A bio-inspired bistable flapping thrust mechanism with flexible suspension

R.L. Harne* and K.W. Wang
Department of Mechanical Engineering, University of Michigan
Ann Arbor, Michigan 48109-2125, USA
*Corresponding author: rhame@umich.edu

Abstract
The nonlinear oscillating wing stroke of diptera has been attributed to a pre-compressed flight motor anatomy. To date, biological evidence has demonstrated the nonlinearity and mathematical evaluations of simplified mechanical wing motor models link empirical observations to characteristic bistable oscillator dynamics. It was concluded that the fast switching between two stable wing orientations leads to high velocity movements, capable of large aerodynamic force generation which is more efficient than sinusoidal motions. While interesting and informative, the investigations have yet to address why the pre-compressed motor anatomy is well-suited to produce flapping aerodynamic forces for various flight conditions. This research seeks to explore the benefits of the dipteran wing motor through a representative structural model to uncover motor components and configurations important for flapping force generation. With such a study, improved design solutions for flapping mechanisms may be proposed that could find application in micro air vehicles. Analytical and experimental investigations suggest that the combination of adjustable axial suspension of the motor and its degree of pre-compression critically regulate the wing motor motions. While bistable motor configurations are shown to produce the most energetic dynamics, the vast range of velocity amplitudes obtained from a variably pre-compressed and stiffened motor exemplify a substantial versatility of flapping force generation is available for advanced flight control. The results suggest that micro air vehicles adopting a pre-compressed wing motor with axial flexure may greatly benefit in incorporating control over axial pre-load and suspension characteristics to enhance the range of flapping forces achievable.

Keywords: Diptera flight mechanism; Pre-compressed beam; Flapping wing; Aerodynamic force

1 Introduction and motivation
Recent interest in micro air vehicles, often for highly maneuverable reconnaissance or surveillance purposes, has fostered a breadth of research (Shy et al., 1999; Pines and Bohorquez, 2006). The technical challenges on this scale make it difficult for fixed wing aircraft to meet performance objectives without excessive weight. In contrast, the flapping of wings is a biologically proven way to improve flight capability as the scale of the flying system reduces (Mueller and Delaurier, 2001). Thus, research attention has turned to opportunities inspired by the principles of flapping flight. Insect flight mechanisms have evolved over longer periods of time than those of birds, in addition to the more relevant scales involved, there is thus strong rationale for the focused research interest in insect wing flapping mechanisms for micro air vehicle development (Ellington, 1999).

Diptera are recognized to possess unique wing motor motions potentially based upon a bistable snap through action as evidenced by various biological data and a structural assessment of wing motor anatomy, material properties, and configuration (Boettiger and Furspan, 1952; Thomson and Thompson, 1977). Figure 1(a) highlights the approximate location of the wing motor on diptera while 1(b) illustrates the wing motor anatomy and corresponding flapping action, as redrawn following Thomson and Thompson (1977). Due to pre-compression provided by axial suspension elements, the motor may exhibit two positions of stable equilibria such that the wings are naturally angled above or below a medial plane. A driving excitation sufficiently strains the axial suspension—a nodum lever at right and the pleurosternal muscle at left—to allow the motor to break through an unstable medial configuration, thereafter propelling the wing to the alternate stable position. A half-cycle of flapping action is illustrated in Fig. 1(b) and proceeds following (1)-(3);
the motions then continue in reverse for the second half-cycle, and so on. While the use of motor pre-compression is thoroughly established, there is some historical controversy in the biological community as to the existence of a permanently buckled wing motor configuration. But, as Dickinson and Tu (1997) comprehensively detail, the wing motor is an active system and if a permanently buckled structure is not the means behind the powerful flapping behaviors, then there is substantial evidence that (using an automotive metaphor) "the transfer of energy from the engine to the wheels [is] regulated by the configuration of the transmission." In other words, the evidence strongly indicates that highly energetic, snap-through-like flapping motions are produced by pre-compression of the motor, whether induced as a result of permanent geometric configuration or by active adjustment of the wing motor through the control muscles.

