118 Long Focal Length Testing for Large Aperture Lens

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Summary

A novel method for long focal length measurement is presented, which is accomplished by obtaining the angle of moiré fringe formed by Talbot effect of Ronchi grating. When the Ronchi grating was illuminated by spherical wave (plane wave pass through a lens), the period of the Talbot image of the grating, which has a relationship with the radius of the spherical wave(focal length of the lens), will change. When put another Ronchi grating at the place of the Talbot image, moiré fringe formed. Correspondingly, the angle of the moiré fringe has a relationship with the focal length. So the focal length can be measured by calculating the angle of the moiré fringe. The formula of calculating the long focal length from the angle of moiré fringe is derived. Error analysis demonstrates that this method can be applied to the real-time testing with high accuracy.

Keywords: Ronchi grating, Talbot effect, Moiré fringe, Focal length measurement

1. Introduction

The characteristics of self-imaging of period object illuminated by spherical wave have been pointed out by general Talbot effect theory [1-3]. According to the theoretic analysis, the Talbot distances are different when the period object is illuminated by plane wave and spherical wave. While we noticed that when the focal length of the lens is very long (>10 meters), their difference is very small, to our measurement it can be neglect. From this point, if two gratings are fixed with a Talbot distance and the moiré fringe will always appear at the second grating and their angle will rotate with the changing of the focal length. Therefore we can obtain the focal length of the lens accurately by calculating the angle.

The conventional method of measuring the angle of the moiré fringe is carried out by manually adjusting collimation lines so that they are parallel to the moiré fringe and recording the rotated angle of the collimation lines. The disadvantage of this method is that it is very slow and might bring some subjective error. This paper presents a digital analysis of the moiré fringe and its angle can be obtained in the frequency domain of the image which is insensitive to noise. After the image is captured by CCD camera, the spectrum image of the moiré fringe can be obtained by image process such as Fourier transform and filter process. By calculating the coordinates of the spectrum points of the stripe, we can get the angle of the moiré fringe rapidly and accurately.

For large aperture lens, the focal lengths will not be identical corresponding to locations within its aperture resulting from fabrication error. In this paper a scanning method is developed to measure the focal lengths at all typical locations.

2. Theory

2.1 Principle of Talbot effect

When a period object is illuminated by a plane wave, the image of the object on the period distance behind the object will be observed. This self-imaging effect is called Talbot effect and the distance called Talbot distance.

The complex amplitude transmittance rate of the Ronchi grating g(x) is presented by

\[ g(x) = \sum_{n=-\infty}^{\infty} A_n \exp\left[ \frac{2\pi}{p} nx \right] \]

for \( n = 0, \pm 1, \pm 2, \ldots \) (1)

where \( p \) is the period of the Ronchi grating.

When the grating is illuminated by monochromatic plane wave, if only \( 0, \pm 1 \) orders diffraction are taken into account, we can get the Talbot
image of the grating at Talbot distance  \( d = mp^2 / \lambda \) for \( m=1,2,3, \ldots \) Equation (2) shows the intensity distribution:

\[
I_n = \left| A_0 + 2A_1 \cos\left( \frac{2\pi}{p} x \right)(-1)^n \right|^2
\]

(2)

When a lens to be tested is inserted in the collimated beam before the Ronchi grating G1 with the distance of s as show in Fig 1, the intensity distribution behind the grating G1 can be expressed as follows:

\[
I(x,z) = A_0^2 + 2A_1^2 + 4A_0A_1 \cos \left[ \frac{2\pi}{p} \left( \frac{f-s}{z} \right) x \right]
\]

(3)

\[
+ 2A_1^2 \cos \left[ \frac{2\pi}{p} \left( \frac{2f-s}{z} \right) x \right]
\]

where \( f \) is the focal length of the lens and \( s \) is the distance between the lens and the grating G1, when \( (f-s)(z+f-s) = \frac{mp^2}{\lambda} \) (for \( m=1,2,3, \ldots \) ). Equation (3) can be simplified as:

\[
I(x,z) = A_0^2 + 2A_1^2 + 4A_0A_1 \cos \left[ \frac{2\pi}{p} \left( \frac{f-s}{z} \right) x \right]
\]

(4)

we can obtain Talbot image G1 of the grating G1 at the location giving by

\[
d = f-s+z = \frac{(f-s)mp^2}{mp^2 + \lambda(f-s)}
\]

(5)

Similarly the period of the Talbot image \( p' \) can also be derived as

\[
p' = \left| \frac{z}{(f-s)} \right| p
\]

(6)

while the orientation of the grating line of the Talbot image keeps the same as the Ronchi grating.

