Application of Hypersingular Integral Equation Method to a Three-dimensional Crack Meeting an Interface

Qin T. Y. 1 and N.A. NODA 2

1 Dept. of Basic Engineering Science, China Agricultural University, Beijing 100083, P. R. CHINA
2 Dept. of Mechanical Engineering, Kyushu Institute of Technology, Kitakyushu, 804-8550, JAPAN

Key Words: Stress intensity factor, Body force method, Crack, Composite material, Hypersingular integral equation.

1 Introduction

In recent decades, the use of new materials is increasing in a wide range of engineering field and the accurate evaluation of interface strength in dissimilar materials becomes very important. Considerable researches have been done to evaluate the stress intensity factors and crack opening displacement for cracks in dissimilar materials [1-3]. However, most of these works are on two-dimensional cases. Lee and Kee [3] evaluated the stress intensity factors of a crack meeting the interface by body force method, but they didn't consider the singularity near the crack front at the interface in their numerical method. Noda and Kobayashi [4] studied mixed modes stress intensity factors of an inclined semi-elliptical surface crack by a body force method, in which the unknown body force densities were approximated by the products of fundamental density functions and polynomials. This numerical method was applied by Wang and Noda [5] to investigate the stress intensity factors of a 3-D rectangular crack using body force method. In the present paper, the hypersingular integral equation based on the body force method is applied to solve the problem of a three-dimensional vertical crack meeting an interface, and the numerical approach suggested by Noda and Kobayashi [5] will be used to obtain a highly reliable numerical results of stress intensity factors.

2 Hypersingular integral equation for a planar crack meeting the bimaterial interface

Consider two dissimilar half-spaces bonded together along the x1-x3 plane. Suppose that the right half-space (x2 plane) is occupied by an elastic medium with elastic constants (u1, v1) and the left half-space (x2 plane) is occupied by an elastic medium with elastic constants (u2, v2). The crack is assumed to be in a plane normal to the x3 axis (Fig. 1), and subjected to a normal load. Based on the body force method [3], the displacements in the right material can be expressed as

\[ u_i(x) = \int \Gamma_{ij}(x, \xi) \bar{u}_j(\xi) d\xi \quad i, k = 1, 2, 3 \quad (1) \]

where \( \bar{u}_j \) is the crack opening displacement, and \( T_{ij}(x, \xi) \) is known function [6]. Using the boundary condition and constitutive relation, the hypersingular integral equation for unknown function \( \bar{u}_j \) can be obtained [3]

\[ \frac{\mu_1}{\pi(x_1 + 1)} \int K_{23}(x, \xi) \bar{u}_j(\xi) d\xi = -\sigma_{23} \quad (2) \]

where \( \int \) is the symbol of the finite-part integral, \( \kappa = 3-4\nu \), and \( K_{35}(x, \xi) \) is known function [3].

3 Stress singularity near the crack front

According to the theory of the hypersingular integral equation, the crack dislocation near a crack front point \( x_0 \) at the interface can be assumed as

\[ \bar{u}_j(\xi) = D(\xi_0) \xi_3^{\lambda} \quad 0 < \Re(\lambda) < 1 \quad (3) \]

where \( D(\xi_0) \) is a non-zero constant related to point \( \xi_0 \), \( \lambda \) is the stress singular index near the crack front meeting the interface. Using the main-part analytical method [7], the stress singular index is determined by

\[ 4A\lambda^2 + 2\cos(\lambda\pi) - A - B = 0 \quad (4) \]

Here \( A = (\mu_1 - \mu_2)/(\mu_1 + \kappa_1\mu_2) \), \( B = (\kappa_1\mu_1 - \kappa_2\mu_1)/(\mu_2 + \kappa_2\mu_2) \). This is coincident with the characteristic equation for the two-dimensional case [2]. The stress intensity factors at the crack front is defined as

\[ K_i = \lim_{r \to 0} \sigma_{ij}(r, \theta) r^{\lambda+1} \quad (5) \]

4 Numerical procedure

Using its behavior near the crack front, the crack
dislocation of a rectangular crack can be written as

$$u_2(\xi_1, \xi_2) = \sum_{n=1}^{N} a_n G_n(\xi_1, \xi_2) \xi_2^k (a^2 - \xi_2^2)(2b - \xi_2)$$ (6)

where $G_n(\xi_1, \xi_2)$ are a power polynomials, and $a_n$ can be obtained from following equations

$$\sum_{n=1}^{N} a_n I_n(x_1, x_2) = -\frac{\pi (x_1 + 1)}{\mu_1} p(x_1, x_2)$$ (7)

in which $I_n(x_1, x_2)$ are integrals, which can be numerically calculated.

5 Numerical results
In the case of homogeneous materials, the numerical results of dimensionless stress intensity factor $F_i = K_i / \sigma_0 \sqrt{b}$ are given in Table 1. In which, the collocation points are $20 \times 20$, and K is the maximum index of the polynomials $G_n(\xi_1, \xi_2)$. It is shown that the results are convergent, and agree with that given by Wang and Noda [5].

When the two materials are different, the numerical results of the stress intensity factors along the interface crack front are given in Fig.2 and Fig.3.

Table 1 Convergence of dimensionless stress intensity factor $F_I$ ($x_2 = 0$, $a/b = 1$)

<table>
<thead>
<tr>
<th>$x_1/a$</th>
<th>0/11</th>
<th>2/11</th>
<th>4/11</th>
<th>6/11</th>
<th>8/11</th>
<th>10/11</th>
</tr>
</thead>
<tbody>
<tr>
<td>K=6</td>
<td>0.7522</td>
<td>0.7468</td>
<td>0.7260</td>
<td>0.6821</td>
<td>0.6085</td>
<td>0.4446</td>
</tr>
<tr>
<td>K=7</td>
<td>0.7539</td>
<td>0.7487</td>
<td>0.7243</td>
<td>0.6803</td>
<td>0.6122</td>
<td>0.4396</td>
</tr>
<tr>
<td>K=8</td>
<td>0.7512</td>
<td>0.7474</td>
<td>0.7260</td>
<td>0.6803</td>
<td>0.6102</td>
<td>0.4451</td>
</tr>
<tr>
<td>K=9</td>
<td>0.7534</td>
<td>0.7462</td>
<td>0.7255</td>
<td>0.6821</td>
<td>0.6090</td>
<td>0.4464</td>
</tr>
<tr>
<td>Wang</td>
<td>0.7534</td>
<td>0.7465</td>
<td>0.7245</td>
<td>0.6828</td>
<td>0.6086</td>
<td>0.4536</td>
</tr>
</tbody>
</table>

Fig.2 Dimensionless stress intensity factors at the interface crack front

Fig.3 Dimensionless stress intensity factors at the interface crack front

Acknowledgment
Financial support from the Inoue Foundation for Science is gratefully acknowledged

References