1A22 Evaluation and optimum design of dual-functional electromagnetic tuned mass dampers

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Abstract
Among various types of Tuned Mass Dampers (TMDs) for the civil structures (such as tall buildings, slender towers and long bridges), dual-functional electromagnetic TMDs have received a lot of attentions due to its enhanced effectiveness in mitigating the vibration of the primary structures and harvest the energy at the same time. This paper quantitatively investigates the effectiveness of dual-functional electromagnetic TMDs to the building performance regarding to building safety, human comfort and energy harvesting when the primary structure is being disturbed by the wind excitation regularly and seismic excitation in an extreme situation. Six corresponding individual Performance Indexes (PIs) are defined and optimized using $H_2$ optimization criterion so as to determine the optimal parameters of dual-functional electromagnetic TMDs, resulting in the best performances of building safety, human comfort and energy harvesting respectively. Moreover, a combinational PI is defined and optimized to evaluate the overall effectiveness of dual-functional electromagnetic TMDs including the priority concerned PIs and the energy harvesting performance. A case study is then performed based on the parameters a real building to verify the effectiveness of the optimized dual-functional electromagnetic TMD. Simulation results of dual electromagnetic TMD are presented in comparison with classic TMD and without TMD case.

Keywords : Energy harvesting, Structural control, Multiple tuned mass dampers, Electromagnetic shunt damping, Resonant circuit, $H_2$ optimization

1 Introduction
Large scale civil structures, such as tall buildings, slender towers and long bridges, disturbed by diverse environmental loadings (such as gusty winds and strong earthquakes) are subjected to large vibration which may lead to damages of building body and discomfort to its human occupant. In order to improve the building safety and people comfort, classic Tuned Mass Dampers (TMDs) consisting of a vibration absorber mass attached to the primary structure with a spring and a damper were used to dissipate vibration energy of primary structures into heat. The classic TMDs have been broadly investigated to extend its effectiveness or robustness by many researchers. Among many optimization criteria or methods, fixed-points method proposed by Den Hartog in 1928 [1] and $H_2$ optimization methods aiming to minimize the Root Mean Square (RMS) value of the displacement of the primary structure disturbed by wind induced excitation proposed by Asami T. et al in 1991 [2] have received exclusive attentions in literature and industry. The effectiveness of classic TMDs in improving building safety and people comfort have been demonstrated in many real buildings such as the Taipei 101 Tower in Taipei [3], the Citicorp Center in New York City [4] and etc.

Besides the classic TMDs, various types of TMDs, such as three elements TMDs [5], parallel TMDs [6], series TMDs [7], are proposed one after another in order to achieve better vibration mitigation. Among these types of TMDs, double-mass series TMDs in which two auxiliary absorbers being connected to the primary system in series, have been proved to be most effective in suppressing vibration by Zuo [7]. However Zuo also pointed out that the motion stroke of series TMDs are
much larger than the classic TMDs, which makes series TMDs barely implementable in real civil structures, especially in those structures with limited space.

Inspired by the principles of double-mass series TMDs and resonant shunt damping in piezoelectric structures, Zuo and Cui [8] recently proposed the series electromagnetic TMDs in which the original viscous damper in the classic TMDs is replaced by an electromagnetic motor shunt with RLC circuit of resonance tuned around that of the primary structure by making use of both mechanical and electrical resonances, this configuration will not induce large additional mechanical motion stroke and meanwhile keep the effectiveness the same as the series TMDs. Instead of dissipating the vibration energy into heat waste as what traditional TMDs do, series electromagnetic TMDs can harvest the vibration energy and achieve dual purpose of vibration mitigation and energy harvesting.

In this paper, we will explore the potential of dual-functional electromagnetic TMD by optimizing different Performance Indexes (PIs), which stand for the effectiveness regarding to building safety, people comfort and energy harvesting under wind or earthquake excitations respectively. Then an overall performance index combining the most concerned PIs is defined and optimized to evaluate the effectiveness of dual-functional electromagnetic TMD for combined excitations. This paper optimizes the parameters of dual-functional electromagnetic TMD by directly minimizing (maximizing) the Mean Square (MS) response of the transfer functions corresponding to each PIs to achieve $H_2$ optimization criteria. The enhanced effectiveness of the electromagnetic TMD will be presented in comparison with classic TMD and structure without TMD case.