![Diagram of wing motor and extension](image)

---

**Fig. 1.** (a) Location of dipteran wing motor is denoted by red circle. (b) Illustrative dipteran wing motor schematic, as redrawn from Thomson and Thompson (1977). The dipteran flexure is composed of flexing element on right side and contractive muscle on left side. (c) Representative structural realization of the wing motor and wing extension, highlighting comparable elements by the background shading with respect to (b). (d) Comparable simply supported buckled beam with end-mounted spring used in this study.

Following the biological investigations, the apparent bistable wing motor has become recent focus of attention in the engineering community where insights for micro air vehicle development are pertinent. Theoretical and experimental findings strongly support energy-enhancing features of the bistable wing motor for flapping flight. Brennan et al. (2003) and Tang and Brennan (2011) found that for many operating conditions the efficiency of such a flapping mechanism was greater than for linear wing motors. A micro air vehicle using a bistable wing motor was constructed by Chin and Lau (2013) that demonstrated improved thrust-to-input-power efficiency as compared to a linear counterpart. However, these findings alone do not immediately justify adoption of a bistable wing motor design to enhance performance for future micro air vehicles under various flight conditions. Therefore, the aim of this study is to probe more deeply into the structural composition of the dipteran wing motor to uncover the elements that ensure persistent generation of flapping aerodynamic forces of various types, which may then serve as design guidelines for improved micro air vehicle development.

In the previous investigations, a structural element that was not explored is the axial suspension that supports the wing motor. As shown in Fig. 1(b), the suspension is composed of a flexing notum lever supporting one side of the motor.
while the pleurosternal muscle supports the opposing side; in this way, the suspension is active by way of the pleurosternal muscle. These components are likewise critical to the pre-compression of the motor since their pre-strained configuration contributes to the shape of the thorax that governs the motor axial pre-load (Williams and Williams, 1943). The motivation to investigate the axial suspension is the comparable evidence in structural dynamics that axial springs can dramatically alter static and dynamic buckling characteristics of pre-compressed beams (Mayers and Wrenn, 1964; Virgin, 2007). In the prior studies of the dipteran wing motor, a lumped parameter model was formulated which did not include lateral motions of the system related to axial flexure (Brennan et al., 2003; Tang and Brennan, 2011); and the experimental study did not evaluate the influence of suspension elements as relate to generating aerodynamic forces (Chin and Lau, 2013). Moreover, the biological evidence indicates that bending and torsional stiffnesses of wing motor elements are not trivial (Dickinson and Tu, 1997) and thus lumped parameter modeling or prototype development may not suitably replicate the biological system. Additionally, bistable structures are strongly nonlinear and may exhibit aperiodic or chaotic behaviors which are not observed in biological data of dipteran wing motions; thus, an indiscrete treatment of a bistable wing motor model can lead to predictions lacking physical meaning. Therefore, in the present work, a distributed parameter model formulation and corresponding experimental system are utilized, and focus is directed to the influences of axial suspension and its pre-load adjustment on the generation of useful flapping motions for flight.

To probe the composition and dynamics of the dipteran wing motor more closely for design cues beneficial for micro air vehicle development, the following sections describe the comparable structural model of the wing motor and the investigation of the critical influence of the axial suspension using both analytical and experimental results.

