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PFC design method based on frequency response fitting -Assurance of minimum phase property by stability theorem of descriptor system-

Masaya TANEMURA*, Yuichi CHIDA** and Yuichi IKEDA**

* Graduate School of Science and Technology, Shinshu University, Japan

4-17-1 Wakasato, Nagano-shi, Nagano, 380-0928, Japan

E-mail: 13tm124f@shinshu-u.ac.jp

** Faculty of Engineering, Shinshu University, Japan

4-17-1 Wakasato, Nagano-shi, Nagano, 380-0928, Japan

Abstract

Simple adaptive control (SAC) is known as a control method that keeps control performance even if plant properties have changed. However, there is a problem such that the vibratory output occurs in the transient response when SAC is applied to a vibration system which includes anti-resonance modes. The reason why the vibratory output occurs is the structure of SAC with the output caused by the vibratory input corresponding to the anti-resonance frequency. In order to overcome the problem, it is shown that designing parallel feedforward compensator (PFC) appropriately is effective. One of the PFC design methods for continuous-time LTI SISO system has been proposed. In the method, the PFC is designed such that the gain of the augmented system is matched to the one of the desired model. Furthermore, the PFC must give the ASPR property the augmented system. These conditions are described using LMI/BMI conditions. The LMI/BMI conditions are solved by the iterative procedure. However, the leading coefficient of the PFC must be given preliminarily to guarantee the ASPR property. Thus, all coefficients of the PFC could not be design parameters in the conventional method. This paper proposes an alternative method to overcome the problem by using the stability theorem of the descriptor system. By this method, the value of the leading coefficient of the PFC is not required in advance. The effectiveness of the proposed method is verified by numerical simulations.

Key words : Simple Adaptive Control, Parallel Feedforward Compensator, Frequency Response Fitting, Descriptor System, LMI, Mechanical System

1. Introduction

Adaptive control is known as a control method that keeps control performance even if plant properties have changed. Especially, simple adaptive control (SAC) includes high robustness and also it can be designed easily for applications. By applying SAC to many objects, the effectiveness of SAC has been shown experimentally (Hino, et al., 1992), (Kyoizumi, et al., 2001), (Ohtomo, et al., 1997). However, there is a problem such that the vibratory output occurs in the transient response when SAC is applied to a vibration system which includes anti-resonance modes (Yamashiro and Chida, 2012). The reason why the vibratory output occurs is the structure of SAC with the output caused by the vibratory input corresponding to the anti-resonance frequency. In order to overcome the problem, it is shown that designing parallel feedforward compensator (PFC) in appropriately is effective (Yamashiro and Chida, 2012). The solution for the problem is to increase the gain of the PFC in the anti-resonance frequency band to the plant gain such that the augmented system which consists of the plant and PFC does not include the anti-resonance frequency. In such case, it is required to design the PFC which guarantees the gain property as well as several conditions for SAC design such as the almost strictly positive real (ASPR) conditions. One of the PFC design methods for continuous-time LTI SISO system is proposed in (Tanemura, et al., 2013). In the method, the PFC is designed such that the gain of the augmented system is matched to the one of the desired model. The matching condition is described using LMI/BMI conditions, and the PFC is obtained by

solving an optimization problem described by LMI/BMI conditions. The restrictions for the ASPR property are included in the LMI/BMI conditions. In the restriction, the minimum phase property of the augmented system is described by the Lyapunov inequality in (Tanemura, et al., 2013). In such case, there is an assumption that the leading coefficient of the numerator of the PFC is known. However, it cannot be given preliminarily in some cases. This paper proposes an alternative method to overcome the problem by using the stability theorem of the descriptor system. By the method, it is not required the value of the leading coefficient of the numerator of the PFC in advance. The effectiveness of the proposed method is verified by numerical simulations.