The paper is organized as following. In Section 2, a brief introduction to the dual-functional electromagnetic TMD and its dynamics. And transfer functions relating with building safety, people comfort (equipment protection) and energy harvesting are defined. In Section 3, Individual performance indexes, overall performance index and optimization algorithm are introduced. In Section 4, a numerical case study based on real civil structure parameters is performed. The simulation results of dual functional electromagnetic TMD will be presented in comparison with classic TMD and structure without TMD. Section 5 gives the conclusions.

2 Model for analysis and corresponding transfer functions

2.1 Concept of dual-functional electromagnetic TMD

Zuo [8] proposed the concept of series TMD, shown on Figure 1(a), and reported its enhanced effectiveness and robustness. The evaluation, in terms of performance of vibration mitigation, of series TMD to wind-excited tall buildings was reported by Tao's paper [9] which concluded that two masses with 1.62% total mass ratio can attain the vibration mitigation effect of the classic TMD with 2% mass ratio; However, the motion stroke of series TMD is six times larger than classic TMD's which makes the series TMD barely implementable in the regular building structure.

In order to achieve the enhanced effectiveness and simultaneously keep the motion stroke on a par with that of classic TMD, Zuo [8] proposed the concept of dual-functional electromagnetic shunt TMD, shown on Figure 1(b), in which the original mechanical resonance of the second vibration absorber $m_2$ of series TMD is replaced by an electrical resonance of an electromagnetic transducer shunt with RLC circuit. In Figure 1(b) the electromagnetic motor of coil inductance $L$ and resistance $R$ is connected between absorber $m_1$ and primary structure $m_s$, and the energy harvesting circuit is modelled as a

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resister R. To point out, the electromagnetic transducer can be either a linear motor or a rotational motor with motion transmission. However for a large damping force application such as TMDs installed in the large scale civil structures, rotational one is much preferred.

2.2 Model for analysis

The energy regulation and storage package can be equivalently modeled as a pure resistive load [10] which makes the auxiliary resonator circuit a standard RLC circuit. Figure 2 shows the dual-functional electromagnetic TMD model for the optimization purpose.

The main principle of dual-functional electromagnetic TMD is that the vibrating motion of primary structure \( m_p \) is mitigated by first amplifying the motion of TMD \( m_1 \) and then serially by the resonance of shunting the RLC circuit. The dynamic features of the optimization-oriented model can be summarized below:

(a) The relative motion between the absorber and the primary structure produce an induced voltage \( e_{\text{EMF}} \) proportional to their relative velocity where the proportional gain \( k_v \) \([V/(m/s)]\) is the voltage constant of the electromagnetic transducer,

\[
e_{\text{EMF}} = k_v (\dot{x}_i - \dot{x}_e)
\]

(b) The electrical current in the electromagnetic transducer will produce a force \( f_{\text{EMF}} \) proportional to the electrical current where the proportional gain \( k_f \) \([N/A]\) is the force constant. The constants \( k_v \) and \( k_f \) are only determined by transducer itself. Moreover the relation \( k_v = k_f \) is held for an ideal transducer without energy loss.

\[
f_{\text{EMF}} = k_f i
\]

(c) In the resonator circuit, the close-loop voltage drop has to be zero according to Kirchhoff voltage law

\[
e_{\text{EMF}} + L \frac{di}{dt} + Ri + C \int i \, dt = 0
\]

(d) The model is externally disturbed by wind or seismic excitations respectively. The wind excitation is applied on the primary structure with a force of \( f_w = m_w x''_w \) \([N]\), where \( x''_w \) is a notation standing for the normalized force. Differently, the seismic excitation is applied on the primary structure and vibration absorber with forces of \( m_w x''_g \) \([N]\) and \( m_1 x''_g \) \([N]\) respectively.

The dynamic governing equation is therefore given by

\[
\begin{align*}
\dot{m}_w \ddot{x}_w + c_x \dot{x}_w + k_q \dot{q} + (k_r + k_1) x_1 - k_q \dot{x}_1 &= m_w \ddot{x}_w - m_w \ddot{x}_g \\
m_1 \dddot{x}_1 - k_q \dot{q} - k_r q + k_1 \dddot{x}_1 &= -m_1 \dddot{x}_g \\
L \dddot{q} - k_q (\dot{x}_1 - \dot{x}_e) + R \dot{q} + C q &= 0
\end{align*}
\]

Some remarks on equation (4) are listed below:
Remark 1. \( x_x, x_y, x_z \) are the relative displacements of the primary structure and TMDs to ground. 

Remark 2. The variable \( q \) is defined as the electrical charge flowing through resonator circuit. Compared with equation (3), we have \( q = i \).

Remark 3. \( x'_w \) and \( x'_g \) represent normalized wind excitation and ground acceleration excitation, respectively. They are both irrelevantly exogenous excitations disturbing the model shown in Figure 2.