2 Dynamic wing motor model

2.1 Derivation of governing equation

The dipteran wing motor anatomy is shown in Fig. 1(b). The comparable structural system studied here is illustrated in Fig. 1(c) showing a simply supported beam having a wing extension on one side and axial spring pre-compression on the alternate side. Biologically, the pleurosternal muscle and notum contribute to the axial stiffness; the pre-compression force is determined from combination of the thorax and motor geometries and the pleurosternal muscle pre-tension. In the structural system, the two axial suspension components are combined to a single influence via a linear spring of stiffness $k_{\Delta}$ which is pre-compressed by a distance $\Delta$. For completeness, it is be recognized that a still greater number of factors influence the dipteran wing motor axial stiffness because the system is immersed amongst a variety of muscles and biological sub-systems (Williams and Williams, 1943; Dickinson and Tu, 1997), but past research has singled out the primary axial stiffness influences to be related to the pleurosternal muscle and notum elements (Thomson and Thompson, 1977) which will be the perspective adopted in this study. The wing dynamics influence the motor through a dissipative aerodynamic force proportional to the square of wing angular velocity (Ellington, 1984b; 1999), and by increasing the net inertia. As a result, the structural model may be reduced to that shown in Fig. 1(d), where the wing extension is neglected and its influence is accounted for by inclusion of nonlinear damping force (not shown in sketch) and rotational inertia of mass $2m_a$ that is distributed at the two simple supports. The beam is excited by harmonic motions of its surrounding frame $a(t) = A_x \cos \Omega t$, where $A_x$ is acceleration amplitude and $\Omega$ is excitation frequency, which is equivalent to a distributed modal force as derived using the present model formulation. Fig. 1(d) illustrates a half-cycle of the structural system fundamental buckling mode dynamics from (1)-(3) as compared to those of the biological system in 1(b).

Considering the beam transverse $w(x, t)$ and axial $u(x, t)$ displacements are functions of the beam length coordinate $x$ and time $t$, the kinetic energy $T$, the potential energy $U$, and the dissipation function $D$ may be expressed as (Mayers and Wrenn, 1964; Virgin, 2007)

$$T = \frac{1}{2} \rho_b A_b \int_0^L (\dot{w} + \dot{u})^2 dx + \frac{1}{2} \left( \frac{1}{2} m_a x_a^2 \right) \left( \frac{\partial w}{\partial x} \right)^2 \bigg|_{x=0} + \frac{1}{2} \left( \frac{1}{2} m_a x_a^2 \right) \left( \frac{\partial w}{\partial x} \right)^2 \bigg|_{x=L_a}$$  \hspace{1cm} (1)
\[ U = \frac{1}{2} E_b A_b \int_0^L \left[ u_x + \frac{1}{2} w_x^2 \right] dx + \frac{1}{2} E_b I_b \int_0^L \left[ w_{xx}^2 + w_{xx} w_x^2 \right] dx + \frac{1}{2} k_d u_x^2 |_{x=L_b} + k_d \int_0^{L_b} u_x dx \]  

\[ D = \frac{1}{2} c_b \int_0^L w_x^2 dx \]  

The operators (') and (\cdot)\_x indicate differentiation with respect to time and beam length coordinate, respectively. Other parameters are as follows, where subscript \( b \) indicates the term is related to the beam: volumetric density \( \rho_b \); area \( A_b \); natural beam length \( L_b \); rotational mass radius \( r_o \); Young’s modulus \( E_b \); moment of inertia \( I_b \); spring pre-compression distance \( \Delta \); viscous damping per beam length \( c_b \). Inclusion of aerodynamic dissipation on beam motions is not provided in Eq. (3) because aerodynamic effects are related to the wing extension dynamics (Ansari et al., 2006) which are not specifically accounted for here; instead, aerodynamic damping force will be introduced following reduction of the continuum model below. Although, like many other periodic biological oscillations, dipteran wing motions may exhibit asymmetry (Thomson and Thompson, 1977; Ellington, 1984a), for sake of simplifying the investigation to carefully evaluate the principal roles of the axial flexure, asymmetries related to the structure itself are here neglected.