2.2 Principle of moiré fringes

When two gratings G1 and G2 are put together and illuminated by collimated wave, moiré fringe will appear with its width \( w \) and angle \( \varphi \) written by:

\[
w = \frac{p_1p_2}{(p_1^2 + p_2^2 - 2p_1p_2 \cos \theta)^{1/2}}
\]

(7)

\[
\sin \varphi = \frac{p_1 \sin \theta}{(p_1^2 + p_2^2 - 2p_1p_2 \cos \theta)^{1/2}}
\]

(8)

where \( \theta \) is the angle of the grating line between G1 and G2, \( p_1 \) and \( p_2 \) are the periods of the two gratings respectively.

3. Principle and system of the measurement

According to the principle of the Talbot effect mentioned above, a collimated beam illuminating on a Ronchi grating G1 will make the Talbot image G1' of the grating G1 appear at the Talbot distance. When we place another grating G2, which has the same period as G1, at the location of Talbot image G1', the moiré fringe will appear. From Equation (7), the width of the moiré fringe can be represented as:

\[
w = \frac{p}{2 \sin(\theta/2)}
\]

(9)

where \( \theta \) is the angle between the grating lines of the two gratings.

When a lens is placed before grating G1, the plane wave will become spherical wave after passing through the lens, so the wave illuminating on G1 is spherical wave. From the above results, at the location \( d = mp^2(f-s)/[\lambda(f-s) + mp^2] \) Talbot image can be observed. In our practical measurement, \( \lambda=632.8nm, p=0.02mm, s=20mm, m=100, f \) is about 10 meters, so we can estimate:

\[
\frac{\lambda(f-s)}{mp^2} >> 1, \quad \lambda(f-s) >> mp^2
\]

so the Equation (5) can be simplified as: \( d = mp^2 / \lambda \). So at the same location, we can still get Talbot
image $G_1'$. However the period of the image has changed, assume $p'$ is the period of $G_1'$, then

$$p' = \frac{f - s - d}{f - s} p$$

(10)

Because of the change of the image period, the angle of the moiré fringe formed by $G_1'$ and $G_2$ will change, assume the change of the angle is $\varphi$, from equation (8) and (10), we obtain a corresponding relationship between $\varphi$ and $f$, which is given by

$$f = s + \frac{wd}{1 - \cos \theta + \sin \varphi \cdot \varphi}$$

(11)

where $s$ is the distance between grating $G_1$ and lens. In this case $\theta$ is very small ($< 1^\circ$), so $\cos \theta = 1, \sin \theta = 2 \sin (\theta / 2)$. Inserting (9) into (11), we obtain

$$f = s + \frac{wd}{p}$$

(12)

where $s, d, p, w$ are measurable, if we can get the angle of the moiré fringe, we can calculate the focal length of the lens. Figure 2 shows the measurement system.

![Figure 2](image-url)

**Fig.2** Figure of automatic measurement system principle of long focal length

4. Scanning Measurement system

As we known long focal lens usually has large aperture. For large aperture lens, it is hard to produce large parallel beam and large area grating is difficult to fabricate and also the focal lengths will not be identical corresponding to different locations within its aperture, so we developed a scan method to measure large aperture lens as shown in figure 3.

![Figure 3](image-url)

**Figure.3** Figure of scanning measurement system

As shown in figure 4, the incident spherical wave on the grating $G_1$ is not always perpendicular to the grating surface except the collimated laser beam is illuminated in the middle of the measured lens. An angle $\alpha$ between the vertical line of the grating and the spherical wave might affect the fringe pattern, however, theoretical analysis shows that the angle $\alpha$ will not bring errors to our measurement.
Assume the spherical wave front before the grating G1 is:

\[ u_0(x, y, R) = \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha)^2 + y^2}{2R}\right)\right] \]

where \( R = f - s \), as Fig.4 shows, after the grating G1, the wave front becomes:

\[ u(x, y, R) = \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha)^2 + y^2}{2R}\right)\right] \sum_{\pm} A_{\pm} \exp\left(\frac{i2\pi R}{p} \mp x\right) \]

if only \(-1, 0, +1\) order diffractions are taken into consideration, the wave front can be written as:

\[ u(x, y, R) = A_0 \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha)^2 + y^2}{2R}\right)\right] + A_1 \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha + \lambda/p)^2}{2R}\right)\right] - \frac{R}{2} \frac{\lambda^2}{p^2} + \frac{2\lambda\sin\alpha}{p} \]

from the above equation, we can see that after passing through the grating G1 the incident spherical wave become three spherical waves.

