Simultaneous Structure-Control Optimization of a Steering Wheel

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This paper presents both simulation and experimental investigations into the simultaneous design of a steering wheel and its control system for vibration suppression. The research can be divided into two stages. In the first stage, the simultaneous design is formulated as a nonlinear programming problem in which the \( \mathcal{H}_\infty \) norm of the closed-loop system is minimized with respect to both the structural and control parameters. For convenience of using existing control design tools, a nested approach in which only the structural parameters are treated as explicit design variables is used to solve the original problem. The design obtained by simultaneous optimization is compared with the original structure and the effectiveness of the simultaneous optimization is shown by simulation. In the second stage, experiments are performed to evaluate the design of simultaneous optimization. To consider the effects of modeling error and system uncertainty, \( \mu \) synthesis instead of \( \mathcal{H}_\infty \) control is adopted in the experimental evaluation. The experimental result shows that the optimized structure is still superior to the original one after the controllers are redesigned in the framework of \( \mu \) synthesis.

Key Words: Simultaneous Optimization, Vibration Control, \( \mu \) Synthesis

1. Introduction

In traditional design practice of an actively controlled structural system, the structure and its controller are designed separately. Firstly, the structure is optimized and the mathematical model of the final structure is simplified to form the plant for control design. Then, the optimal control system is synthesized for the given plant. Such method cannot yield the overall optimal design, since it is possible to further enhance the performance by simultaneous structure-control optimization which takes advantage of interactions between the two subsystems. Due to more stringent performance requirements in advanced applications, simultaneous optimization has been receiving increasing attention from researchers\(^1\)\(^2\)\(^3\).

In this paper, an actively controlled steering wheel is adopted as the object of studying structure-control simultaneous optimization. The study consists of two stages: the design stage and the experimental stage.

In the design stage, for simplicity, standard \( \mathcal{H}_\infty \) control is used to control the vibration of the wheel caused by environmental disturbance. Several geometric parameters of the wheel are chosen as structural design variables and the simultaneous design problem is posed as a constrained nonlinear programming in which the closed-loop \( \mathcal{H}_\infty \) norm is minimized over the structural and control parameters. To solve the original problem, a nested approach in which only the structural parameters are treated as explicit design variables is used. The simulation result shows that the simultaneously optimized system has smaller vibration level and needs less control effort than the traditionally designed system.

In the experimental stage, to consider the effects of the uncertainties in plant modeling, sensing and actuation, the control systems of the optimized and the original structures are redesigned using \( \mu \) synthesis. Although the control law is changed from \( \mathcal{H}_\infty \) into \( \mu \), the experimental results show that compared with the original system, the optimized one needs less convergence time, requires smaller steady-state control input, and has lower steady-state vibration level.

2. Simultaneous Design of the Steering Wheel

2.1 General Formulation of Simultaneous Optimization

Generally, simultaneous optimization of a structure-control system can be posed as a nonlinear programming problem in which a certain objective function \( \mathcal{F} \) is minimized over the structural parameters \( p_s \) and control parameters \( p_c \):

\[
\min_{p_s, p_c} \mathcal{F}(p_s, p_c)
\]

It is clear that Eq. (1) has the same solution with the following one

\[
\min_{p_s} \min_{p_c} \mathcal{F}(p_s, p_c)
\]

Thus, the simultaneous optimization can be transformed to a structure optimization nested with control optimizations as sub-processes.

\[
\min_{p_c} \mathcal{F}(p_c)
\]

where

\[
\mathcal{F}(p_c) \triangleq \min_{p_s} \mathcal{F}(p_s, p_c)
\]

develops the control optimization using a certain control law. In such nested approach, since the structural and control design variables are treated separately and the control design is well encapsulated, various existing structural and control design techniques can be employed without any difficulty.

