A Robust Measurement for Tooth Form Grinding Works of a Helical Gear

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Summary

Error estimation and compensation algorithm for tooth form grinding works of a helical gear is proposed. Tooth profile ground by a formed wheel is modeled by truing parameters and setting parameters of the grinding wheel. Actual tooth form is measured by a touch probe on the machine. The truing and setting parameters are estimated by fitting the measured surface to the measurement data as the distances between the modeled surface and the measurement points coincide with the probe radius. Error propagation analysis from measurement errors to the estimation errors of the parameters is applied and suggests a robust measurement algorithm. The proposed method was applied to the actual grinding process. The truing and setting errors were measured and compensated within 10 micrometers.

Keywords: Robust measurement, Least square method, Error propagate analysis, Tooth Form, Grinding Works

1. Introduction

Gear grinding works improve tooth face roughness and straight strains by heat treatment, which contribute to lowering noise and high efficient gear transmissions. Especially, form grinding using a formed wheel on a NC grinding machine is one of the most flexible gear finishing process. However, there are several disturbances for accurate machining such as abrasion of truing dresser, positional error of grinding wheel by the mechanical alignment error of NC machine, and so on.

Some researchers have been constructing system for modeling and measuring errors in manufacturing process of gears[1][3]. The basic idea of their measuring method derives from an actual gear cutter location by fitting a modeling machine to the actual tooth profile measured by touch probe on a coordinate measurement machine (CMM). However, it is difficult to apply this method to the form grinding of helical gears by the following reasons.

(i) Profile of the grinding wheel is expressed by trigonometric functions restricted by implicit condition. It means that the fitting algorithm and calculating processes become more complex.
(ii) Because of the abrasion of the truing dresser, profile of the grinding wheel does not only coincides with the desired profile but also alters every truing process.
(iii) It takes much time to release, carry, set and adjust the work piece from the NC machine to the CMM. The on-machine measurement is effective in shortening the time, but loses the accuracy of the measurement.

In this paper, a robust measurement method for tooth form grinding works of a helical gear is proposed. At first, the tooth profile ground by the formed wheel is modeled including truing parameters. The truing and setting parameters of the grinding wheel are estimated by fitting the modeled surface to the actual tooth surface measured by touch probe. Next, an error propagation from measurement errors to the estimation errors of the parameters is applied and suggests a robust measurement algorithm. The proposed algorithm was applied to the actual grinding process. The truing and setting errors were estimated and compensated within 10 micrometers.

2. Error estimation algorithm for tooth form grinding works

2.1 Definition of coordinate systems

Figure 1 shows the schematic view of grinding and measuring of a helical gear. This system can form the grinding wheel, grind the tooth face, measuring machining errors, and feedback the errors continuously. Coordinate systems around grinding wheel and workpiece are defined as showed in Fig.2.

Let Σg be the coordinate systems which fixed on the workpiece. Coordinate system Σc is set as its z-axis coincides with the Σg, while translated "z" and rotated θ along helix of the gear. Coordinate transformation matrix from Σg to Σc is expressed as

\[ A_c(\theta) = \text{Trans}(0, 0, \xi) \cdot \text{Rot}(\zeta, \theta) \]

\[ \xi = (r_0 / \tan \beta_0) \theta \]

Where, Trans and Rot denote the translational and rotational matrices[3], respectively. Variables \( r_0 \) and \( \beta_0 \) denote
pitch circle radius and helix angle of the gear, respectively.