\[ u(x, y, z) = A_0 \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha)^2 + y^2}{2R}\right)\right] + A_1 \exp\left[\frac{i2\pi}{\lambda} \left(\frac{(x-R\sin\alpha + \lambda/p)^2}{2R}\right)\right] - \frac{R}{2} \frac{\lambda^2}{p^2} + \frac{2\lambda\sin\alpha}{p} \]

Assume \( A_1 = A_1 \), the distribution of the intensity in the propagation of \( d = f - s + z \) behind the grating G1 can be expressed as:

\[ I(x, y, z) = A_0^2 + 2A_1^2 + 4A_0A_1 \cos\left[\frac{2\pi}{\lambda} \left(\frac{R\lambda x}{p} - \frac{R^2\lambda\sin\alpha}{p} + \frac{RA\sin\alpha}{p}\right)\right] \cos\left[\frac{2\pi}{\lambda} \left(\frac{R^2\lambda^2 x}{2p^2} - \frac{R\lambda\sin\alpha}{2p}\right)\right] \]

+ \( 2A_1^2 \cos\left[\frac{2\pi}{\lambda} \left(\frac{2R\lambda x}{p} - \frac{2R^2\lambda\sin\alpha}{p} + \frac{RA\sin\alpha}{p}\right)\right] \cos\left[\frac{2\pi}{\lambda} \left(\frac{R^2\lambda^2 x}{2p^2} - \frac{R\lambda\sin\alpha}{2p}\right)\right] \]

From the above Equation, when \( \frac{2\pi}{\lambda} \left(\frac{R^2\lambda^2 x}{2p^2} - \frac{R\lambda\sin\alpha}{2p}\right) = m\pi \), we can get the Talbot image of the grating, the Talbot distance is:

\[ d = f - s + z = \frac{(f - s)mp^2}{mp^2 + \lambda(f - s)} \]

the period of the image is:

\[ p' = \frac{1}{\sqrt{L^2}}/(f - s) \]

So we can conclude that the Talbot distance and the period of the Talbot image are the same with the grating illuminated perpendicularly by the spherical wave. Experiments also demonstrate that the moiré fringe will move with fixed interval and orientation when the mirror slightly tilts. Therefore the scanning method is feasible and practical.

5. Result and Error Analysis

In our experiment, as shown in Fig.5, moiré fringe was imaged to a CCD camera by image system and its digital
image acquisition was realized by a frame grabber. The angle of the moiré fringe could be accurately calculated by the spectrum of the moiré fringe image as shown in Fig.6 and Fig.7. Since the measurement error of the angle could be easily controlled within 0.025°, the accuracy of the focal length measured could reach 0.15%. Experiment demonstrated that this method was an effective way to the long focal length measurement.

When the experiment setup was fixed, \( \frac{wd}{p} \) was a constant from equation (10). Assume \( k = \frac{wd}{p} \), then Equation can be simplified as:

\[
    f = s + ktg\varphi
\]  

(18)

\( k \) can be defined as a calibrated coefficient obtained by a standard long focal lens, and it can reduce the error caused by \( w, d \) and \( p \) respectively.

In our experiment the measured lens was a concave lens as shown in Fig.8. The first surface of the lens is a plane and the radius of the second surface \( R \) is 9772 mm. The aperture of the lens \( D \) is 150mm. And the refractive index of the glass is 1.53.

From equation:

\[
    \frac{1}{f} = (n - 1)(\frac{1}{R_1} - \frac{1}{R_2})
\]  

(19)

we can get the theoretical focal length of the lens is \( f = 18438 \text{mm} \).

Fig.9 and Fig.10 are the measured focal length by our measurement system in different locations of the lens.

From Fig.9 the average focal length is \( f = 18427 \text{mm} \) and the deviation is \( \sigma = 0.12\% \) and from Fig.10 the average focal length is \( f = 18335 \text{mm} \) and the deviation is \( \sigma = 0.13\% \).

To the measured lens, from the theoretical analysis the focal length difference can be neglect within the aperture because of its long focal length. The measured focal length difference in the paraxial domain and marginal domain may be caused by the fabrication of the lens.
6. Conclusions

This paper presents a new method for the measurement of long focal lens. Because of the neglect of the variation of the Talbot distance after the lens to be measured was put before the grating, this method is most suitable for long focal length measurement. During the measurement, it is no need to adjust the distance between the two gratings. When the lens is inserted in the collimated beam before the first grating, we can get the focal length of the lens in several seconds by the computer.

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References