THREE-DIMENSIONAL NUMERICAL ANALYSES ON LIQUID-METAL MAGNETOHYDRODYNAMIC FLOW THROUGH CIRCULAR PIPE IN MAGNETIC-FIELD OUTLET-REGION

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ABSTRACT Three-dimensional numerical calculations have been performed on liquid-metal MHD flow through a circular pipe in the outlet region of a magnetic field. The continuity equation, the momentum equation and the induction equation have been solved numerically. The induction equation is derived from Maxwell's equations and Ohm's law in electromagnetism. The discretization of the equations is performed by the finite difference method, and the solution procedure follows the MAC method. Calculation results have shown that along the flow axis, i.e. the circular pipe axis, the pressure decreases steeply as a fully-developed MHD flow, drops more sharply in the magnetic-field outlet-region, and finally decreases slowly as a normal non-MHD flow. The flow velocity distribution changes from a flat profile of a fully-developed MHD flow to a parabolic profile of a non-MHD laminar flow in the outlet region of the magnetic field.

Keywords: Numerical Analysis, Magnetohydrodynamics, MHD, Liquid Metal, Outlet Region

1. INTRODUCTION

Liquid metal flow through a magnetic field is used for various power engineering equipments such as an electromagnetic pump, a magnetohydrodynamic (MHD) power generator and a fusion reactor blanket (for cooling), due to its high heat capacity and thermal conductivity in a power engineering system. However, there exists a large MHD pressure drop arising from the interaction between the liquid-metal flow and the magnetic field. In particular, the MHD pressure drop may become considerably larger in the inlet and outlet regions of the magnetic field than in the fully-developed region inside the magnetic field for the reason mentioned below.

Figure 1 shows the coordinate system, the applied magnetic field and the induced electric currents in the outlet region of the magnetic field. The applied magnetic field is imposed in the y direction, having a constant value for $z=0 \sim z1$, a linear decrease from $z=z1 \sim z2$, and a value of zero for $z=z2 \sim z0$, as shown in Fig. 1(a).

In the region of fully-developed MHD flow near $z=0$, the induced electric current which is produced by the vector product of flow velocity and applied magnetic field flows in the negative x direction as shown in Figs. 1(b) and 1(c1). The induced current returns by passing through regions very near the walls (in an x - y plane at the same z) where the flow velocity is nearly zero, in the case of insulating walls. (The induced current can also pass through the walls in the case of conducting walls.) The induced current loop has a relatively large electrical resistance, since the current needs to flow in the thin regions near the walls. The Lorentz force which is caused by the vector product of induced current and applied magnetic field acts in the negative z direction and produces a large pressure drop.

Fig. 1 Outlet region of applied magnetic field

In the outlet region of magnetic field from $z \approx z1$ to $z \approx z2$, the induced electric current flows in the negative x direction, as was the case of fully-developed region, as shown in Fig. 1(b). However, the induced current can pass through the large region downstream the magnetic field section (in an x - z plane with the same y) where no magnetic field or small magnetic field is applied. The electric resistance in this region is much smaller than the resistance in the thin region near the walls mentioned above. Hence, the induced current becomes larger in the outlet region than in the fully-developed region. The Lorentz force and thus the pressure drop also may become considerably larger in the outlet region than in the fully-developed region [1-4].

On the other hand, the induced current in the last section of the outlet region near $z=z2$ can flow in the positive x direction as shown in Fig. 1(b). Thus, a smaller Lorentz force may act in the flow direction and thus a small pressure recovery may occur in this section of the outlet region [1-4].

For these reasons, the authors have already performed three-dimensional numerical calculations on the MHD flow through a circular pipe in the inlet region of the magnetic field, including the region of no magnetic field upstream the magnetic field region [4]. In this study, the authors have conducted three-dimensional numerical calculations on the MHD flow in the outlet region of the magnetic field, including the region of no magnetic field downstream the magnetic field region. To the authors' knowledge, there have been no numerical calculations or experimental studies on the MHD flow through a circular pipe in the magnetic-field outlet-region.

2. NUMERICAL CALCULATIONS

Numerical calculations are performed for an MHD flow in a circular pipe with an inner radius of a , shown in Fig. 1. A fully-developed MHD laminar flow enters the calculation domain at $z=0$, and a fully-developed non-MHD flow leaves the domain at $z=z0$. The applied magnetic field is imposed in the y direction as shown in Fig. 1(a), as was stated previously.