Let $\Sigma_x'$ be the coordinate systems which fixed on the grinding wheel. The rotational axis of the grinding wheel coincides with the y-axis of $\Sigma_x'$. Coordinate transformation matrix from $\Sigma_x$ to $\Sigma_x'$ is expressed as

$$\frac{\partial}{\partial}A_x'(c) = Trans(D, 0, 0) \cdot Rot(x, \Gamma) \cdot Trans(0, d, 0)$$

(2)

Where, $D$ is distance along common normal between $z$-axis of $\Sigma_x$ and rotational axis of the grinding wheel, $d$ is offset distance of the grinding wheel along the $y$-axis of $\Sigma_x'$, and $\Gamma$ is the angle set of the grinding wheel. Values of $D$, $d$ and $\Gamma$ correspond with $Z$, $C$ and $Y$ axes of the NC machine, respectively. $\Sigma_x$ is set as its y-axis be coincide with the $\Sigma_x'$, while rotated $\varphi$ along y-axis of $\Sigma_x'$. Coordinate transformation matrix from $\Sigma_x$ to $\Sigma_x'$ is expressed as

$$\frac{\partial}{\partial}A_x(\varphi) = Rot(y, \varphi)$$

(3)

2.2 Profile of the desired tooth form and the grinding wheel

Desired profile of the tooth form on the xy-plane of $\Sigma_x$ is expressed with parameter $\alpha$ as

$$\frac{\partial}{\partial}x_0 = \begin{bmatrix} \frac{\partial}{\partial}x_0 \cdot \gamma(\alpha) \cdot \delta(\alpha) \end{bmatrix}^T$$

(4)

The superscript $t$ means the profile curve is expressed in the $\Sigma_x$. Parameters about the desired value are expressed with subscript 0. The tooth surface in the $\Sigma_x'$ is expressed with $\alpha$ and $\theta_0$ as

$$\frac{\partial}{\partial}x_0(\alpha, \theta_0) = \frac{\partial}{\partial}A_x(\theta_0) \cdot x_0(\alpha)$$

(5)

The tooth surface $\frac{\partial}{\partial}x_0$ is projected on the yz-plane of the $\Sigma_x$ as

$$\frac{\partial}{\partial}x_0(\alpha, \theta_0) = \frac{\partial}{\partial}A_x(\theta_0) \cdot \frac{\partial}{\partial}A_x(\alpha) \cdot \frac{\partial}{\partial}x_0(\alpha)$$

(6)

In the Eq. (6), $\frac{\partial}{\partial}x_0$ and $\frac{\partial}{\partial}z_0$ are elements of $\frac{\partial}{\partial}x_0$. The profile of the grinding wheel is given as envelope of the projected curves. Condition of the envelope is given as

$$f(t(\alpha, \theta_0) = \frac{\partial}{\partial}x_0(\alpha) \cdot \frac{\partial}{\partial}y_0(\alpha) - \frac{\partial}{\partial}z_0(\alpha) = 0$$

(7)

The subscripts $\alpha$ and $\theta_0$ denote partial derivatives by $\alpha$ and $\theta_0$. The Eq. (7) means $\theta_0$ is dependent with $\alpha$. For simplicity, dependent parameters are eliminated in expressions from here.

2.3 Modeling of actual tooth profile

As shown in Fig. 3, truing path for the NC machine is given as reference point; central point of the diamond stone; is traced to the desired profile. By applying tool radius compensate function of the NC machine, the dresser edge traces the modified trajectory distance of the stone’s radius from the desired profile. However, because of an abrasion of the diamond stone $r$ and fixing error of the dresser $z$, as shown in Fig. 4, the desired profile is not true on the grinding wheel. In convenient, $r$ and $z$ are named as truing parameter and collected as a vector $t$. Actual profile of the grinding wheel is modeled by the truing parameters as follows;

$$\frac{\partial}{\partial}x_0(t, \alpha) = \frac{\partial}{\partial}A_x(0, 0, 0) \cdot \gamma(\alpha) + r \cdot \frac{\partial}{\partial}n(\alpha)$$

(8)

$$t = [r, z]^T$$

Where, $\frac{\partial}{\partial}n(\alpha)$ is unit normal vector of the desired profile of the grinding wheel $\frac{\partial}{\partial}x_0(\alpha)$. 