2.3 Transfer Functions (TFs)

The performances of the building include building safety, people comfort, and energy harvesting. They are related with the deformation of the primary structure \( x_p \), the acceleration of the primary structure \( \ddot{x}_p \) and the electrical current in the harvesting circuit \( i \) respectively. Under wind or earthquake excitations, we will have six transfer functions, corresponding the six performance evaluations as listed in Table 1. Their dimensionless forms are explicitly given below.

### Table 1. Transfer functions

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Building Safety</th>
<th>Human Comfort (Equipment Protection)</th>
<th>Energy Harvesting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind</td>
<td>( H_{sw} )</td>
<td>( H_{sw} )</td>
<td>( H_{sw} )</td>
</tr>
<tr>
<td>Earthquake</td>
<td>( H_{se} )</td>
<td>( H_{se} )</td>
<td>( H_{se} )</td>
</tr>
</tbody>
</table>

1. \( H_{sw} \) is the transfer function, from wind acceleration \( \ddot{x}_w \) to relative displacement of primary structure \( x_p \), given by

\[
H_{sw}(\alpha) = \frac{\ddot{x}_w}{x_p} = \frac{\text{Num}(\ddot{x}_w)}{\text{Den}(x_p)},
\]

where

\[
\text{Den}(x_p) = (\alpha)^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + \text{um}(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3
\]

2. \( H_{se} \) is the transfer function, from ground acceleration \( \ddot{x}_g \) to relative displacement of primary structure \( x_p \), given by

\[
H_{se}(\alpha) = \frac{\ddot{x}_g}{x_p} = \frac{\text{Num}(\ddot{x}_g)}{\text{Den}(x_p)},
\]

where

\[
\text{Den}(x_p) = (\alpha)^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right)
\]

3. \( H_{sw} \) is the transfer function, from wind acceleration \( \ddot{x}_w \) to absolute acceleration of primary structure \( \ddot{x}_p \), given by

\[
H_{sw}(\alpha) = \frac{\ddot{x}_w}{\ddot{x}_p} = \frac{\text{Num}(\ddot{x}_w)}{\text{Den}(x_p)},
\]

where

\[
\text{Den}(x_p) = \alpha^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + \text{um}(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3
\]

4. \( H_{se} \) is the transfer function, from ground acceleration \( \ddot{x}_g \) to absolute acceleration of primary structure \( \ddot{x}_p \), given by

\[
H_{se}(\alpha) = \frac{\ddot{x}_g}{\ddot{x}_p} = \frac{\text{Num}(\ddot{x}_g)}{\text{Den}(x_p)},
\]

where

\[
\text{Den}(x_p) = \alpha^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right)
\]

5. \( H_{sw} \) is the transfer function, from wind acceleration \( \ddot{x}_w \) to current \( i \) of resonator circuit, given by

\[
H_{sw}(\alpha) = \frac{i}{\ddot{x}_w} = \frac{\text{Num}(i)}{\text{Den}(\ddot{x}_w)},
\]

where

\[
\text{Den}(\ddot{x}_w) = \alpha^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right)
\]

6. \( H_{se} \) is the transfer function, from ground acceleration \( \ddot{x}_g \) to current \( i \) of resonator circuit, given by

\[
H_{se}(\alpha) = \frac{i}{\ddot{x}_g} = \frac{\text{Num}(i)}{\text{Den}(\ddot{x}_g)},
\]

where

\[
\text{Den}(\ddot{x}_g) = \alpha^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right) + (\alpha)\left(2(1 + \mu)\xi_f f_1 + 4\xi_f f_2 + (1 + \mu)f_2 f_3\right)
\]

where \( j = \sqrt{-1} \),

\[
\text{Den} = (\alpha)^4 + (\alpha)^3(2\xi_x f_1 + 4\xi_x f_2 + 2f_2) + (\alpha)^2(2(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3) + (\alpha)^2(2(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3) + (\alpha)^2(2(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3) + (\alpha)^2(2(1 + \mu)\xi_f f_1 + 2(1 + \mu)\xi_f f_2 + 2\xi_f f_3 + (1 + \mu)f_2 f_3)
\]

and

\[
\omega_n = \sqrt{\xi_x / m_x}, \quad \text{natural frequency of primary structure;}
\]