The basic equations which describe a liquid-metal MHD flow are the continuity equation, the momentum equation and the induction equation. The equations are expressed respectively by:

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \eta \nabla^2 \mathbf{v} + \frac{1}{\mu} (\nabla \times \mathbf{B}) \times \mathbf{B}_0, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}_0) + \frac{1}{\sigma \mu} \nabla^2 \mathbf{B}. \quad (3)$$

The vector $\mathbf{B}$ is an induced magnetic field produced by the induced electric current, and is treated as an unknown variable together with the velocity $\mathbf{v}$ and the pressure p . The induced electric current $\mathbf{j}$ can be calculated by the Ampere equation $\mathbf{j} = (1/\mu)(\nabla \times \mathbf{B})$. The third term in the right-hand side of Eq. (2) represents the Lorentz force. The induction equation, i.e. Eq. (3), is derived from Maxwell's equations and Ohm's law in electromagnetism.

The basic equations are expressed in nondimensional forms by introducing the following nondimensional variables and numbers:

$$t^* = \frac{t}{a/\bar{v}_z}, \quad r^* = \frac{r}{a}, \quad z^* = \frac{z}{a},$$

$$v_r^* = \frac{v_r}{\bar{v}_z}, \quad v_\theta^* = \frac{v_\theta}{\bar{v}_z}, \quad v_z^* = \frac{v_z}{\bar{v}_z}, \quad p^* = \frac{p}{\rho \bar{v}_z^2},$$

$$B_r^* = \frac{B_r}{\bar{v}_z \mu \sqrt{\sigma \eta}}, \quad B_\theta^* = \frac{B_\theta}{\bar{v}_z \mu \sqrt{\sigma \eta}}, \quad B_z^* = \frac{B_z}{\bar{v}_z \mu \sqrt{\sigma \eta}}, \quad (4)$$

$$Re = \frac{\bar{v}_z a}{\nu}, \quad Ha = B_0 a \sqrt{\frac{\sigma}{\eta}}, \quad R_m = \frac{\bar{v}_z a}{v_m}. \quad (5)$$

The final nondimensional basic equations become respectively:

$$\nabla \cdot \mathbf{v} = 0, \quad (6)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{v} + \frac{1}{Re} (\nabla \times \mathbf{B}) \times \mathbf{Ha}, \quad (7)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{Rm} \nabla \times (\mathbf{v} \times \mathbf{Ha}) + \frac{1}{Rm} \nabla^2 \mathbf{B}. \quad (8)$$

Star (*) is omitted to simplify the description in Eqs. (6) through (8) and in the following description. Note that the Hartmann number $\mathbf{Ha}$ is a given (known) vector having only y -component as a given function of z , i.e. $Ha(z)$.

The coordinate system is transformed from the Cartesian coordinate system (x, y, z) to the curvilinear coordinate system (ξ, η, ζ) in order to deal with a channel with an arbitrary flow cross-section in the future, and is thereafter transformed into the cylindrical coordinate system (r, θ, z) as a special case of the curvilinear coordinate system. Considering the symmetry, the numerical calculations are carried out for the region of $0 < r < 1$ and $0 < \theta < \pi/2$. (Note that the inner surface of the wall corresponds to $r=1$, or $x=1$ and $y=1$, in the nondimensional coordinates.)

As the boundary condition on the flow velocities, the inflow boundary condition is adopted at the flow inlet, i.e. at $z=0$, by fixing a fully-developed MHD flow velocity [5]. The outflow boundary condition is given at the flow outlet, i.e. at $z=z0$, by fixing the reference pressure. No-slip condition is given at the wall and the symmetry condition is adopted at $\theta=0$ and $\theta=\pi/2$. As the boundary condition on the induced magnetic fields, $\partial \mathbf{B} / \partial z = 0$ and $\mathbf{B}=0$ are specified at the flow inlet and the flow outlet, respectively. The former reflects the situation that the induced current does not change in the z -direction at the flow inlet, and the latter represents that no induced current exists at the flow outlet. At the wall, $\mathbf{B}=0$ is specified assuming that the walls are electrically nonconducting. The boundary conditions on the induced magnetic fields at the symmetry plane of $\theta=0$ and $\theta=\pi/2$ are not intuitively clear. Hence, by performing a calculation for the whole cross section in the case of small Hartmann numbers, it has been confirmed that the