Moreover, some constraints, such as lower and upper bounds on the design parameters \( p_i^s \) and \( p_i^u \), and other performance requirements \( \{f_i\} \), can be imposed on the optimization

\[
\min_{p_c} \mathcal{F}(p_c)
\]

s.t. \( p_i^s \leq p_i \leq p_i^u \)

\[
f_i(p_c) \leq 0 \quad (i = 1, 2, 3, \ldots, n_f)
\]

2.2 Structural Modeling and Structural Variables

The configuration of the steering wheel is shown in Fig. 1 where the absence in the right-side spokes is used to indicate the locations of bonded piezoelectric actuators. The whole frame consists of a ring, four spokes and a boss plate. As shown in Fig. 2, eight geometric parameters of the spokes are chosen as design parameters

\[
p_s = [l_1, l_2, h_1, h_2, \ldots, h_7]^T
\]

\[
p_i^s = [3, 2.5, 5, 5, 5, 5, 5, 5]^T
\]

\[
p_i^u = [7, 6.5, 15, 15, 15, 15, 15, 15]^T
\]
where \( t_1 \) and \( t_2 \) are the thickness parameters of the front and rear spokes respectively. The four spokes and the output force of the actuators are supposed to be symmetric about the YZ plane.

By finite element analysis and neglecting the high-order modes, the following state space model can be obtained

\[
\begin{align*}
&x = Ax + Bu + B_wu \\
&z = C_1x \\
&y = C_2x
\end{align*}
\]

(7)

where \( A, B_a, B_w, C_1, \) and \( C_2 \) are system matrices, and \( x, y, z, w, u, \) and \( \nu \) are state variable, performance output, sensor output, external disturbance and control input. As shown in Figs. 1 and 3, \( w \) is the \( Y \)-direction ground displacement, \( y = Y_1 \), i.e., the \( Y \)-direction displacement of node 1; \( \nu = [\nu_1, \nu_2]^T \) consists of the front and rear control moments produced by the bonded piezoelectric actuators; \( z = [X_1, y, Z_1]^T \) where \( X_1 \) and \( Z_1 \) are the \( X \)- and \( Z \)-direction displacements of nodes 3 and 4.

In addition, for simplicity, the properties of the actuators is assumed to be linear, i.e., \( u = k_vw \), where \( v_i \) is the applied voltage and \( k \) is a constant which can be determined by the geometrical and material properties of the piezoelectric actuators and the steering wheel.

2.3 Control Design and Simultaneous Optimization

In the nested approach to simultaneous optimization discussed above, the controller will be redesigned wherever the structure is modified. Thus efficient and direct control design method is important. For its simplicity, standard \( H_\infty \) control is used in the following simultaneous optimization

\[
\begin{align*}
&\min \{ J_\infty(p, p_i) \} \\
&\text{s.t. } p_i^l \leq p_i \leq p_i^u \\
&f_i(p_i) \leq 0 \quad (i=1,2,3,\ldots,n_i)
\end{align*}
\]

(8)

where \( J_\infty \) is the closed-loop transfer function from \( w \) to \( z \) (see Fig. 4), and \( f_i \) is the vector of some static stiffness requirements on the structure. The details of \( f_i \) are omitted here.

Furthermore, to reduce the control effort at the same time, the performance output \( z \) is replaced by \( z_\nu \) and the objective function in Eq. (8) is changed to the \( H_\infty \) norm of closed-loop transfer function from \( w \) to \( z_\nu \)

\[
z_\nu = [z^T, au]^T
\]

(9)

where \( \alpha \) is a positive scalar and a smaller \( \alpha \) means better vibration suppression but higher control effort.

2.4 Simulation Results of Simultaneous Optimization

For the simultaneous optimization problem defined in Eq. (8), the sequential quadratic programming method and the Riccati-based \( H_\infty \) design tool are used to optimize the structural and control parameters respectively. Under certain termination criteria, the simultaneous optimization terminated after 299 times of objective evaluations. The convergence profile of the objective function is shown in Fig. 5.

