On the Transmission of Heat through Condenser Tubes.

By

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Member.

1. It is generally considered that in a surface condenser there is clinging to the tubes a thin film of condensed water on the steam side, and a similar layer of water on the water side, and that in these films convection currents do not apparently take place, and a great resistance is offered to the passage of heat, since the heat can pass across those films by conduction only. Some authorities moreover suppose that round the tubes there also tends to form a blanket of air.

The phenomenon that the B.T.U. transmitted per degree temperature difference per unit surface area per unit time varies according to the circumstances, such as the steam velocity, the water speed, or the amount of air present in the exhaust steam, can be explained by considering that these have an influence upon the thickness of one or more layers suggested in the above.

For the case of the radial flow of heat through a tube, Fourier's fundamental conduction equation

$$\frac{\partial \theta}{\partial t} = \frac{k}{c\rho} \left\{ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right\}^*$$

can be reduced to

$$\frac{\partial \theta}{\partial t} = \frac{k}{c\rho} \cdot \frac{1}{r} \cdot \frac{d\left(r \frac{d\theta}{dr}\right)}{dr}.$$

But for a steady flow of heat $\frac{\partial \theta}{\partial t} = 0$.

Therefore
$$\frac{1}{r} \cdot \frac{d\left(r \frac{d\theta}{dr}\right)}{dr} = 0,$$

the solutions of which gives

$$\theta = \frac{(\theta_1 - \theta_2) \log r}{\log r_1 - \log r_2} + \frac{\theta_2 \log r_1 - \theta_1 \log r_2}{\log r_1 - \log r_2},$$

where r_1 = inner radius of the tube,

r_2 = outer radius of the tube.

The rate of flow of heat per unit area in the direction of r will therefore be given by

$$H = k \frac{d\theta}{dr} = \frac{k(\theta_1 - \theta_2)}{r \log r_1 - r \log r_2} = \frac{k(\theta_2 - \theta_1)}{r \log \frac{r_2}{r_1}}. \dots\dots\dots (1)$$

Now let

r = outer rad. of the tube,

δ_1 = thickness of the air film,

δ_2 = thickness of the water film on the outside of the tube,

t = thickness of the tube wall,

δ_3 = thickness of the water film on the inside of the tube,

k_a = conductivity of air,

k_w = conductivity of water,

k_m = conductivity of metal,

Then the total resistance to the transmission of heat thorough the entire path will be

* in which θ = temperature, t = time, k = conductivity, c = specific heat, ρ = density.

$$R = \frac{r \log \frac{r + \delta_1 + \delta_2}{r + \delta_2}}{k_a} + \frac{r \log \frac{r + \delta_2}{r}}{k_w} + \frac{r \log \frac{r}{r - t}}{k_m} + \frac{r \log \frac{r - t}{r - t - \delta_3}}{k_w} \dots \dots \dots (2)$$

The data given in the Engineering, Nov. 27, 1914, p. 648, enable us to assume for ordinary conditions

$$\frac{r \log \frac{r + \delta_1 + \delta_2}{r + \delta_2}}{k_a} : \frac{r \log \frac{r + \delta_2}{r}}{k_w} : \frac{r \log \frac{r}{r - t}}{k_m} : \frac{r \log \frac{r - t}{r - t - \delta_3}}{k_w} = 0.2 : 0.3 : 0.07 : 0.5.$$

Now taking $\left. \begin{array}{l} k_a = 0.000048 \\ k_w = 0.00136 \\ k_m = 0.3 \end{array} \right\} \dots \dots \dots (3)$

we get

$$\log \frac{r + \delta_1 + \delta_2}{r + \delta_2} : \log \frac{r + \delta_2}{r} : \log \frac{r}{r - t} : \log \frac{r - t}{r - t - \delta_3} = 0.000096 : 0.00408 : 0.21 : 0.0068.$$

Hence for 20 mm. dia. tubes whose thickness is 1 mm., we shall have

$$\left. \begin{array}{l} \delta_1 = 0.0005 \text{ mm.} \\ \delta_2 = 0.022 \text{ mm.} \\ \delta_3 = 0.030 \text{ mm.} \end{array} \right\} \dots \dots \dots (4)$$

2. Regarding the transference of heat through condenser tubes we have a well-known formula

$$S = \frac{Q}{U} \log \frac{\theta_s - \theta_t}{\theta_s - \theta_o}, \dots \dots \dots (5)$$

where

S = cooling surface in sq. ft.,

Q = quantity of cooling water in lbs. per hour,

θ_s = temperature of steam in deg. Fahr.,

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θ_i =temperature of inlet water in deg. Fahr.,

θ_o =temperature of outlet water in deg. Fahr.,

U =heat transmitted in B.T.U. per sq. ft. of cooling surface per hour per deg. Fahr. of temperature difference between steam and water=coef. of heat transmission.