\[
\xi_x = c_x / 2m_x \omega_n, \quad \text{dimensionless damping of primary structure;}
\]
\[ \mu = m_2/m_1, \text{ mass ratio}; \]
\[ \omega_1 = \sqrt{k_1/m_1}, \text{ natural frequency of TMD}; \]
\[ \omega_0 = \sqrt{1/LC}, \text{ natural frequency of resonator circuit}; \]
\[ \xi = R/2L\omega_0, \text{ damping of resonator circuit}; \]
\[ f_1 = \omega_1/\omega_0, \text{ frequency ratio of TMD and primary structure}; \]
\[ f_2 = \omega_2/\omega_0, \text{ frequency ratio of resonator circuit and primary structure}; \]
\[ \mu_k = (k_qk_f)/(Lk_1), \text{ electromagnetic mechanical coupling coefficient (stiffness ratio)}; \]
\[ \alpha = \omega/\omega_0, \text{ frequency ratio of external disturbance and primary structure (normalized frequency)}; \]

Among these dimensionless parameters, \( \mu_k \) is called as electromagnetic mechanical coupling coefficient [8]. It is actually a stiffness ratio, which plays a similar role as the mass ratio in the classic TMD.

3 Performance indexes and optimizations

3.1 Individual Performance Indexes (PIs)

As mentioned before, the effectiveness of dual-functional electromagnetic TMD is evaluated by 3 kinds of Performance Indexes (PIs) which respectively represent performances of building safety, people comfort and energy harvesting. The related TFs are correspondingly \( H_{aw} \) (or \( H_{AE} \)), \( H_{aw} \) (or \( H_{AE} \)), \( H_{aw} \) (or \( H_{AE} \)) under wind (or earthquake) disturbance which have been explicitly given in the previous session. If the disturbance is assumed as a broad bandwidth input, minimizing the \( H_2 \) norms of \( H_{aw} \) and \( H_{aw} \) leads to the minimum Root Mean Square (RMS) value of the relative displacements or absolute acceleration of primary structures (harvestable power) respectively. Since power is resistance time square of current, the energy harvesting will be related with \( H_{aw} \) or \( H_{AE} \), which will explain later.

3.1.1 Building safety. Building safety is related with the structure deformation \( x_n \). The PIs of building safety, which are the \( H_2 \) norms of corresponding TFs of \( x_n \) under wind or earthquake excitation, are defined as:

1. Under wind excitation

\[ P_{lew} = \|H_{aw}(\omega)\|^2 = \|\frac{x_n}{x_{ro}}\|^2 = \frac{1}{\omega_0^2} (x_2^2) \]  \hspace{1cm} (11)

where \( (x_2^2) \) is the Mean Square (MS) response of the primary structure's deformation (or relative displacement) under the Gaussian white noise input with unit Power Spectrum Density (PSD) and is given by:

\[ (x_2^2) = \int_{-\infty}^{\infty} \left[ \frac{x_n}{x_{ro}} \right]^2 dw = \frac{1}{\omega_0^2} \int_{-\infty}^{\infty} \left| H_{aw}(\omega)x_n \right|^2 d\omega = \frac{1}{\omega_0^2} \int_{-\infty}^{\infty} \left[ \frac{\text{PSD}(\omega)}{\text{PSD}(0)} \right] d\omega \]  \hspace{1cm} (12)

After plugging (12) into (11), we can obtain:

\[ P_{lew} = \frac{1}{\omega_0^2} \int_{-\infty}^{\infty} \left[ \frac{\text{PSD}(\omega)}{\text{PSD}(0)} \right] d\omega \]  \hspace{1cm} (13)

2. Under earthquake excitation, similarly

\[ P_{leq} = \|H_{ae}(\omega)\|^2 = \|\frac{x_n}{x_{ro}}\|^2 = \frac{1}{\omega_0^2} \int_{-\infty}^{\infty} \left[ \frac{\text{PSD}(\omega)}{\text{PSD}(0)} \right] d\omega \]  \hspace{1cm} (14)

An optimal set of decision dimensionless parameters of dual-functional electromagnetic TMDs leading to the best building safety performance under wind (or earthquake) excitations are obtained by minimizing \( P_{lew} \) (or \( P_{leq} \)).