The forced, periodic response of the beam, i.e., the wing motor, is primarily harmonic at the same frequency of the excitation and is assumed to be proportional to the fundamental mode of a simply supported beam by the Ritz method,

\[ u(x,t) = f_x(t)x + f_z(t) \sin[2\pi x/L_b] \]  

\[ w(x,t) = \hat{g}(t) \sin[\pi x/L_b] \]  

where \( \mathbf{q} = [f_x, f_z, \hat{g}]^\mathbf{T} \) are generalized coordinates of the axial and transverse responses. Note that \( \hat{g} \) is related to the amplitude of wing tip motions through a moment arm expression and is therefore related to aerodynamic forces (Ellington, 1984b). Following substitution of Eqs. (4, 5) into (1-3) and employing Lagrange’s equations

\[ \frac{\partial}{\partial t} \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathbf{L}}{\partial \mathbf{q}} = 0 \]  

where \( \mathbf{L} = T - U \), leads to direct determination of coefficients \( \mathbf{f}_1, \mathbf{f}_2 \)

\[ \mathbf{f}_1 = - \left( \frac{k_d A_b}{E_b A_b} \right) \frac{1}{k + 1} + \left( \frac{\pi}{L_b} \right)^2 \frac{1}{k + 1} \hat{g}^2 \]  

\[ \mathbf{f}_2 = - \frac{1}{\left( \frac{\pi}{L_b} \right)^2} \hat{g}^2 \]  

where the axial spring stiffness ratio is \( \kappa = k_d L_b / E_b A_b \) which is the ratio of the axial spring to beam axial stiffnesses. Nondimensional motor displacement \( g = \hat{g}/L_b \) and time \( \tau = \omega_0 t \) are introduced where \( \omega_0 \) is the uncompress beam fundamental natural frequency expressed in Eq. (A1). Continuing application of Lagrange’s equations to coordinate \( \hat{g} \) and further simplification leads to the governing equation for the transverse beam response:

\[ g'' + \eta g' + \left[ 1 - \frac{k_d A_b}{E_b A_b} \right] g + \left[ \beta + \gamma / (k + 1) \right] g^3 = z \cos \omega \tau \]  

where key terms include: \( P_{cr} \), critical buckling load; \( z \), nondimensional excitation level; \( \omega \), nondimensional excitation frequency. The operator \( (\cdot)' \) indicates differentiation with respect to nondimensional time \( \tau \). The remaining terms and further details for the prior are given in the Appendix. When assuming small deflections and slopes (\( \beta \to 0 \)), note that the case of no axial spring (i.e., \( k_d \) and thus \( \kappa \) both \( \to 0 \)) reduces Eq. (9) to satisfy classical linear beam response. The limiting case therefore indicates the beam is free to move axially. This point is interesting because free axial beam motion is “inherent in the derivation of Timoshenko beam theory” (Mayers and Wrenn, 1964) although such boundary conditions are difficult to accurately realize in experiments for axially compressed beams having doubly clamped or simply supported ends.
The final derivation step for the present study is to introduce a nonlinear damping term to account for aerodynamic effects:

$$g'' + \eta g' + \eta_a g |g'| + \left[ 1 - \frac{k_d \Delta}{P_{cr}} \right] g + \left[ \beta + \gamma \frac{\kappa}{\kappa + 1} \right] g^3 = \omega \cos \omega t$$

(10)

where $\eta_a$ is an aerodynamic dissipation factor (Malatkar and Nayfeh, 2003).

The nonlinear governing equation for the wing motor, Eq. (10), deserves attention prior to analytically or numerically exploring its characteristics. The term $k_d \Delta/P_{cr}$ is the axial pre-load ratio, defined as the ratio of pre-loading force to the fundamental critical buckling load. Should $k_d \Delta/P_{cr} > 1$, the linear stiffness term may become negative, giving rise to two positions of stable equilibria (Mayers and Wrenn, 1964; Virgin, 2007). However, one effect of the axial spring is to increase the axial load supportable by the beam prior to buckling: in other words, even if $k_d \Delta/P_{cr} > 1$, the multiplicative factor $1/(1 + \kappa)$ may prevent negative values of the linear stiffness, although it is still greatly reduced.

The axial spring therefore stabilizes the beam. However, it is also seen that the cubic nonlinear influences related to the axial spring may then strongly influence system response since linear stiffness is dramatically reduced. In the dipteran wing motor, the spring stiffness $k_d$ and compression distance $\Delta$ are actively adjusted by the pleurosternal muscle and thus a biological control determines under what conditions the wing motor exhibits bistability, which occurs when $k_d \Delta/[P_{cr}(1 + \kappa)] > 1$.

