Refinement of subloading surface model formulation

Member: Koichi HASHIGUCHI, JW Research Inst., Osaka university, Miho-ga-oka, Itabagi-shi, Osaka-fu

Abstract

Refined formulation of the subloading surface model is given for the rates of the anisotropic hardening variable and the elastic-core, i.e. similarity-center of the yield and the subloading surfaces and for the difference of curvatures depending on initial, reloading and inverse loading curves. It would provide the pertinent description of the cyclic loading behavior.

Key Words: Constitutive equation, Plasticity, Elastic-core, Reloading loading, Subloading surface model

1. Introduction

The subloading surface model\(^1\) has been extended to describe the cyclic loading behavior by introducing the translation rule of similarity-center of the subloading surface to the normal-yield surface\(^2\). Here, the similarity-center is regarded as the internal variable exhibiting the stress point at which the deformation is induced most elastically, so that it be called the elastic-core. However, the existing translation rule\(^3\) of the elastic-core is mathematically complex and cannot be incorporated into the constitutive equation in the hyperelastic-based plasticity\(^4\) due to the multiplicative decomposition. Further, the difference in curvatures of initial, reloading and inverse loading curves cannot be described pertinently by the present formulation.

In this article, the refined evolution rules of the anisotropic hardening variable, the elastic-core and the normal-yield ratio for the description of the difference in curvatures of initial, reloading and inverse loading curves are given for the pertinent description of cyclic loading behavior.

2. Strain rate and elastic constitutive relation

The strain rate \(\dot{\varepsilon}\) is additively decomposed into the elastic strain rate \(\dot{\varepsilon}^e\) and the plastic strain rate \(\dot{\varepsilon}^p\) as

\[
\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p
\]

First, assume that the elastic constitutive relation is given by the hypereelastic relation, i.e.

\[
d^e = E : \dot{\varepsilon}
\]

where \(\sigma\) is the Cauchy stress, \((\cdot)^e\) designating the proper corotational rate, and \(E\) is the elastic modulus tensor.

3. Plastic strain rate

3.1 Yield surface

The yield surface with the isotropic hardening variable \(H\) and the kinematic hardening variable \(\alpha\) is described as

\[
f(\dot{\varepsilon}) = F(H)
\]

where

\[
\dot{\varepsilon} = \sigma - \alpha
\]

3.2 Fundamental postulate of elastoplasticity: Subloading surface concept

In order to describe the plastic strain rate induced by the rate of stress inside the yield surface, we adopt the following postulate. **Fundamental postulate of elastoplasticity:** Subloading surface concept

The plastic strain rate is induced when the stress approaches the yield surface but only the elastic strain rate is induced when the stress parts from the yield surface. In other words, the stress approaches the yield surface when a plastic strain rate is induced but parts from the yield surface when only an elastic strain rate is induced.

Then, in order to introduce the measure of approaching degree to the yield surface, renamed the normal-yield surface, let the following subloading surface passing through the current stress and keeping the similarity to the normal-yield surface be introduced, which plays the general measure of approaching degree of the stress to the normal-yield surface (see Fig. 1).

\[
d^p = \lambda \eta
\]

where \(\lambda > 0\) designating the magnitude and the direction of plastic strain rate by \(\lambda\) and \(\eta\), respectively.

3.3 Associated flow rule for subloading surface

The associated flow rule for the subloading surface is adopted:

\[
d^p = \lambda \eta
\]

3.4 Evolution rule of normal-yield ratio

Based on the above-mentioned fundamental postulate of elastoplasticity, the rate of the normal-yield ratio must satisfy the following conditions.

\[
R = \begin{cases} \rightarrow \alpha < 0 & \text{for } d^p = 0 \\ 0 < R < 1 & \text{for } d^p = 0 \\ = 1 & \text{for } R = 1 \\ < 1 & \text{for } R > 1 \end{cases}
\]

\[
R = \begin{cases} 0 & \text{for } d^p = 0 \\ < 0 & \text{for } d^p = 0 \end{cases}
\]
Here, the rate of the normal-yield ratio evolves with the plastic strain rate, obeying Eq. (9) but it is calculated from the equation of the subloading surface by substituting a stress changing by the elastic constitutive relation with fixed internal variables into Eq. (5) when only the elastic strain rate is induced. Then, it follows that

\[ R = U(R)\|\text{d}\sigma\| \text{ for } \text{d}\sigma = 0 \]

\[ R = \frac{f(R)}{F} \text{ for } \text{d}\sigma = 0 \]

where \( U(R) \) is the monotonically-increasing function of \( R \) fulfilling the conditions.

\[ U(R) \rightarrow +\infty \text{ for } R = 0 \text{ (quasi-elastic state)} \]

\[ > 0 \text{ for } R < 1 \text{ (subyield state)} \]

\[ = 0 \text{ for } R = 1 \text{ (normal-yield state)} \]

\[ < 0 \text{ for } R > 1 \text{ (over-normal-yield state)} \]

The explicit examples of the function \( U(R) \) are as follows:

\[ U(R) - u_1 \ln R \]

where \( u \) is the material parameter.