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People comfort (Equipment protection). People discomfort is mainly caused by large absolute acceleration of the primary structures in which people stay in a human acceleration sensitive range of 0.063 to 1 Hz [11]. Also this absolute acceleration may cause a major damage to the equipment, especially to those vibration-sensitive precision instruments, inside civil structures. According to [12], the natural frequencies of general equipment are much less than hundred times of building's natural frequency $100\omega_s$. Hence the priority concern about the building absolute acceleration will be in the frequency range of $[0, 100\omega_s]$. With the consideration of the frequency range sensitive to the human and equipment, the performance indexes of people comfort (equipment protection) are defined as:

3. Under wind excitation

$$P_{1\omega w} = \| H_{w\omega}(\alpha) \mathcal{E}(\alpha) \|_2 = \left[ \int_{-\infty}^{\infty} \mathcal{L}^2 \right]^{1/2} = \int_{-\infty}^{\infty} \left[ \frac{\omega_s}{\omega_s} \mathcal{L}^2 \right]^{1/2} \omega d\omega$$

where $\mathcal{L}(s)$ is a low pass filter with cutoff frequency $100\omega_s$.

$$\mathcal{L}(s) = \frac{100}{s^2 + 100}$$

The corresponding $\mathcal{E}(\alpha)$ is therefore written as

$$\mathcal{E}(\alpha) = \frac{100}{\omega_s + 100}$$

4. Under earthquake excitation, similarly

$$P_{1\omega e} = \| H_{e\omega}(\alpha) \mathcal{E}(\alpha) \|_2 = \left[ \int_{-\infty}^{\infty} \mathcal{L}^2 \right]^{1/2} = \int_{-\infty}^{\infty} \left[ \frac{\omega_s}{\omega_s} \mathcal{L}^2 \right]^{1/2} \omega d\omega$$

An optimal set of decision dimensionless parameters of dual-functional electromagnetic TMDs leading to the best people comfort (equipment protection) performance under wind (or earthquake) excitations are obtained by minimizing $P_{1\omega w}$ (or $P_{1\omega e}$).

3.1.3 Energy harvesting. For energy harvesting, we need to maximize the average harvestable electrical power on the resistive load $R$. The instant power is

$$P(t) = \mathcal{R}(t)^2$$

With a white noise disturbance input, the average electrical power is given by

$$\mathcal{P} = \mathcal{R}(\mathcal{I})$$

where $\langle \mathcal{I}^2 \rangle$ is the mean square response of the resonator circuit's current $i$ under the Gaussian white noise input with unit Power Spectrum Density (PSD). Therefore the performance indexes of energy harvesting performance are defined as

5. Under wind excitation

$$P_{1\omega w} = \mathcal{R}\| H_{w\omega}(\alpha) \|_2 \mathcal{R} \left[ \frac{\omega_s}{\omega_s} \mathcal{L}^2 \right]^{1/2} \omega d\omega$$

$$= \frac{\omega_s}{\omega_s} \mathcal{R} \left[ \frac{\omega_s}{\omega_s} \mathcal{L}^2 \right]^{1/2} \omega d\omega$$

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where \( \frac{m}{k_h k_w} \) are constants for a particular primary structure system.

6. Under earthquake excitation, similarly

\[
P_{\text{la}} = R \|H_\omega(\alpha)\|_s^2 = R \left( \frac{1}{s - 2\omega_\alpha} \right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \left[ \frac{\sinh(\nu s)}{s} \right]^2 ds
\]

An optimal set of decision dimensionless parameters of dual-functional electromagnetic TMDs leading to the best energy harvesting performance under wind (or earthquake) excitations are obtained by maximizing \( P_{\text{lw}} \) (or \( P_{\text{lg}} \)).

### 3.2 Overall combined performance index

Optimizing one individual performance index unlikely will result in an omnipotent optimal result that is best for other performance index simultaneously. Therefore, a combinational performance index is desired to represent an overall effectiveness of dual-functional electromagnetic TMD. For practice, building safety is the highest priority if extreme situations happen otherwise more attentions are paid on people comfort (equipment protection) and energy harvesting. In other words, among 6 individual performance indexes, appropriately weighted \( P_{\text{ld}}, P_{\text{lw}} \) and \( P_{\text{la}} \) compose of the overall performance index. The overall performance index is therefore defined as:

\[
P_{\text{overall}} = W_1 \frac{P_{\text{ld}}}{r_{\text{ld}}} + W_2 \frac{P_{\text{lw}}}{r_{\text{lw}}} + (1 - W_1 - W_2) \frac{P_{\text{la}}}{r_{\text{la}}}
\]

where \( P_{\text{ld}}^{\text{opt}}, P_{\text{lw}}^{\text{opt}} \) and \( P_{\text{la}}^{\text{opt}} \) are the optimal value of individual performance indexes respectively defined in the last section. \( W_1, W_2 \) and \( 1 - W_1 - W_2 \) are constant weighting factors of building safety, people comfort (equipment protection) and energy harvesting respectively. They hold the relations as below:

\[
W_1 \geq 0; \quad W_2 \geq 0; \quad W_1 + W_2 \leq 1
\]

If more concerns over the building safety, people comfort or energy harvesting, the corresponding weighting factor can be increased so that all the weighting factors are redistributed to redefine the new \( P_{\text{overall}}^{\text{opt}} \). An optimal set of decision dimensionless parameters of dual-functional electromagnetic TMDs leading to the best balance among the effectiveness of building safety, people comfort (equipment protection) and energy harvesting are obtained by minimizing \( P_{\text{overall}}^{\text{opt}} \).