conditions are given by:

$$\theta = 0: \quad A(-\theta) = -A(\theta), B(-\theta) = B(\theta), C(-\theta) = -C(\theta), \quad (9a)$$

$$\begin{aligned} \theta = \pi/2: \quad A(\pi/2 - \theta) &= -A(\pi/2 + \theta), \\ B(\pi/2 - \theta) &= B(\pi/2 + \theta), \\ C(\pi/2 - \theta) &= C(\pi/2 + \theta) \end{aligned} \quad (9b)$$

where A , B and C are the x , y and z components of $\mathbf{B}$, respectively.

The discretization of the equations is carried out by the finite difference method. The calculations are performed using a non-uniform expanding $15 \times 15 \times 30$ grid with grid elements closely spaced near the channel wall of $r=1$ and the region between or around $z=z1$ and $z=z2$. The first-order accurate upwind differencing is adopted for the fluid convection terms in Eq. (7). The solution procedure follows the MAC method that is widely used in numerical calculations.

Even for the fully-developed region, it is difficult to obtain a stable numerical solution for large Hartmann numbers [5]. In the present three-dimensional calculations, stable numerical solutions have been obtained for Hartmann numbers up to 100 and Reynolds numbers up to 1000 by applying the following means or procedures. (1) The grids are arranged closely near the wall of $r=1$, i.e. at $r=0.0, \dots, 0.95, 0.97, 0.99, 0.995, 1.0$, on referring to a velocity profile of the classical Hartmann flow, i.e. fully-developed MHD flow in infinite parallel plates [5]. (2) Simultaneous linear equations on the pressure, i.e. Poisson equation, are solved not by the iterative method but by the elimination method. (3) First, a solution is obtained for Re (Reynolds number) of 0.01, and thereafter Re is increased gradually to a final value, i.e. 1000.

3. CALCULATION RESULTS

3.1 Pressure along Flow Axis

Numerical calculations have been performed for a circular pipe with insulating wall under the conditions of a Reynolds number (Re) of 1000, a Hartmann number (Ha) of 100 (for the fully-developed region) and a magnetic Reynolds number (Rm) of 0.001. The Hartmann number (relating to the applied magnetic field) is 100 from $z=0$ to $z1$, decreases linearly from $z=z1$ to $z2$, and is zero from $z=z2$ to $z0$ (See Fig. 1(a)). The values of $z1/z2$ are changed from 10/20 to 10/10.05. (Note that both $z1$ and $z0-z2$ are fixed to 10 in all the cases.) These values for the nondimensional numbers and parameters are selected in order to simulate those typical to laboratory scales and conditions.

Figure 2 shows calculated pressures along the flow axis, i.e. the z -axis, for the cases of $z1/z2$ from 10/20 to 10/10.05. From $z=0$ to $z \approx z1$, the pressure decreases steeply following the pressure drop of a fully-developed MHD flow. From $z \approx z1$ to $z \approx z2$, the pressure decreases more sharply than in the region of $z < z1$, since a large Lorentz force is produced in the negative z direction as was explained previously and will be mentioned in Sec. 3.3. In $z > z2$, the pressure decreases slowly, representing the frictional pressure drop as a non-MHD laminar flow.

The steeper the gradient of the applied magnetic field becomes, the more sharply the pressure decreases from

Fig. 2 Pressures along z -axis for various magnetic field gradients

$z \approx z1$ to $z \approx z2$. However, the pressure drop through the magnetic-field outlet-region becomes saturated for the steeper gradient of the magnetic field (See the cases of $z1/z2=10/10.1$ and $10/10.05$). For the slower gradient of the magnetic field, the effect of the length along the flow axis (i.e. z -axis) contributes more to the pressure drop through the outlet region than the effect of the outlet region (Compare the cases of $z1/z2=10/20$ and $10/15$).

The small pressure recovery, which was pointed out in Chap. 1, is observed (but is not seen clearly) in the region near $z \approx z2$.