### 3.5 Unified nonlinear anisotropic hardening rule

The evolution rule of anisotropic (e.g. kinematic and rotational) hardening variable \( \rho(u) = u - \rho = 0 \) is given as

\[ \rho \text{ is a function of } u \text{ and } \rho \]

\[ \rho \text{ is a function of } u \text{ and } \rho \]

where \( \rho \) and \( \rho \) are the material properties. The function \( \rho \) is defined by the deviatoric part.

### 3.6 Translation rule of elastic-core

The most elastic deformation behavior is induced in the state that the stress lies on the similarity-center, i.e. \( \sigma = c \) leading to \( R = 0 \). Then, the similarity-center \( c \) is interpreted as the most elastic stress state so that let it be called the elastic-core or elastic-center. Here, note that the elastic-core \( c \) approaches the normal-yield surface, following the stress \( c \) in the plastic loading process. However, the elastic core should not approach the normal-yield surface without limitation.

Now, let the following elastic-core surface be introduced, which always passes through the elastic-core \( c \) and keeps the similarity to the normal-yield surface with respect to the kinematic-hardening variable \( c \).

\[ f(c) = \rho_c F(H) \text{, i.e. } \rho_c - f(c) / F(H) \]

\[ \rho_c \text{ designates the ratio of the size of the elastic-core surface to the normal-yield surface (see Fig. 1) so that let it be called the elastic-core yield ratio} \]

Then, let it be postulated that the elastic-core cannot reach the normal-yield surface designating the fully-plastic stress state so that the elastic-core does not go over the following limit elastic-core surface.

\[ f(c) \leq \chi F(H) \text{, i.e. } \rho_c \leq \chi \]

where \( \chi \) is the material parameter and the following inequality must be satisfied.

\[ f(c) \leq \chi F(H) \text{, i.e. } \rho_c \leq \chi \]

Let the translation rule of elastic-core be formulated as follows:

\[ \hat{c} - c (\text{d} \sigma - \rho_c / \text{d} \sigma) \text{d} \sigma = -c \frac{\hat{\sigma}}{\sigma} (\hat{\sigma} - \rho_c / \sigma) \]

\[ \text{where } c \text{ is a material constant or function in general and } \hat{\sigma} = \frac{\partial f}{\partial c} / \| \partial f / \partial c \| \text{ (|\hat{\sigma}| = 1)} \]

It follows from Eq. (19) that

\[ \hat{\sigma} : c (\text{d} \sigma - \rho_c / \text{d} \sigma) \text{d} \sigma < 0 \text{ for } \rho_c > \chi \]

Therefore, the elastic-core is automatically pulled-back to the limit elastic-core surface when it goes out from that surface.

### 3.7 Difference of curvatures in initial, reloading and inverse loading curves

The unique relation \( \sigma^p - \sigma^q = f(R - R_\rho) \) holds in the monotonic loading process if \( U \) in Eq. (11) is the function of only the normal-yield ratio \( R \). Therefore, \( \sigma^p \) induced during a certain change of \( R \) in the monotonic loading process is identical irrespective of initial, reloading and inverse loading. This property results in the description that the returning of the reloading stress-strain curve to the previous loading curve is unrealistically gentle. Then, in what follows, let us consider the formulation for the pertinent description of the difference of the curvatures in the reloading and the inverse loading curves.

First, note the following facts:

1) The difference between the curvatures in the reloading and the inverse loading curves becomes larger as the plastic deformation proceeds continuously.

2) The similarity-center corresponding to the most elastic stress state approaches the normal-yield surface, following the current stress, as the plastic deformation proceeds continuously, and the approaching degree of the similarity-center to the normal-yield surface is expressed by the elastic-core yield ratio \( \rho_c \) in Eq. (16).

3) The transition from the elastic to plastic state is more abrupt (i.e. greater for a larger value of the material parameter \( u \) in the function \( U \) in Eq. (14). Therefore, the increase in the curvature of stress-strain curve can be described by giving a larger value to the material parameter \( u \).

4) By the facts 1)-3), the difference between the values of \( u \) for the reloading and the inverse loading states should be greater for the larger value of \( \rho_c \).

5) The difference from reloading to unloading in the plastic loading process can be judged by how the direction of the stress is near to the outward-normal \( \hat{\sigma} \) of the similarity-center surface. Then, it can be described by the scalar product \( c_\sigma \) of these unit tensors, i.e.

\[ c_\sigma = \hat{\sigma} \cdot \hat{\sigma} = 1 \]

\[ \text{Eventuall, introducing the variables } \rho_c \text{ and } c_\sigma, \text{ let the material parameter } u \text{ in Eq. (14) be extended as follows:} \]

\[ u = \hat{u} \exp(u_c \rho_c c_\sigma) \]

\[ \text{where } \hat{u} \text{ (average value of } u \text{ ) and } u_c \text{ are the material constant. } 0 \text{ is the continuous function of the variables } \rho_c \text{ and } c_\sigma \text{. The function } u \text{ for the particular states are shown in the bracket. } c_\sigma = 1 \text{. 0 and } -1 \text{ designate the states that the current stress lies outward-normal, tangential and inward-normal directions, respectively, of the similarity-center surface. } \rho_c = \chi \text{ and } 0 \text{ designate the states that the elastic-core lies on the limit elastic-core surface and on the back stress point, respectively.} \]