Lastly, the stable configurations of the wing motor are found by determining the fixed points of Eq. (10):

$$g = 0 \text{ and } g = \pm \sqrt{\frac{k_d \Delta}{P_{cr}} - 1}$$

(11)

Because $\beta$, $\gamma$, and $\kappa$ are non-negative, when the pre-load ratio ($k_d \Delta/P_{cr} - \kappa > 1$) the system is monostable and only has an equilibrium at $g = 0$. However, should ($k_d \Delta/P_{cr} - \kappa < 1$), the system will exhibit two stable configurations positioned symmetrically about an unstable equilibrium at $g = 0$.

2.2 Analytical solution method for predicting wing motor response

In this study, Eq. (10) is solved via approximate analytical means. For harmonic excitation of the motor, a suitable approximation of the response $g$ is a Fourier series expansion (Virgin, 2007). Given that wing motions are primarily sinusoidal with respect to the frequency of excitation (single periodic motions), a one-term Fourier expansion may be employed:

$$g(t) = g_0(t) + g_1(t) \cos(\omega t + \psi(t))$$

(12)

Substitution of Eq. (12) into (10), eliminating higher-order harmonic terms, and assuming slowly-varying coefficients leads to an equation system given by

$$-\eta g_0' = A g_0 + B g_0 \left[ \frac{3}{2} g_1^2 \right]$$

(13)

$$-\eta g_1' + 2 \omega g_1 \psi' = -\omega^2 g_1 + A g_1 + B g_1 \left[ 3 \frac{3}{2} g_1^2 + \frac{9}{2} g_1^2 \right] - \omega \cos \psi$$

(14)

$$2 \omega g_1' + \eta g_1 \psi' = -\eta \omega g_1 - \frac{\delta}{g_1^2} \omega^2 g_1^2 - \omega \sin \psi$$

(15)

where $A = 1 - k_d \Delta/[P_{cr}(\kappa + 1)]$, $B = \beta + \gamma \kappa/(1 + \kappa)$, and reduction of the nonlinear damping influences follows that as demonstrated by Ravindra and Mallik (1994). Assuming steady-state response, the constant response bias $g_0$ may be solved in terms of the motion amplitude $g_1$ using Eq. (13). It is then found that either $g_0^2 = 0$ or $g_0^2 = -A/B - 3g_1^2/2$. Depending on the pre-compression of the motor as quantified by the linear stiffness term $A$, one or three response solutions are determined. For example, if $A$ is positive, then the steady-state solution to Eq. (13) is only physically meaningful for the result $g_0^2 = 0$: in other words, the system is monostable because the motor is not buckled.
If the linear stiffness is negative then the motor is bistable and all three equilibria exist $g_0 = 0, \pm [-A/B - 3g_1^2/2]^{-1/2}$, where the case of $g_0 = 0$ is an unstable configuration. Then, substituting $g_0$ into Eqs. (14,15), the two equations may be squared and summed to yield a sixth-order polynomial equation in terms of $g_1$. Having computed the roots of the polynomial, the coefficients $g_0$ and $\psi$ may be determined.

Not all mathematically predicted response coefficient sets $a = [g_0, g_1, \psi]^T$ are stable and physically meaningful. Stability may be checked by considering Eqs. (13-15) expressed in the form $Pa' = Ga$ where $P$ and $G$ are matrices determined accordingly. Given a solution set $a^*$, stable responses are those for which the eigenvalues of the Jacobian $J = [d(P^{-1}G)/da]|_{a=a^*}$ are negative and real (Nayfeh and Mook, 1995).

The stable responses predicted from this approach, for a given system and excitation parameter set, represent the steady-state beam (wing motor) center point displacements. Note that by this solution method, flapping wing forces are proportional to the response through the square of wing velocity amplitude, $|\omega g_1|^2$, and the arm moment between the motor center to wing extension tip (Ellington, 1984b; 1999).