In addition to the above notations, let

d_i =internal diameter of tube in ft.,

d =external diameter of tube in ft.,

l =efficient length of tube,

v =velocity of circulating water in ft. per sec.,

N =no. of tubes,

w =weight of water in lbs. per cub. ft.,

then

$$Q = N \cdot w \cdot \frac{\pi}{4} \cdot d_i^2 v \times 3,600 \dots \dots \dots (6)$$

$$S = N \pi d l \dots \dots \dots (7)$$

(6), (7) in (5) gives

$$\frac{U}{3,600 \times w} \cdot \frac{1}{v} \cdot \frac{l}{\frac{\pi d_i^2}{4}} = \log \frac{\theta_s - \theta_i}{\theta_s - \theta_o} \dots \dots \dots (8)$$

Put

$$d = a d_i \dots \dots \dots (9)$$

then

$$\frac{U}{3,600 \times w} \cdot \frac{l}{v} \cdot \frac{l}{m} a = \log \frac{\theta_s - \theta_i}{\theta_s - \theta_o}$$

Again put

$$\frac{a}{3,600 \times w} = \beta \dots \dots \dots (10)$$

(8) now reduces to

$$\beta \cdot \frac{U}{v} \cdot \frac{l}{m} = \log \frac{\theta_s - \theta_i}{\theta_s - \theta_o}$$

Transforming this we have

$$\frac{\theta_s - \theta_i}{\theta_s - \theta_o} = e^{\frac{\beta U l}{vm}}$$

$$\theta_o = \theta_s - (\theta_s - \theta_i) e^{-\frac{\beta U l}{vm}}$$

Let

$$\eta = \frac{\theta_o - \theta_i}{\theta_s - \theta_i}, \dots\dots\dots(11)$$

then

$$\begin{aligned} \eta &= \frac{\theta_s - (\theta_s - \theta_i) e^{-\frac{\beta U l}{vm}} - \theta_i}{\theta_s - \theta_i}, \quad \theta_s > \theta_i, \\ &= 1 - e^{-\frac{\beta U l}{vm}} \dots\dots\dots(12) \end{aligned}$$

On rough approximation

$$\eta = 1 - \frac{1}{1 + 1.72 \frac{\beta U l}{vm}} = \frac{l}{\frac{m}{1.72 \frac{\beta U}{v}} + l} \dots\dots\dots(13)$$

which, if we put $\frac{1}{1.72 \frac{\beta U}{v}} = r$, becomes

$$\eta = \frac{l}{rm + l} \dots\dots\dots(14)$$

It is of interest to note that this equation is of the same form as Prof. Perry's empirical formula* for the efficiency of boiler tubes which he deduced from the results of a number of experiments.

3. It is at present a well-known fact that the coefficient of heat transmission varies to a considerable extent according to circumstances, and it has been attempted by not a few experimenters to determine this. In 1913, when he was a student in this College

* Perry : Steam Engine. p. 433.

of Engineering, the writer also was able to conduct a series of experiments in this line, and what may appear more or less now of the conclusions drawn from those experimental results may be briefly stated as follows:—

1° Mr. Row * insisted on that the pressure of water lowers considerably the rate of heat transmission, but this statement, though true for steam tubes in which the circulation of water is slow, is in no measure applicable to the case of water tubes.

In the steam tube, the bubbles formed on the water side of the tube plays a great part in the transmission of heat, and it is because the water pressure influences on the movement of those bubbles of steam that the heat transmission is affected by the pressure of water. In the water tube, however, the circulation of water is so rapid that steam bubbles, if formed, would be immediately carried away by it, and thus the steam bubbles would have no time to affect anything on the transfer of heat.

2° External vibrations have no slightest effect on the transmission of heat in a condenser, whilst in steam boilers, they help the action of heating surface to a considerable extent.

For this result the writer may find his explanation as follows:—
The increased efficiency of locomotive boilers is due to the fact that the vibrations help the vapour bubbles forming on the water side of the tubes to detach themselves from the tubes. In water tubes heated by steam on the outside, on the other hand, the temperature of the water flowing through those tubes being not so high, the formation of steam bubbles will be very little, compared with the case of smoke tubes in steam boilers, and even if such bubbles be formed, the rapidly circulating water would immediately destroy them, without letting them have no time to oppose the

* Pro. Inst. Heating & Ventilating Engineers, London, Feb., 1912.

conduction of heat by forming a non-conducting layer of themselves between the tube surface and the water.

3° The size of the tube has a marked effect on the rate of heat transmission. Generally speaking, the smaller the tube the higher the efficiency.

The probability of the chance of water particles in a tube coming into contact with the tube side will be some function of the ratio.

$$\frac{\text{perimeter of the tube in contact with water}}{\text{area of the water way}} = \frac{1}{m},$$

increasing and decreasing therewith.