The pressure drops in the fully-developed region of $z < z1$, $-\Delta p/\Delta z$, are almost the same for all the cases. The pressure drops agree nearly with a value calculated numerically by the authors for the fully-developed region, $-\Delta p/\Delta z \approx 0.123$ [5], and also a value predicted by Schercliff's theoretical approximate equation, $-\Delta p/\Delta z \approx 0.118$ [6], for the case of $Ha=100$ and $Re=1000$.

3.2 Velocity Distribution

Figure 3(a) shows calculated velocity v_z distribution at $z=5$ for the case of $z1/z2=10/20$. There is no significant difference among velocity distributions from $z=0$ to $z=10$. The velocity profile is a flat one having a peak value of 1.26. The velocity distribution in this region agrees nearly with a profile calculated by the authors for the fully-developed MHD region [5].

Figure 3(b) present calculated velocity distribution at $z=15$. The distribution is still flat. Figure 3(c) shows calculated velocity profile at $z=19$. The peak value of the velocity is larger than that shown in Fig. 3(a). The velocity distribution changes from a profile typical to an MHD flow to a profile typical to a non-MHD flow in the region of $z \approx 20$, since the applied magnetic field becomes zero in this region.

Figure 3(d) shows a calculate velocity profile at $z=25$. No significant difference exists among the velocity profiles from $z=21$ to $z=30$. The velocity distribution in Fig. 3(d) is a typical one of a non-MHD laminar flow with a peak value of ~ 2 .

(a) At $z=5$

(b) At $z=15$

(c) At $z=19$

(d) At $z=25$

Fig. 3 Velocity v_z distributions for the case of $z_1/z_2=10/20$

Fig. 4 Induced current distribution in x - z plane at $y=0$ for the case of $z_1/z_2=10/20$

3.3 Induced Current Distribution

Figure 4 illustrates induced electric current distribution in the x - z plane at $y=0$ for the case of $z_1/z_2=10/20$. Figures 5(a), (b) and (c) give induced current distributions in the x - y planes at $z=5$, 10 and 20, respectively, for the same case. On the right side of each figure, is shown the magnitude of the (nondimensional) induced current vector in the each coordinate direction. In Figs. 5(a) through (c), the induced current vector is reduced by a factor of 5 for two vectors from the wall at each circumferential angle, in order to make the figures compact.

In the fully-developed region from $z=0$ to $z \approx 8$, the induced current, flowing mainly in the negative x direction, does not change in the z -direction, as shown in Fig. 4. This constant induced current produces constant Lorentz force (acting in the negative z direction) and results in constant pressure drop along the z -axis as shown in Fig. 2. The induced current returns by passing in an extremely thin region very near the wall, as shown in Fig. 5(a). Almost no induced current flows in the region from $z \approx 22$ to $z=30$, as shown in Fig. 4, since no magnetic field is applied.

In the magnetic-field outlet-region from $z \approx 8$ to

Fig. 5 Induced current distributions in x - y plane for the case of $z_1/z_2=10/20$

$z \approx 22$, the induced current forms a loop mainly in the x - z plane, as shown in Fig. 4. The induced current is larger in the outlet region than in the fully-developed region. This is because the electric resistance of the induced current loop in the outlet region is much smaller than the resistance of the loop in the fully-developed region. The induced current can return in the large downstream region in the outlet region, although the current needs to return only in the extremely thin region near the wall in the fully-developed region.

Fig. 6 Inlet and outlet regions of magnetic field

The induced current flows mainly in the negative x direction from $z \approx 8$ to $z \approx 19$. Hence, in this region, a larger Lorentz force than in the fully-developed region acts in the negative z direction, and a larger pressure drop is produced along the z -axis as shown in Fig. 2. On the other hand, the induced current flows mainly in the positive x direction from $z \approx 19$ to $z \approx 22$. Thus, the Lorentz force is exerted in the positive z direction, and a small pressure recovery along the z -axis happens in this region as shown in Fig. 2 (This small recovery is clearer at $z \approx 15$ in the case of $z_1/z_2=10/15$). Also in the outlet region, there exists an induced current loop which returns in an extremely thin region near the wall, as shown in Figs. 5(b).