2. Problem establishment

2.1. Plant

In this paper, the following stable continuous-time LTI SISO plant is considered:

$$G_p(s) \begin{cases} \dot{\mathbf{x}}_p(t) = \mathbf{A}_p \mathbf{x}_p(t) + \mathbf{b}_p u_p(t) \\ y_p(t) = \mathbf{c}_p \mathbf{x}_p(t) \end{cases} \quad (1)$$

$$\mathbf{A}_p \in \mathbb{R}^{n_p \times n_p}, \mathbf{b}_p \in \mathbb{R}^{n_p \times 1}, \mathbf{c}_p \in \mathbb{R}^{1 \times n_p}$$

2.2. Constitution of SAC

The following stable reference model which requires the plant output to follow is considered:

$$G_m(s) \begin{cases} \dot{\mathbf{x}}_m(t) = \mathbf{A}_m \mathbf{x}_m(t) + \mathbf{b}_m u_m(t) \\ y_m(t) = \mathbf{c}_m \mathbf{x}_m(t) \end{cases} \quad (2)$$

$$\mathbf{A}_m \in \mathbb{R}^{n_m \times n_m}, \mathbf{b}_m \in \mathbb{R}^{n_m \times 1}, \mathbf{c}_m \in \mathbb{R}^{1 \times n_m}$$

The following assumptions on the plant and reference model are assumed.

Assumptions(Mizumoto and Iwai, 2001)

- 1) The plant is ASPR.
- 2) The plant and the reference model have CGT solutions.
- 3) $u_m(t)$ is differentiable.

The control objective is to achieve the calculation of Eq.(3) under these assumptions.

$$\lim_{t \rightarrow \infty} e(t) = 0, \quad e(t) = y_p(t) - y_m(t) \quad (3)$$

A constitution of SAC used in this paper is shown in Fig.1. SAC controllers are described by Eq.(4)-(8). Where, $k_u(t)$ and $k_x(t)$ are adaptive feedforward gains, $k_e(t)$ is an adaptive feedback gain. The PI-type adaptive law with σ -modification method is used as the adaptive adjusting law. Γ_P and Γ_I are adaptive adjusting gain matrixes. An initial value of $k_I(t)$ is a design parameter.

$$u_p(t) = \mathbf{k}(t)^T \mathbf{z}(t) \quad (4)$$

$$\mathbf{z}(t) = \begin{bmatrix} e_a(t) & \mathbf{x}_m(t)^T & u_m(t) \end{bmatrix}^T$$

$$\mathbf{k}(t) = \begin{bmatrix} k_e(t) & \mathbf{k}_x(t)^T & k_u(t) \end{bmatrix}^T$$

$$\mathbf{k}(t) = \mathbf{k}_P(t) + \mathbf{k}_I(t) \quad (5)$$

$$\mathbf{k}_P(t) = -\Gamma_P \mathbf{z}(t) e_a(t) \quad (6)$$

$$\dot{\mathbf{k}}_I(t) = -\Gamma_I \mathbf{z}(t) e_a(t) - \sigma(t) \mathbf{k}_I(t) \quad (7)$$

$$\sigma(t) = \frac{e_a^2(t)}{1 + e_a^2(t)} \sigma_1 + \sigma_2 \quad (8)$$

$$\Gamma_P = \Gamma_P^T > 0, \Gamma_I = \Gamma_I^T > 0, \sigma_1 > 0, \sigma_2 \geq 0$$

Since few plants have the ASPR property in general, a PFC as depicted by the transfer function, $G_f(s)$, is introduced in Fig.1. The PFC is designed to ensure that the augmented system, Eq.(9), is ASPR. Henceforth SAC is applied for the augmented system.

$$G_a(s) = G_p(s) + G_f(s) \quad (9)$$

Fig. 1 Block diagram of SAC system with the PFC

2.3. Vibratory input problem

This section considers the problem such that the vibratory input occurs when SAC is applied to the system including the anti-resonance frequency, by which the vibratory output is generated. The reasons for the vibratory input and the method for suppressing the vibration are clarified as follows (Yamashiro and Chida, 2012). First, in Fig.1, the transfer function $G_u(s)$, from $u_m(t)$ to $u_p(t)$, is derived. Next, an influence of the anti-resonance appearing in $G_u(s)$ is clarified. The plant, PFC and reference model are described by $G_p(s)$, $G_f(s)$ and $G_m(s)$ as follows,

$$G_p(s) = \frac{n_p(s)}{d_p(s)}, \quad G_f(s) = \frac{n_f(s)}{d_f(s)}, \quad G_m(s) = \frac{n_m(s)}{d_m(s)}.$$