But we know, on the other hand, that the total absorption of heat by the water is greater, the greater the number of particles of water at the original temperature which are brought into contact with the transmitting surface during a given time, and greatest in any case, if the particles of the total amount of water used are in contact with the transmitting surface an equal length of time. Hence it would be reasonable to assume that the heat transmission ratio increases with a decrease in the size of the tube according to some law which can be determined by experiments only.

4° The law that the water speed increases the rate of heat transmission is true, but the influence of the former on the latter is greater at lower speeds, and for any particular tube, there is a certain velocity, after which any further increase in the speed would not increase the transmission rate at all. Broadly speaking, the larger the tube the lower this limiting velocity. It has also been found that the increase in the transmission rate is very slow in the neighbourhood of the above limiting velocities; consequently it will be important that in practice the velocity of water should be kept much below the limiting value. The writer may recommend

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the following values as the maximum speeds which might be economically adopted in practice :

12 ft. per sec. for $\frac{1}{2}$ in. tube,

9	„	$\frac{3}{4}$	„
5	„	$\frac{7}{8}$	„
1.5	„	$1\frac{5}{16}$	„
1	„	$2\frac{1}{8}$	„

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4. Recent investigations by Weighton, Weir and others have revealed the fact that it is of prime importance for an efficient working of a surface condenser that the steam should take such a course that it will flow over all the tubes, and at a uniform velocity, so that all the tubes in it should be equally efficient. To meet these requirements it is now a current practice to reduce the cross-sectional area of the condenser as we approach the bottom. There seems, however, to have been nobody who has ever determined what is the best curve for the periphery of the reduced section.

The following rough theory has been worked out with view to apply to a surface condenser, in which no horizontal baffles are placed, and the exhaust steam flows in straight paths from the exhaust inlet to the condensate outlet, perpendicularly to the axes of the tubes.

Let

$n_0, n_1, n_2, \dots, n_{q-1}, n_q, \dots, n_{r-1}$ = the no. of tubes in the 1st, 2nd, 3rd, ..., q th, $(q+1)$ th, ..., r th row, r being the total number of rows,

$$N = \text{total no. of tubes} = \sum_{q=0}^{q=r-1} n_q$$

l = length of tubes in ft,

d =outside dia. of tubes in ft.,

p =horizontal pitch of tubes in ft.,

λ =condensation rate in lbs. per sq. ft. of tube surface per sec.,

σ =specific volume of steam in cub. ft.,

v_{so} =vol. of steam entering the condenser in cub. ft. per sec.,

v_{ao} =vol. of air entering the condenser in cub. ft. per sec.,

v_s =vol. of steam passing through any row of tubes in cub. ft. per sec.,

v_a =vol. of air passing through any row of tubes in cub. ft. per sec.,

v =velocity of exhaust vapour passing through any row of tubes in ft. per sec.

Then in order that all the tubes in the condenser may be equally efficient, it will be necessary that the following relations hold for any row of tube. (m -th row, say.)

$$v_s = (N - \sum_{q=0}^{q=m-1} n_q) l \pi d \lambda \sigma, \dots \dots \dots (15)$$

$$v_s + v_a = n_{m-1} l (p - d) v \dots \dots \dots (16)$$

(16) - (15)

$$v_a = n_{m-1} l (p - d) v - (N - \sum_{q=0}^{q=m-1} n_q) l \pi d \lambda \sigma,$$

or
$$n_{m-1} \{ l \pi d \lambda \sigma - l (p - d) v \} = v_a + (N - \sum_{q=0}^{q=m-1} n_q) l \pi d \lambda \sigma \dots \dots \dots (17)$$

But l , d , and p =const. in one and the same condenser. Let us further assume λ and σ =const.

Then remembering that the air is non-condensable, accordingly $v_a = v_{ao}$ =const., and putting the condition v =const. in (17), we get

$$n_{m-1} \cdot A = v_{ao} + (N - \sum_{q=0}^{q=m-1} n_q) B, \dots \dots \dots (18)$$

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where

$$A = l\pi d\lambda\sigma + l(p-d)v \dots\dots\dots(19)$$

$$B = l\pi d\lambda\sigma \dots\dots\dots(20)$$

Subtracting the equation for the m -th row from that for the $(m+1)$ th row, we have

$$(n_m - n_{m-1})A = -n_{m-1}B,$$

$$n_m = n_{m-1} \left(1 - \frac{B}{A}\right).$$

Similarly

$$n_{m-1} = n_{m-2} \left(1 - \frac{B}{A}\right), \quad \& \text{ so on.}$$

Hence

$$\begin{aligned} n_m &= n_{m-1} \left(1 - \frac{B}{A}\right) \\ &= n_{m-2} \left(\frac{B}{A}\right)^2 \\ &\dots\dots\dots \\ &= n \left(1 - \frac{B}{A}\right)^m \dots\dots\dots(21) \end{aligned}$$

