the numerical model will be proposed.

REFERENCES


Collard, B., Pisapia, S., Bellizzi, S., Witters, F., 2003, “PWR fuel assembly modal testing and analysis”, Symposium of Flow-Induced Vibration ASME PVP Conference, Cleveland, Ohio, USA.