3 Influence of axial suspension characteristics on wing motor velocity and response transitions

According to the governing equation for wing motor responses, Eq. (10), changing axial suspension stiffness and pre-load may influence the system in several ways. The aim of this section is to investigate these dependencies for insight into the maintenance of useful flapping motions for aerodynamic force generation using a nonlinear, axially compressed wing motor. An experimental system, illustrated in Fig. 1(d), is constructed in the laboratory and its identified parameters are provided in Table 1. The axial spring element is achieved using a cantilever beam whose length is adjusted in order to tailor the equivalent linear spring stiffness. Modifying the pre-load force is accomplished by changing the compression distance $\Delta$ via a micrometer. According to the experimental system parameters and measurements, the fundamental natural frequency of the uncompressed structure is approximately 32 Hz. In the following studies, the ratio of excitation frequency to the natural frequency, $\omega = \Omega/\omega_0$, is selected within the range of approximately 0.20 to 0.5. This selection is supported by comprehensive biological evidence that a large variety of insects utilize wingbeat frequencies of comparable frequency ratio range (Ha et al., 2013) and by numerous studies indicating that aerodynamic performance is improved when flapping occurs at frequencies less than structural resonance $\omega < 1$ (Ramananarivo et al., 2011).

### Table 1. Experimental system parameters.

<table>
<thead>
<tr>
<th>Young's modulus $E_o$, N.m$^{-2}$</th>
<th>density $\rho_o$, kg.m$^{-3}$</th>
<th>length $l_b$, mm</th>
<th>width $h_b$, mm</th>
<th>thickness $t_b$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>180×10$^6$</td>
<td>8000</td>
<td>147</td>
<td>12.1</td>
<td>0.457</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>spring stiffness $k_d$, kN.m$^{-1}$</th>
<th>mass $m_{ar}$, g</th>
<th>radius $r$, mm</th>
<th>mechanical loss $\eta$</th>
<th>aerodynamic loss $\eta_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[30.3, 108, 263]</td>
<td>62.5</td>
<td>10</td>
<td>0.07</td>
<td>1.0</td>
</tr>
</tbody>
</table>

A series of experiments were then conducted and, from amongst the results, two excitation frequencies (12 and 16 Hz) using the same pre-load ratio $k_d\Delta/P_e=1.01$ were selected to demonstrate the crucial dependence of axial suspension parameters and excitation level on wing motor velocity amplitudes. The uncompressed natural frequency of the motor is approximately 32 Hz, and thus excitations at 12 and 16 Hz represent normalized excitation frequencies of $\omega \approx 0.37$ and 0.50, respectively. In the following studies, the influences of axial suspension stiffness and excitation level are considered. In the experiments, to minimize the non-stationarity of the excitation and capture a large range of levels, the excitation level is slowly swept up and down at constant excitation frequency. In contrast, analytical results represent stationary and steady-state behaviors. The analytical model results of wing motor velocity amplitude are provided in Figs. 2(a,b) while corresponding experimental data are shown in (c,d). Because of the pre-load ratio and axial spring stiffnesses considered, some of the responses represent monostable and some bistable wing motor configurations. Thus, in Figs. 2(a,b), responses which are symmetric about a central position are indicated by solid...
lines (whether the system is mono- or bistable) while model predictions indicating intrawell response (specific to bistable configurations) are shown as dashed curves. Experimental data points are connected by lines for the sake of visualization in observing where response forms underwent large transitions in amplitude.

In the analytical results of Figs. 2(a,b), the velocity amplitudes for the un compressed motors $k_dA/P_{cr}$=0 are provided as a single curve because the predictions across the range of axial suspension stiffness are nearly identical. It is evident from comparing un compressed to pre-compressed responses that pre-compressed wing motors have the advantage of obtaining much greater range of velocity amplitudes. The pre-loading reduces the contribution of linear stiffness with respect to the nonlinear influences, and yields the low and high amplitude response forms that are characteristic of strongly nonlinear systems. Thus, the axial suspension alone does not produce beneficial performance variability, but the combination of axial suspension and pre-load is needed for dramatic adjustment of wing motor velocity.