### 3.3 Optimization algorithm

The goal of optimization is to achieve the best effectiveness of dual-functional electromagnetic TMD by tuning decision parameters of stiffness \( k_1 \), capacitor \( C \), resistor \( R \) and inductor \( L \) for given of motor constants \( k_w \) and \( k_r \), building parameters \( m, c, k \) and TMD mass \( m_1 \). Or from performance indexes point of view, how to choose dimensionless decision parameters mechanical tuning ratio \( f_s \), electrical tuning ratio \( f_e \), electrical damping ratio \( \xi_e \) and electrical-mechanical coupling \( \mu_e \) for given parameters structural nature frequency \( \omega_\alpha \), structural damping ratio \( \xi \) and mass ratio \( \mu \) to respectively achieve the optimal performance index value \( P_{\text{ld}}^{\text{opt}} \) of \( P_{\text{ld}} \) which stands for all the predefined performance indexes \( P_{\text{ld}}, P_{\text{la}}, P_{\text{lw}}, P_{\text{lg}}, P_{\text{lw}}, P_{\text{lg}} \) and \( P_{\text{overall}} \). The optimization criteria set are given by

\[
\frac{dP}{d\xi} = 0, \quad \frac{dP}{df_s} = 0, \quad \frac{dP}{df_e} = 0, \quad \frac{dP}{d\xi_e} = 0
\]

By solving (25), we may find a series of extreme values of \( P_{\text{ld}} \). For performance indexes \( P_{\text{ld}}, P_{\text{lg}}, P_{\text{lg}}, P_{\text{lw}}, P_{\text{lg}} \) and \( P_{\text{overall}} \), the corresponding optimal \( P_{\text{ld}}^{\text{opt}} \) is reached at minimum extreme value \( P_{\text{ld}}^{\text{min}} \). For the energy harvesting performance indexes \( P_{\text{lw}} \) and \( P_{\text{lg}} \), \( P_{\text{lg}}^{\text{opt}} \) is reached at maximum extreme value \( P_{\text{lg}}^{\text{max}} \).
4 Case studies and numerical analysis

In this session, the case study is processed to evaluate the effectiveness of the dual-functional electromagnetic TMDs, optimized for multiple performance indexes, applied to a real existing 38-story building. Simulation results of dual-functional electromagnetic TMDs are presented in comparison with building with classic TMD and without TMD. The geometry dimension of the building is 137 meters high from ground to the roof and covers around 450 square meters site area. With 25,000 metric tons (25,000,000kg) primary structure mass, the building has a 3.6-second first natural period and 1% internal damping ratio. In order to protect the building from vibration, a classic TMD, with a 1% mass ratio, has been installed in the building. We modeled this building as one degree freedom modeling in introduced in section 2. The classic TMD's spring stiffness and damping ratio is obtained by using Den Hartog's fixed point method [2] which is given by

\[
\begin{align*}
& f_{opt} = \frac{\omega_0}{\omega_n} \frac{\sqrt{m_0/m_n}}{1 + \xi_n^2} \\
& \xi_{opt} = \frac{\xi_n}{\sqrt{1 + \xi_n^2}}
\end{align*}
\]

(26)

The parameters of building and its optimal classic TMD are listed on Table 2.

### 4.1 Individual performance indexes optimization

Individual performance indexes are optimized respectively to achieve the most effective performances, including the building safety, people comfort (equipment protection) and energy harvesting ability under wind or earthquake excitations. In order to study the difference among sets of decision parameters optimized for different PIs and evaluate the effectiveness of dual-functional electromagnetic TMDs, the corresponding frequency response and optimal value of each PI are numerically and graphically presented in Table 3 and Figure 3 in comparison with classic TMD and without TMD case.

From the Table 3, it presents that the largest difference of optimal parameters is between that of PIs optimized for building safety under earthquake excitation $P_{dE}$ and human comfort under daily wind excitation $P_{aw}$ which are normally priority concerns for the building designers. Due to this phenomenon, an overall performance index is necessary to be well defined so as to make a balanced trade-off between $P_{dE}$ and $P_{aw}$. The same conclusion is graphically shown by Figure 3.