The results of the \( H_\infty \) simultaneous design are shown in Table 1, in which the closed-loop \( H_\infty \) norms and structural variables of the original and the optimized structures are compared. It can be seen from the table that a significant (about 75%) decrease of the objective function is achieved by the simultaneous design. When considered separately, the displacement output norm and the control input norm are reduced to about 43% and 26% of the original values respectively. As listed in the table, most structural parameters (except \( t_2 \) and \( h_2 \)) of the optimized wheel reach the lower bounds on them (see Eq. (6)). Thus the optimized structure is more flexible than the original one. The considerable decrease of the necessary control moment can be regarded as the consequence of the stiffness change in the spokes. Since one parameters \( h_3 \) is equal to its upper bound after optimization, we cannot conclude that desirable design will be obtained by simply reducing the values of the structural variables to their lower bounds. Slender spokes result in small control moments but do not mean that good vibration suppression can be achieved. The shape of the optimized wheel is sketched in Fig. 7.

The closed-loop frequency response functions of the original and optimized systems are compared in Fig. 6. It can be seen from the figure that the \( H_\infty \) performance of both displacement output and control inputs are improved considerably. Since the comparison is made between the two closed-loop systems, the improvement due to the simultaneous optimization should not be considered as small.

3. Experimental Evaluation

In the above section, the steering wheel and its \( H_\infty \) controller are optimized simultaneously and as the result, the optimal structure (in the sense of having minimum \( H_\infty \) closed-loop norm) is obtained. In the optimization, no consideration has been given to the effects of modeling errors and sensing and actuation uncertainties. However, these factors usually cannot be neglected if a control system is to be realized in a real structural system. Since \( \mu \) synthesis is a systematic robust control method to analysis and quantify the effects of uncertainty, in this section, original and optimized design will be evaluated in the framework of \( \mu \).

3.1 Refinement of the structural models

Specimens of the original and the optimized steering wheels are manufactured and simple vibration test is carried out. Before the control design using \( \mu \) synthesis, the finite element models for the two structures are modified according to the test results. Several parameters which determine the boundary conditions are used as parameters for identification in which the difference between the computed and the experimental natural frequencies are minimized. The results are shown in Table 2. It can be seen from the table that the first six frequencies obtained by FEM agree with the experimental data very well after the identification.

3.2 Redesign of the Controller Using \( \mu \) Synthesis

The refined open-loop plant \( S \) is augmented as shown in Fig. 8(a) for \( \mu \) synthesis. The sensor noise signal is denoted by \( d \) and added to the displacement \( y \) to form a polluted sensor output \( y' \). The white noise signals \( w' \) and \( d' \) are shaped by corresponding weighting filters \( W_{ds} \) and \( W_{rn} \) to produce the realistic external disturbance \( w \) and sensor noise \( d \). The performance output \( z \) is also shaped by \( W_{r} \) to meet different design specifications at different frequencies. The output of actuators are denoted by \( u' \). Moreover, the uncertainty of the whole system is represented by an uncertainty block \( \Delta \) filtered by \( W_{\Delta} \). The four weighting functions except \( W_{ds}(s) \) are shown in Fig. 9.

The details of \( \mu \) synthesis will not be discussed here. However, for our design program, some necessary information is given briefly. By introducing a fictitious block \( \Delta_{\mu} \) as shown in Fig. 8(b), the following block-diagonal complex uncertainty matrix can be obtained

\[
\Delta = \text{diag}(\Delta_{\mu_1}, \Delta_{\mu_2})
\]

\[
\Delta_{\mu} \in C^{n_\mu}, \quad \Delta_{\mu} \in C^{n_\mu}
\]

(10)

The objective of \( \mu \) design can be expressed as follows (see Eq. 8)

\[
\min \max_{C} \mu_{\mu}(P(j\omega, C))
\]

(11)

where \( \mu_{\mu}(P) \) is defined as

\[
\mu_{\mu}(P) = \frac{1}{\min_{\Delta}(\sigma(\Delta)); \det(I - P\Delta) = 0)
\]

(12)

unless no \( \Delta \) makes \((I-P\Delta)\) singular, in which case \( \mu_{\mu}(P)=0 \). The optimization defined in Eq. (11) is solved approximately by the “D-K” iteration method.