![Fig. 2. Comparison of analytical (top row) and experimental (bottom row) results for wing motor velocity for pre-load ratio $k_dA/P_{cr}$=1.01. (a,c) Excitation at $\omega$=0.37 (12 Hz). (b,d) Excitation at $\omega$=0.5 (16 Hz). In (a,b), results for uncompressed motors, $k_dA/P_{cr}$=0, are provided as a single curve because results across the range of axial suspension stiffness are nearly coincident.](image)

As shown throughout Fig. 2, decreasing the spring stiffness $k_d$ while retaining the same pre-compression load ratio yields steadily increasing peak wing motor velocities. Moreover as $k_d$ is reduced, a greater level of excitation is necessary to obtain the symmetric, higher velocity motions beneficial for flapping force generation. It is apparent that for the same driving excitation force, the adjustment of only axial suspension stiffness yields dramatic changes in wing motor velocity and hence aerodynamic force. Analytical and experimental results agree on the many key trends and influences of changing axial suspension, as well as the general quantitative range of velocity amplitudes. Perfect quantitative comparison is not intended in the present study, primarily due to difficulty in exactly identifying experimental system parameters and eliminating asymmetries. Furthermore, in the case of 12 Hz excitation, the experimental system exhibited aperiodic responses for the case of $k$=0.0045 shown by the fluctuating curves in the excitation level range 40 to 70 mN in Fig. 2(c). Nevertheless, the overall match of trends and amplitude ranges between analytical and experimental results in Fig. 2 support the primary influences observed regarding axial suspension.

For each pre-compressed motor configuration considered in Fig. 2, theoretical co-existence of low and high amplitude velocities occurs over a range of excitation level. For levels less than this band, the responses trend towards the lower

© 2014 The Japan Society of Mechanical Engineers
amplitude (and vice versa) and the difference between low and high velocity amplitudes is increased as the suspension stiffness reduces. Depending on the axial spring stiffness with respect to the pre-load ratio, the transition may be either from low to high amplitude monostable responses or from bistable intra- to interwell oscillations. For example, consider Fig. 2(b) where the excitation frequency is 16 Hz. Due to the axial suspension ratios studied here, the configurations $\kappa=0.016$ and 0.039 are monostable while $\kappa=0.0045$ is bistable. Corresponding experimental results of displacement and velocity are shown in Fig. 3 for the configurations $\kappa=0.0045$ and 0.016 at excitation levels $\sim 21$ and $\sim 11$ mN, respectively. The excitation levels are chosen because measurements indicated co-existence of low and high amplitude responses for such levels, as observed in Fig. 2(d). Figure 3(a) shows the time series of motor displacements while (b) presents phase portraits of displacement and velocity. The lower amplitude time response for $\kappa=0.016$ is nearly sinusoidal, which is further exemplified by a mostly elliptical phase portrait. The phase portrait of the larger amplitude response for $\kappa=0.016$ is strongly asymmetric. Note that the case of pre-load ratio 1.01 and axial spring $\kappa=0.016$ indicates the system is on the verge of buckling but still remains monostable. It is known that the influences of imperfections in dynamic systems become significantly magnified when the structure is configured near the buckling threshold leading to response asymmetries (Virgin, 2007; Harne and Wang, 2014). These results suggest that designing a comparable micro air vehicle wing motor near the buckling threshold may necessitate more sensitive motor control to maintain viable and consistent flapping motions. For the softer axial suspension $\kappa=0.0045$ using the same pre-load ratio of 1.01, the system is bistable. This is notably observed in Fig. 3 by the lower amplitude intrawell response which has a biased displacement to a non-zero equilibrium and the very large amplitude snap through motions that co-exist for the same excitation conditions.