![Fig. 1 Tooth form grinding and on machine measurement](image1)

![Fig. 2 Coordinate systems of the wheel](image2)
Surface of the grinding wheel is projected to curves on the xy-plane of $\Sigma_i$ as
\[
\chi(x,t,\alpha,\phi) = A_q\zeta(x,t,\alpha,\phi) = A_qA_f(c(\phi)) x(t,\alpha)
\]
\[
\zeta(x,t,\alpha,\phi) = \sigma_z
\]
Actual tooth profile is given as envelopes of curves with the following condition.
\[
f_2(x,t,\alpha,\phi) = y_{\alpha'}x_{\alpha'} - y_{\alpha'}x_{\phi} = 0
\]
(10)

2.4 Estimation of the parameter
Actual tooth surface is measured by a touch probe on the NC machine as shown in Fig. 4. Measurement data of the tooth form $\tilde{x}(x_m)(i=1,...,m)$ are central points of the probe sphere in the coordinate system of the NC machine $\Sigma_m$. Cross section of cylindrical part on the work piece is measured by the touch probe and stored as $\tilde{x}(x_m)(i=1,...,m)$. Coordinate system of the work piece $\Sigma_m$ is set by the data as its origin be on the plane $\pi_{x}$. $z$-axes aligns on the central line of the work piece, and $xy$-axes coincide with $ZY$-axes of the NC machine. As shown in the Fig. 4, tooth form on the $\pi_{x}$-xy-plane of the $\Sigma_m$ equals to rotated form of the $\tilde{x}(x)$ around $z$-axis with angle $\theta_n$.

In order to measure the truing and setting errors, $\theta_n$ is also collected in the measurement parameters. In convenient $\Sigma_i$, the coordinate system fixed to the work piece. The matrix $A_{q}$ is redefined as
\[
A_{q}(c) = \text{Rot}(\pi_{x},\theta_n) \text{Trans}(D,0,0) \text{Rot}(\pi_{y},\Gamma) \text{Trans}(0,d,0)
\]
(11)

Tooth surface $\tilde{p}$ is given as helix flute by actual tooth profile as
\[
\tilde{p}(x,t,\alpha,\lambda) = A(q)\tilde{x}(x,t,\alpha)
\]
(12)

The tooth form data $\tilde{x}(x_m)$ are transformed to the coordinate systems as $\tilde{x}(x_m)$, Surface parameters $\alpha$ and $\lambda$ at foot of the perpendicular from the measurement data $\tilde{x}(x_m)$ is given by solving the following equations.
\[
f_1 = (\tilde{p}(x_m,\alpha,\lambda)-\tilde{p}(x_m,\alpha,\lambda)) = 0
\]
\[
f_2 = (\tilde{p}(x_m,\alpha,\lambda)-\tilde{p}(x_m,\alpha,\lambda)) = 0
\]
(13)

As a result, $l_i$; distance between the measurement data $\tilde{x}_m$ and actual tooth surface; is given a function of the truing parameters $t$ and the setting parameters $c$ as
\[
l_i = (\tilde{x}_m - \tilde{x})\tilde{p} = (\tilde{p}_c,\tilde{p}_\alpha,\tilde{p}_\lambda)
\]
(14)
The truing parameters $t$ and the setting parameters $c$ are calculated by nonlinear least square method as each $l_i$ corresponds to the probe radius. Flow diagram of the algorithm is shown in Fig. 5.

3. Robust measurement method for tooth form grinding works
3.1 Evaluation of the robustness

Error propagate analysis, which describes relationship between the measurement errors and the estimation errors is introduced.

For simplicity, the truing parameters $t$ and the setting parameters $c$ are collected as machining parameters $h$, 

$$h = \{ D, d, \Gamma, \theta_n, r, z \}^T$$  \hfill (15)

At the measurement point where the touch probe contacts the tooth surface, the flowing equation is formed.

$$\Delta L, (t^e x_m, h) = l(t^e x_m, h) - r_p = 0$$  \hfill (16)

Where, $r_p$ is radius of the touch probe. Total derivative of the Eq. (16) yields to 

$$\Delta L(t^e x_m, h) = \Delta x_m \Delta x + \Delta h = 0$$

$$\Delta h = \left[ \begin{array}{c} \Delta D \\ \Delta d \\ \Delta \Gamma \\ \Delta \theta_n \\ \Delta r \\ \Delta z \end{array} \right] = \left[ \begin{array}{c} \Delta x_m \\ \Delta y_m \\ \Delta z_m \end{array} \right]$$  \hfill (17)