When k_x is assumed as $k_x = 0$ for simplify, $G_u(s)$ is given by

$$\begin{aligned} G_u(s) &= \frac{k_u(1 - k_e G_f(s)) - k_e G_m(s)}{1 - k_e(G_p(s) + G_f(s))} \\ &= \frac{d_p}{d_m} \frac{k_u(d_f d_m - k_e n_f d_m) - k_e n_m d_f}{d_p d_f - k_e(n_f d_p + n_p d_f)} \\ &= \frac{d_p}{d_m} \frac{k_u\{d_f d_m - k_e n_f d_m\} - k_e n_m d_f}{\{d_1 + d_2 + d_3\}}, \end{aligned} \quad (10)$$

$$d_1 = d_p \cdot d_f, \quad d_2 = -k_e \cdot n_f \cdot d_p, \quad d_3 = -k_e \cdot n_p \cdot d_f,$$

$$d_d = d_1 + d_2 + d_3.$$

In Eq.(10), d_3 which is the denominator of $G_u(s)$ has $n_p(s)$ including the anti-resonance. Therefore, the resonance appears at frequency response of $G_u(s)$ and generates the vibratory input. The above is confirmed by the following numerical example,

$$G_p(s) = \frac{115.7(s^2 + 0.01193s + 57.14)}{(s^2 + 4.906s + 47.76)(s + 39.0382)(s + 3.5456)},$$

$$G_{f1}(s) = \frac{0.001}{0.1989s + 1}, \quad G_m(s) = \frac{25.27}{s^2 + 7.108s + 25.27},$$

$$k_e = -20, \quad k_u = 1.$$

Figure 2 shows gain characteristics of d_1 , d_2 , d_3 , d_d and $G_u(s)$. The closed circle, the broken line, the dotted line, the open circle and the solid line show d_1 , d_2 , d_3 , d_d and $G_u(s)$, respectively. According to Fig.2, the anti-resonance of d_d causes the resonance of $G_u(s)$. The resonance of $G_u(s)$ generates the vibratory input, $u_p(t)$. It generates the anti-resonance of d_d that the gain of d_3 is larger than that of others in the anti-resonance frequency band.

A method for suppressing the vibration is shown as follows. In the anti-resonance frequency band, increasing the gain of d_1 or d_2 produces to decrease the influence of the anti-resonance. Here, $n_f(s)$ included in d_2 is considered. By increasing the gain of $n_f(s)$, that makes the gain of d_2 larger than that of d_3 . When the following equation is used as the PFC, gain characteristics of d_1 , d_2 , d_3 , d_d and $G_u(s)$ are shown in Fig.3.

$$G_{f2}(s) = \frac{1}{0.1989s + 1}$$

Figure 3 shows that the anti-resonance of d_d has been suppressed; consequently, the resonance of $G_u(s)$ is suppressed. Accordingly, increasing the gain of the PFC improves the vibratory input problem. In other words, designing the PFC such that the augmented system, $G_a(s)$, does not include the anti-resonance frequency improves the problem.

Fig. 2 Bode plot of $G_u(s)$ and denominator of $G_u(s)$, $G_{f1}(s)$ Fig. 3 Bode plot of $G_u(s)$ and denominator of $G_u(s)$, $G_{f2}(s)$

2.4. Considering the problem

As mentioned above, designing the PFC such that $G_a(s)$ does not include the anti-resonance frequency improves the problem. Furthermore, the PFC must ensure the ASPR property of $G_a(s)$. Besides, ensuring stability of the PFC is desirable. This paper has the purpose to design degrees (n, m) and coefficients (a_i, b_i) of the PFC described by Eq.(12) to satisfy the above requirement for the plant described by Eq.(11).