Inserting the value of B given by (20) in (18) for the 1st. row of tubes, we obtain

$$A = \frac{v_{ao}}{n_o} + \frac{NB}{n_o} = \frac{v_{ao}}{n_o} + \frac{N}{n_o} l\pi d\lambda\sigma \dots\dots\dots(22)$$

But

$$v_{so} = Nl\pi d\lambda\sigma.$$

Hence

$$l\pi d\lambda\sigma = \frac{v_{so}}{N} \dots\dots\dots(23)$$

(23) in (22) gives

$$A = \frac{v_{ao}}{n_o} + \frac{N}{n_o} \cdot \frac{v_{so}}{N} = \frac{1}{n_o} (v_{ao} + v_{so}) \dots\dots\dots(24)$$

Also from (20) and (22),

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$$B = l\pi d\lambda\sigma = \frac{v_{so}}{N} \dots\dots\dots (25)$$

Hence (21) can now be written thus

$$\begin{aligned} n_m &= n_o \left\{ 1 - \frac{v_{so}}{v_{so} + v_{ao}} \cdot \frac{n_o}{N} \right\}^m \\ &= n_o \left\{ 1 - \frac{1}{1 + \frac{v_{ao}}{v_{so}}} \cdot \frac{n_o}{N} \right\}^m \dots\dots\dots (26) \end{aligned}$$

Put

$$\frac{v_{ao}}{v_{so}} = \mu, \dots\dots\dots (27)$$

and we finally obtain

$$n_m = n_o \left\{ 1 - \frac{1}{1 + \mu} \cdot \frac{n_o}{N} \right\}^m \dots\dots\dots (28)$$

in which n_m = no. of tubes in the $(m+1)$ th row,

n_o = „ „ 1st row,

N = total no. of tubes in the condenser,

μ = vol. ratio of air to steam when entering the condenser.

Suppose the tubes are to be arranged in symmetry with respect to the vertical centre line of the condenser. Then if we take the centre line of the condenser as the axis of x , and the horizontal line passing through the centres of the tubes in the 1st row as the axis of y , and if we take the vertical pitch as the unit of length, then the locus of the centres of the outermost tubes in each row will be represented by

$$y = y_o \left\{ 1 - \frac{1}{1 + \mu} \cdot \frac{2y_o}{N} \right\}^{\frac{\sqrt{3}}{2}x} \dots\dots\dots (29)$$

in which y_o = half the width of the 1st row; in absolute value,

this will be equal to $\frac{1}{\sqrt{3}} \times$ no. of tubes in the 1st row,

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$\frac{\sqrt{3}}{2}$ = usual ratio of the horizontal to the vertical pitch.

This equation represents a curve which is convex toward the axis of x .

We are thus led to a very interesting conclusion:—The curve of the periphery of the section of the ideal uniflx condenser should be inside convex instead of being outside convex as is in Weir's Uniflux condenser.

Now we will proceed to find the value of μ which occurs in practice. Thomas C. McBride says that 4.0 volumes of air in 10,000 volumes of steam represents the upper limit of the proportion of air to exhaust steam anticipated by American practice in air pump guarantees.

Hence put

$$\mu = \frac{4}{10,000},$$

then since

$$\frac{1}{1 + \frac{1}{10,000}} = 1 \quad \text{practically,}$$

(28) and (29) can be reduced to

$$n_m = n_o \left(1 - \frac{n_o}{N}\right)^m \dots \dots \dots (30)$$

$$y = y_o \left(1 - \frac{2y_o}{N}\right)^{\frac{\sqrt{3}}{2}x} \dots \dots \dots (31)$$

So far we assumed the volume of water formed by the condensation to be nil. This may be approximately true at the top portion of the condenser where the volume ratio of steam to water is enormously great. But as we approach the lower end, the volume of water becomes not negligible, and in order to keep the flow of steam uniform throughout the condenser, it will be neces-

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sary in practice to allow some more passage for the steam flow than the amount calculated by the foregoing formula. This, however, admits of no calculation, because the velocity, with which the water would fall down, is quite unknown.

Moreover, in the above, we assumed that the specific volume of steam is constant throughout the condenser. But in actual condenser, as there is more or less temperature difference between the top and the bottom being unavoidable, the change of the specific volume of steam may be not inconsiderable. It becomes therefore more and more necessary that the passage for steam should be made larger than that calculated by the foregoing equation, or that the condenser should be made much wider in its cross-section at the lower part than according to the same equation. But in any way the curve of the periphery would remain inside convex, because the amount of the foregoing correction must increase in such a way that the change of increase will be greater as we approach the bottom.

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—THE END—