3.4 Comparison with Magnetic-Field Inlet-Region

Figure 6 shows the applied magnetic field, the induced currents including the directions of Lorentz force and the pressure along z -axis, in the inlet region and the outlet region of the magnetic field. The larger Lorentz force acts and thus the larger pressure drop occurs in the inlet and outlet regions than in the fully-developed MHD region for the reason mentioned previously. On the other hand, a smaller Lorentz force may act in the flow direction and thus a small pressure recovery may occur in the first section of the inlet region and in the last section of the outlet region also for the reason mentioned previously. The pressure drop behavior is not completely symmetric, since the fully-developed non-MHD flow enters the calculation domain in the inlet-region of magnetic field while the fully-developed MHD flow enters the domain in the outlet region.

Figure 7 presents pressures along z -axis for the magnetic-field inlet-region, calculated by the authors and reported in Ref. [4]. The calculation parameters for Fig. 7 are the same as for Fig. 2, except for the Hartmann number

Fig. 7 Pressures along z -axis for magnetic-field inlet-region

change along z -axis. The Hartmann number (relating to the applied magnetic field) is 0 from $z=0$ to $z1$, increases linearly from $z=z1$ to $z2$, and is 100 from $z=z2$ to $z0$.

In Fig. 7, the pressure decreases slowly following the drop of a non-MHD laminar flow from $z=0$ to $z \approx z1$. The pressure recovery appears clearly in the region near $z \approx z1$. This is because the induced current flows in the positive x direction and then the Lorentz force acts in the positive z direction. The pressure decreases more rapidly in the region from $z \approx z1$ to $z \approx z2$ than in the fully-developed MHD region of $z > z2$. The reason is that the induced current in the negative x direction and thus the Lorentz force in the negative z direction are larger in the region from $z \approx z1$ to $z \approx z2$ than in the fully-developed MHD region of $z > z2$. The pressure decreases rapidly following the drop of a fully-developed MHD flow in the region of $z > z2$.

The pressure recovery does not appear clearly in the magnetic-field outlet region. The reason may be that the mechanical energy added by the Lorentz force in the flow direction is used to increase the velocity in the central (bulk flow) region in the outlet-region while the mechanical energy is used to increase the pressure due to the decrease in the velocity in the inlet-region. However, the pressure loss through the magnetic-field outlet region is nearly the same as that through the inlet region.

In future, following improvement in computer performance, it is desirable to examine the effect of the number of grid elements on calculation results.

4. CONCLUSIONS

The following conclusions have been obtained from the calculation results.

- Along the flow axis, i.e. the circular pipe axis, the pressure decreases steeply as a fully-developed MHD flow, drops more sharply in the magnetic-field outlet-region, and finally decreases slowly as a normal non-MHD flow.
- The pressure recovery that was observed in the magnetic-field inlet-region does not appear clearly in the magnetic-field outlet-region. However, the pressure drop through the magnetic-field outlet-region is nearly the same as that through the magnetic-field inlet-region.

(c) The flow velocity distribution changes from a flat profile of a fully-developed MHD flow to a parabolic profile of a non-MHD laminar flow in the outlet-region of the magnetic-field.

NOMENCLATURE

- $A, B, C = x, y, z$ components of induced magnetic field
 a = radius of circular pipe
 $\mathbf{B}$ = induced magnetic field vector
 $B_r, B_\theta, B_z = r, \theta, z$ components of induced magnetic field
 $\mathbf{B}_0$ = applied magnetic field vector
 B_0 = applied magnetic field
 Ha = Hartmann number vector
 Ha = Hartmann number
 $\mathbf{j}$ = induced electric current vector
 $j_x, j_y, j_z = x, y, z$ components of induced electric current
 p = pressure
 Re = Reynolds number
 Rm = magnetic Reynolds number
 r, θ, z = coordinate
 t = time
 $\mathbf{v}$ = velocity vector
 $v_r, v_\theta, v_z = r, \theta, z$ component of velocity
 $\bar{v}_z$ = average of v_z
 x, y, z = coordinate
 η = viscosity
 μ = magnetic permeability
 ν = kinematic viscosity
 $\nu_m = 1/\sigma\mu$, magnetic kinematic viscosity
 ρ = density
 σ = electric conductivity
 * (Star) = nondimensional

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