By collecting the Eq. (17) about each measurement data in matrix form, 

$$J_e \Delta x_m + J_h \Delta h = 0$$

$$J_e = \left[ \begin{array}{ccc} \Delta D \\ \Delta d \\ \Delta \Gamma \\ \Delta \theta_n \\ \Delta r \\ \Delta z \end{array} \right] = \left[ \begin{array}{c} \Delta x_m \\ \Delta y_m \\ \Delta z_m \end{array} \right]$$  \hfill (18)

is derived. By using the pseudo-inverse matrix $J_e^+$, Eq. (18) is solved as 

$$\Delta h = J_e^+ J_e \Delta x_m = \Gamma h^{-1} \Delta x_m$$  \hfill (19)

The measurement errors propagate to the estimation error of the machining parameter through the error propagate matrix $\Gamma h$. Values of the error propagate matrix alter according to positions and numbers of the measurement data. It means that influence of the error propagation can be controlled by measurement method.

In order to evaluate error propagation matrix, singular values of each columns of the matrix $\Gamma h$ is introduced. For example, influence of the measurement errors to the parameter $D$ is defined as $s_D$, which corresponds to the singular values of the first row of the matrix $\Gamma h$.

In the following sections, a robust measurement method, which makes less the propagation of the measurement errors to the model parameters $D, d, \Gamma, \theta_n, r$ and $z$, is derived by using previous criterion of evaluation. Tooth form as its module, number of teeth, pressure angle and helix angle are 3mm, 36, 20 degree and 15 degree, respectively is used as model case.

3.2 Regions of tooth form for measurement

Influence of robustness according as regions of tooth form for measurement are examined. In the form grinding of the tooth form, in addition to left and right teeth flanks, tooth root can be used for the measurement regions. Jacobian matrix $J$ in the Fig. 5, which is used for the least square method, does not have full rank when only one side of tooth flank used for the measurement region. It is necessary to use at least both sides of the tooth flanks.

Table 1 shows the singular values $s$ of model parameters in accordance with regions of measurement. In the table, "right 5 left 5" means left and right teeth flanks are measured with each 5 points. As shown in the table 1, the robustness of the measurement about $D, \Gamma, r$ and $z$, are improved when the tooth root is used as the measurement region to together with the left and right teeth flanks. On the other hand, measurement of the tooth root slightly makes less the robustness about $d$ and $\theta_n$.

3.3 Reduction of numbers of the estimated parameters

As shown in the Table 1, $D$ and $z$ are sensitive about measurement errors. These parameters are dependent with each other. Parameter $z_i$ is eliminated by measuring the diameter of the grinding wheel directory. Concretely, $z_i$ equals $r$, if the truing path is modified as the diameter of the actual grinding wheel coincides with the desired diameter. Table 2 shows the singular values $s$ when $z_i$ is eliminated from the estimated parameters. Robustness
about D, Ω and *, are greatly improved by the reduction of numbers of the parameters. On the other hand, robustness about d and θ are not so improved. Relations about robustness and regions of the measurement are same as the previous section. The robustness of the measurement about D, Ω, and * are improved when the tooth root is used. On the other hand, measurement of the tooth root slightly makes less the robustness about d and θ.

3.4 Robust measurement method for tooth form grinding works

In accordance with the analysis of the previous sections, a robust measurement method for tooth form grinding works is proposed as follows.

(1) The vertical position of the dresser and distance of the grinding wheel to the work piece are not independent. By measuring the diameter of the grinding wheel directory, the vertical position of the dresser is excluded from the parameters to be estimated.

(2) The sensitivity of parameter estimation error is reduced by using measurement data of the tooth root in addition to the both left and right teeth flanks.

(3) After estimating the parameters in the previous procedure, parameter d and Ω are re-estimated by using data of both left and right teeth flanks, excluding the data of the tooth root.