Figure 4 shows the sensitivity analysis for decision parameters $k_1$, $C$, $L$, and $R$ to investigate their influence on the corresponding $H_2$ norms optimized for different PIs. From the Figure 4, it is clear to see that $k_1$ and C are highly sensitive to the $H_2$ norms which means a little change of $k_1$ and $C$ from their optimal value lead to a reduced performance of building safety, human comfort (equipment protection) or energy harvesting. On the other side, due to the low sensitivity, an inaccurate $L$ and $R$ will not make much difference on the corresponding performance.

### Table 2. Parameters

<table>
<thead>
<tr>
<th>System</th>
<th>Mass [kg]</th>
<th>Angular natural frequency [rad/s]</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building without TMD</td>
<td>$m_0 = 2.5 \times 10^7$</td>
<td>$\omega_0 = 1.7455$</td>
<td>$\xi = 0.01$</td>
</tr>
<tr>
<td>Building with classic TMD $\mu = 0.01$</td>
<td>$m_0 = 2.5 \times 10^7$</td>
<td>$\omega_0 = 1.7455$</td>
<td>$\xi = 0.01$</td>
</tr>
<tr>
<td>Building with dual-functional electromagnetic TMD $\mu = 0.01$</td>
<td>$m_0 = 2.5 \times 10^7$</td>
<td>$\omega_0 = 1.7455$</td>
<td>$\xi = 0.01$</td>
</tr>
</tbody>
</table>

Motor constants: $k_t = 150$ [N/A]

$k_e = 150$ [V/(m/s)]

Decision parameters: $f_1, f_2, \xi, \mu (k_t, L, R)$

### Table 3. Optimal decision parameters and optimal value of PIs

| Transfer Functions | $\frac{X_1}{X_1} |$ | $\frac{X_2}{X_2} |$ | $\frac{X_3}{X_3} |$ | $\frac{X_4}{X_4} |$ | $\frac{X_5}{X_5} |$ | $\frac{X_6}{X_6} |$ | $\sqrt{R} \frac{f_1}{X_1} |$ | $\sqrt{R} \frac{f_1}{X_2} |$ |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Without TMD       | 2.1566          | 2.1566          | 11.4605         | 6.6190          | N/A             | N/A             | N/A             | N/A             |
| Classic TMD       | 1.1992          | 1.2096          | 10.0543         | 3.6808          | N/A             | N/A             | N/A             | N/A             |
| Dual-functional   | $P_{dE}$        | 1.135           | 1.1433          | 9.9843          | 3.4732          | 4256.6          | 4280.5          |
| Electromagnetic   | $P_{aw}$        | 1.1394          | 1.1389          | 9.9982          | 3.4867          | 4244.1          | 4280.5          |
| TMD which is      | $P_{dE}$        | 1.135           | 1.1564          | 9.9795          | 3.4868          | 4256.6          | 4267.9          |
| optimized for     | $P_{aw}$        | 1.135           | 1.1433          | 9.9842          | 3.4732          | 4256.6          | 4280.5          |
| $P_{dE}$          | 1.1361          | 1.1488          | 9.9907          | 3.4766          | 4258.2          | 4275.8          | 4282.1          |

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Figure 3. Frequency response of electromagnetic TMD with parameters optimized for different performance indexes in comparison with classic TMD and building without TMD under wind or seismic excitation. (a) and (b): building safety, (c) and (d): people comfort (equipment protection), (f) and (g): energy harvesting. (Black dash: building without TMD; black solid: building with classic TMD; blue dash: electromagnetic TMD with parameters optimized for $P_A$; blue solid: for $P_I$; red dash: for $P_B$; red solid: for $P_B$; green dash: for $P_C$; green solid: for $P_C$.)
4.2 Overall performance index optimization

As is explained, the overall performance index is to make a trade-off between the most concerned performance indexes. With a consideration of energy harvesting ability, the overall performance index is defined by (23). Here we take $W_1 = 0.3$ and $W_2 = 0.5$ as the weighting factors for the $Pls$ of building safety under earthquake excitation and people comfort (equipment protection) under wind excitation respectively. Shown on Figure 5, the parameters of dual-functional electromagnetic TMD optimized for overall performance index $PI_{overall}$ make a balance among different performance indexes. Meanwhile its effectiveness is better than classic TMD and building without TMD for all the performance of building safety, people comfort (equipment protection) and energy harvesting.