The histories of “D-K” iteration are shown in Table 3. After 5 and 2 iterations, the peak \( \mu \) values of the original and the optimized systems reached their minima of 6.131 and 4.684 respectively. The theory of robust performance says that if Eq. (11) is equal to \( \mu_{\mu} \) for all stable and rational transfer function \( \Delta_{\mu}(s) \)
which satisfies \( |\Delta_{\text{opt}}| \leq 1/\mu_{\text{opt}} \), the \( H_\infty \) norm of the perturbed closed-loop system \( P_\text{c} \) (see Fig. 8(b)) is guaranteed to be less than or equal to \( \mu_{\text{opt}} \). Hence, the improvement of \( \mu_{\text{opt}} \) has two benefits that the allowed uncertainty size is increased and at the same time, the degradation of the closed-loop performance due to uncertainty is decreased. The final frequency-dependent \( \mu \) values are shown in Fig. 10. It can be seen from the figure that the optimized structure has smaller \( \mu \) values than the original one at most frequencies, especially around 70Hz. The characteristics of the redesigned \( \mu \) controllers are compared in Fig. 11. It can be found that at most frequencies, the optimized structure needs smaller control gain than the original.

### 3.3 Experimental Results

Experiments are carried out to evaluate the closed-loop performance of the original and the optimized steering wheels with their \( \mu \) controllers designed in the above section. The control system consists of a gap sensor, a two-channel amplifier, piezoelectric actuator groups, a master PC, and DSP board system. The digital controller has 12 states and a sampling frequency of 10kHz.

The open-loop and closed-loop frequency responses from the disturbance \( w \) to the displacement output \( y \) are shown in Fig. 12, where the vertical dotted lines indicate the natural frequencies. As shown in the figure, with control, the optimized structure has smaller resonant peaks than the original. The magnitudes of the resonant peaks are listed in Table 4. It can be seen that further considerable reductions of the vibration level have been achieved for all three peaks by the simultaneous optimization.

Figure 13 shows the time responses at the resonant frequencies for both the original and the optimized systems. It can be found from the figure that the vibrations are successfully suppressed after the control started. Moreover, compared with the original system, the optimized needs less convergence time, requires smaller steady-state control input, and has lower steady-state vibration level.

### 4. Conclusion

Simultaneous structure-control optimization, including experimental investigation, is applied to the steering wheel for vibration suppression under environmental disturbance.

The simultaneous design problem is formulated as a constrained nonlinear programming problem, and solved using the nested approach in which the structural parameters are treated in the main optimization and the control parameters are designed in the nested sub-process of control optimization. For reducing computational cost, relatively simple standard \( H_\infty \) control is adopted in the simultaneous optimization. While in the experiment, to consider the modeling error and system uncertainty and improve the robustness of the closed-loop system, the controller is redesigned using \( \mu \) synthesis. Although different control laws are used in the design and experimental stages, large performance enhancement due to the simultaneous optimization is demonstrated by both the simulation and experiment results. After the simultaneous optimization, the \( H_\infty \) norms of the closed-loop displacement response obtained by simulation and experiment are reduced about 57% and 45% respectively. The merit of the simultaneous optimization and the effectiveness of the proposed approach are verified.

### References

Table 2 Result of identification

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<th>No.</th>
<th>Exp.</th>
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Optimized structure

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<td>6</td>
<td>239.2</td>
<td>239.37</td>
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Fig. 7 Optimized structure

Fig. 8 Blocks diagrams for \( \mu \) design

Table 3 History of \( D-K \) iteration

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<tr>
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<td>6.131</td>
<td></td>
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<tr>
<td>6</td>
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</table>

Fig. 9 Weighting functions

Fig. 10 \( \mu \) values

(a) For original structure
(b) For optimized structure

Fig. 11 Frequency response of \( \mu \) controllers

Fig. 12 Experimental frequency response from \( w \) to \( y \)

(a) Original structure
(b) Optimized structure

Table 4 Peak closed-loop response

<table>
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<tbody>
<tr>
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<tr>
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<td>3</td>
<td>10.34</td>
<td>5.90</td>
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</table>

(a) Orig. 81.6Hz
(b) Opt. 72.6Hz
(c) Orig. 141.6Hz
(d) Opt. 120.1Hz
(e) Orig. 238.4Hz
(f) Opt. 238.9Hz

Fig. 13 Time response of the original and optimized systems