![Fig. 3. Experimental responses for excitation at 16 Hz and $k_d\Delta/P_{\text{cr}}=1.01$. For $\kappa=0.0045$ the excitation level is $\sim 21$ mN. For $\kappa=0.016$ the excitation level is $\sim 11$ mN. (a) Time series of displacements. (b) Corresponding phase portraits. For this pre-load ratio and the springs selected, $\kappa=0.016$ represents a monostable motor configuration while $\kappa=0.0045$ is bistable.](image)

Whether through adjustment of the axial spring stiffness $k_d$ or by modifying the pre-compression distance $\Delta$ to change pre-load ratio, the analytical and experimental results demonstrate that a broad range of motor velocity amplitudes and wing motion responses are achievable for a given driving excitation level. The range vastly exceeds that possible for uncompressed motor designs as shown in Figs. 2(a,b) and is a clear indicator of variable aerodynamic force generation. Diptera are well-known for a dramatic adjustability of wing stroke amplitude to tailor thrust and lift and to finely control flight, accomplished without modifying driving energies (Boettiger and Furshpan, 1952; Dickinson and Tu, 1997). Judging from prior biological research insights (Thomson and Thompson, 1977) and compared to the present results, a pre-compressed wing motor utilizing actively adjustable axial suspension may provide a suitable means to achieve such range of performance. Current micro air vehicle control frameworks often employ advanced algorithms for propulsion and guidance and thus incorporation of a controlled suspension is realistic in light of existing architectures. Collectively, the analytical and experimental findings present useful design strategies for the development of a pre-compressed flapping mechanism for micro air vehicles to achieve versatile flapping force generation far exceeding that of linear motor mechanisms.

© 2014 The Japan Society of Mechanical Engineers
4 Conclusions

This study sought insight into the means by which a pre-compressed nonlinear flapping wing mechanism is favorably adjusted to achieve large range of aerodynamic force generation as observed in diptera, and began considering the findings in the context of micro air vehicle development. Through analytical and experimental methods, the axial suspension that supports the wing motor is probed for its critical dependence upon wing motion responses. It is found that when the motor is pre-compressed, adjustment of axial stiffness could dramatically alter the resultant wing velocities, and hence aerodynamic forces, without resorting to changing driving excitations. The suspension and preload characteristics are biologically adjustable in diptera; thus, it may be anticipated that micro air vehicle frameworks adopting these features should likewise enhance flight control versatility.

Appendix

Additional variables include: \( A_b = h_b t_b \), beam area; and \( I_b = h_b t_b^3 / 12 \), beam moment of inertia.

\[
\omega_0^2 = \frac{\frac{1}{2} E_b l_b h_b \left( \frac{\pi}{l_b} \right)^4}{\frac{1}{2} \rho_b A_b l_b + m a \left( \frac{\pi}{l_b} \right)^2}
\]

(A1)

\[
\omega = \Omega / \omega_0
\]

(A2)

\[
\eta = \frac{\frac{1}{2} h_b l_b}{\left( \frac{1}{2} \rho_b A_b l_b + m a \left( \frac{\pi}{l_b} \right)^2 \right) \omega_0}
\]

(A3)

\[
K = \frac{1}{2} E_b l_b h_b \left( \frac{\pi}{l_b} \right)^4
\]

(A4)

\[
P_{tr} = E_b h_b \left( \frac{\pi}{l_b} \right)^2
\]

(A5)

\[
\beta = \frac{1}{2} h_b \left( \frac{\pi}{l_b} \right)^2
\]

(A6)

\[
\gamma = \frac{3 l_b^2}{l_b^2}
\]

(A7)

\[
z = \frac{48}{\pi} \frac{\rho_b}{E_b h_b l_b} \left( \frac{h_b}{l_b} \right)^3 A_a
\]

(A8)

References


Mayers, J. and Wrenn, B.G., Combined influence of higher-order linear effects and nonlinear effects on the lateral vibration behavior of solid and sandwich beams, SUDAER 208 (1964).