Figure 4. Sensitivity of the dual-functional electromagnetic TMD to the changes of tunneling parameters under wind or seismic excitation. (Blue: stiffness $k_i$; red: capacitor $C$; black: inductor $L$; green: resistor $R$). (a) and (b): building safety, (c) and (d): people comfort (equipment protection), (f) and (g): energy harvesting.

Figure 5. Frequency response of electromagnetic TMD with parameters optimized for overall performance index in comparison that with parameters for individual performance index, classic TMD and building without TMD. (a): building safety under seismic excitation, (b): people comfort (equipment protection).
5 Conclusions

This paper investigates the effectiveness of dual-functional electromagnetic TMDs to the building performance regarding to building safety, human comfort and energy harvesting when the primary structure is being excited by the wind loads regularly and seismic motion in an extreme situation. By defining and optimizing six individual Performance Indexes (PIs) using \( H_0 \) optimization criteria, it is found that the largest difference of optimal parameters is between that of PIs optimized for building safety under earthquake excitation and human comfort under daily wind excitation both of which are often priority concerns for the building designers. It is also concluded that dual-functional electromagnetic TMDs lead to enhanced effectiveness compared to classic TMDs in improving building safety, human comfort and energy harvesting under various environmental dynamic loadings as shown in the frequency responses of various types of TMDs. By defining and optimizing the overall performance index \( P_{overall} \), dual-functional electromagnetic TMDs designs can make trade-off between effectiveness of building safety, human comfort and energy harvesting by choosing different weighing factors, which give some design flexibility.

Acknowledgement

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Nomenclatures

\( m_1, m_2, m_3 \) masses [kg] of the primary structure and TMDs (first TMD and second TMD if applicable).
\( k_1, k_2, k_3 \) stiffness [N/m] of the primary structure and TMDs.
\( c_1, c_2, c_3 \) damping coefficient [N·s/m] of the primary structure and TMDs.
\( x_1, x_2, x_3 \) displacements [m] of the primary structure and TMDs relative to ground.
\( R_{equiv}, R_i \) equivalent resistance [\( \Omega \)] of electricity converter and storage, internal resistance [\( \Omega \)] of inductor.
\( R, L, C \) equivalent resistance [\( \Omega \)], inductance [\( \text{H} \)] and capacitance [\( \text{F} \)] of the electrical resonator circuit.
\( k_f \) back electromotive voltage constant [V/(m/s)] and force constant [N/A].
\( q, i, C \) charge [C] and current [A] of the resonator circuit.
\( x_w, \dot{x}_w \) external wind acceleration [m/s\(^2\)] excitation and ground acceleration [m/s\(^2\)] excitation.

\( H_{xx} \) dimensionless transfer functions.
\( P_{Ixx} \) individual performance indexes.
\( P_{overall} \) overall performance index.
\( L \) low-pass filter with cutoff frequency 100\( \omega_p \).
\( P(\omega), P \) instant power and average power on the resistive load \( R \).
\( W_1, W_2 \) weighting factors of \( P_{Ixx} \) and \( P_{overall} \).
\( P_{opt} \) optimal value of corresponding performance indexes.
\( \lambda, \| \mathbf{I} \|_2 \) mean value and system \( H_2 \) norm.
\( e, e \) gradient and partial gradient.

Appendices

By using the residue theorem or the table in [13], the MS response integration of the 6th order system can be obtained as:

\[
I_0 = \int_{0}^{\infty} \frac{\left( a_1 x_{10} + a_2 x_{20} + a_3 x_{30} - a_4 x_{10}^2 - a_5 x_{20}^2 - a_6 x_{30}^2 + a_7 x_{10} x_{20} + a_8 x_{10} x_{30} + a_9 x_{20} x_{30}\right)}{a_{10} x_{10} + a_{11} x_{20} + a_{12} x_{30}} \, dx = \pi \frac{\text{Res}_{a_{12}}}{a_{12}}
\]

Where

\[
\text{Res}_{a_{12}} = -B_2(\dot{\mathbf{A}}_1 - \dot{\mathbf{A}}_2 - \mathbf{A}_1 - \mathbf{A}_2 - 2\mathbf{A}_1 \mathbf{A}_2 + 2\mathbf{A}_1 \mathbf{A}_2 - \mathbf{A}_1 \mathbf{A}_2 - \mathbf{A}_1 \mathbf{A}_2 + \mathbf{A}_1 \mathbf{A}_2 + \mathbf{A}_1 \mathbf{A}_2 - \mathbf{A}_1 \mathbf{A}_2 - \mathbf{A}_1 \mathbf{A}_2)
\]

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