4. Experimental result of estimation and compensation of the machining errors

4.1 Experimental setup

One example of experimental result is shown in the following section. Specifications of the tooth form are same as the previous section; module, number of teeth, pressure angle and helix angle are 3mm, 36, 20 degree and 15 degree, respectively.

4.2 Estimation of the truing and the setting parameters

A tooth form was ground with finishing allowance of 1 mm. On about 2 cross sections perpendicular to the axis of the workpiece, teeth flanks and tooth root were measure by the touch probe. The truing and the setting parameters are estimated as shown in Fig. 6 (a) and Table 3 (a). In the table, ΔD, Δd and Δr, show the differences from the desired values. Columns (A&B) means that the parameters were estimated by using measurement data of the both cross sections A and B. Rows A and B show that the parameters were measured by using data of the cross sections A and B, respectively. Absolute difference of each estimated value between measured with data A and with data B is shown in abs(A-B) row. In this row, the abs(A-B) about parameter d is larger comparing with the other parameters. It means that the parameter d is sensitive against the measurement errors. It was implied in the table 2. The singular value s is larger than the other parameters.

4.3 Compensation of the truing and setting errors

The errors ΔD, Δd and Δr, estimated by measurement data (A & B) were feedback to the finishing truing and grinding. The grinding wheel is attached towards the C-axis of the NC machine with quite a lot of overhanging as shown in the Fig. 1. In order to avoid influence of another errors excluded in this research such as alignment error of the C-axis of the NC machine, set angle of the grinding wheel Ω was not compensated. Instead, in the finishing grinding process, Ω was set as the value which was estimated in the previous section.

The results are shown in the Fig. 6 (b) and Table 3 (b). Errors in D and r, were compensated within accuracy of 1 μm. However, there still remains 10 μm errors in d; the offset position of the grinding wheel. This is because parameter d is sensitive against the measurement errors comparing with D and r, as discussed in the

<table>
<thead>
<tr>
<th>Table 1 Robustness of full parameters measurement</th>
<th>Table 2 Robustness with reduction numbers of the parameters</th>
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<tbody>
<tr>
<td>s (-)</td>
<td>right 5 left 5</td>
</tr>
<tr>
<td>ΔD (mm/mm)</td>
<td>1.91E5</td>
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<tr>
<td>Δd (mm/mm)</td>
<td>57.32</td>
</tr>
<tr>
<td>ΔΩ (rad/mm)</td>
<td>54.14</td>
</tr>
<tr>
<td>Δθm (rad/mm)</td>
<td>1.08</td>
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<tr>
<td>Δr (mm/mm)</td>
<td>582.0</td>
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<tr>
<td>Δr (mm/mm)</td>
<td>1.90E5</td>
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<tr>
<td>ΔD (mm/mm)</td>
<td>201.3</td>
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<tr>
<td>Δd (mm/mm)</td>
<td>57.32</td>
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<tr>
<td>ΔΩ (rad/mm)</td>
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<td>1.08</td>
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<tr>
<td>Δr (mm/mm)</td>
<td>68.51</td>
</tr>
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</table>
In addition, there exist another disturbances excluded in this research such as thermal deformation of the machine, deformations of the machine, the workpiece and the grinding wheel by strains in the grinding process, and so on. Those unmodelled disturbances may occurred machining errors in the finishing process. In order to achieve accurate machining, it is obvious to examine about accuracy of the machine tools. This is our incoming problem.

5. Conclusion

(1) The tooth profile ground by the formed wheel is modeled by 2 truing parameters; edge radius and vertical position of the dresser, and 3 setting parameters; distance, offset, and set angle of the grinding wheel towards the work piece.

(2) The truing and the setting parameters are estimated by fitting the modeled surface to the actual tooth surface measured by touch probe as the distances between the modeled surface and the measurement points be coincide with the probe radius, by using nonlinear least square method.

(3) Error propagation from measurement errors to the estimation errors of the parameters is applied and suggests a robust measurement algorithm.

(4) The suggested robust measurement method is applied to the actual grinding process. The truing and setting errors are measured and compensated within 10 micrometers.

References