$$G_p(s) = \frac{n_p(s)}{d_p(s)} = \frac{b_{pmp} s^{mp} + b_{pmp-1} s^{mp-1} + \dots + b_{p0}}{s^{np} + a_{pnp-1} s^{np-1} + \dots + a_{p0}} \quad (11)$$

$$G_f(s) = \frac{n_f(s)}{d_f(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_0} \quad (12)$$

3. Conventional method and problem

3.1. Outline

By describing the design conditions of the PFC using LMI/BMI, an optimization problem is solved, besides the PFC is designed in the precedent study (Tanemura, et al., 2013). In this section, a PFC design method and a problem are represented. First, a LMI condition which is required so that $G_a(s)$ does not include the anti-resonance is derived. Next, a BMI condition to ensure the ASPR property of $G_a(s)$ is derived. Finally, the problem caused by the condition to ensure the ASPR property is explained. The stability condition of $G_f(s)$ is dealt with when the problem is considered.

3.2. Condition for the anti-resonance not to appear in $G_a(s)$

The desired model described as $G_r(s)$ which does not include the anti-resonance is designed for fitting frequency responses of $G_a(s)$ and $G_f(s)$. By describing the matching condition using LMI, the PFC design problem becomes the convex optimization problem. $G_r(s)$ is designed not to include the anti-resonance and to have the ASPR property. An error transfer function (Chida and Nishimura, 2008) is described as

$$E(\omega) = M(\omega)G_r(j\omega) - M(\omega)(G_p(j\omega) + G_f(j\omega)). \quad (13)$$

Where, $M(\omega)$ is a frequency dependent weighting function. Equation (13) shows the matching error between $G_a(j\omega)$ and $G_r(j\omega)$. Equation (13) is, however, not affine about parameters a_i, b_i . Thus, $M(\omega)$ is specified such that $M(\omega) = d_f(j\omega)/d(j\omega)$. Here, $d(j\omega)$ is a known polynomial of the n -th degree. Substituting $M(\omega) = d_f(j\omega)/d(j\omega)$ into Eq.(13) gives

$$\begin{aligned} E(\omega) &= (G_r(j\omega) - G_p(j\omega)) \frac{d_f(j\omega)}{d(j\omega)} - \frac{n_f(j\omega)}{d(j\omega)} \\ &= [\Phi_0^r(\omega) + \Phi_a^r(\omega)\mathbf{a} + \Phi_b^r(\omega)\mathbf{b}] + j[\Phi_0^i(\omega) + \Phi_a^i(\omega)\mathbf{a} + \Phi_b^i(\omega)\mathbf{b}] \\ &= E^r(\omega) + jE^i(\omega), \\ \mathbf{a} &= [a_0, \dots, a_{n-1}]^T, \quad \mathbf{b} = [b_0, \dots, b_m]^T. \end{aligned} \quad (14)$$

Where, Φ_0^r and Φ_0^i are independent terms of $\mathbf{a}$ and $\mathbf{b}$, Φ_a^r and Φ_a^i are terms of coefficients of $\mathbf{a}$, Φ_b^r and Φ_b^i are terms of coefficients of $\mathbf{b}$. Assuming that $\mathbf{x}^T = [\mathbf{a}^T, \mathbf{b}^T]$ derives the affine equation for $\mathbf{x}$ as follows,

$$\begin{bmatrix} E^r(\omega) \\ E^i(\omega) \end{bmatrix} = \begin{bmatrix} \Phi_0^r(\omega) \\ \Phi_0^i(\omega) \end{bmatrix} + \begin{bmatrix} \Phi_a^r(\omega) & \Phi_b^r(\omega) \\ \Phi_a^i(\omega) & \Phi_b^i(\omega) \end{bmatrix} \mathbf{x} \\ = \beta(\omega) + \alpha(\omega)\mathbf{x}. \quad (15)$$

Equation (15) is intractable because Eq.(15) is the continuous function for ω . Accordingly, errors at the discrete frequency data ω_k which are sampled at N-points are considered. Taking the discrete frequency data ω_k simplifies the problem. The following error objective function J is introduced by using ω_k .

$$J(\mathbf{x}) = \sum_{k=1}^N \begin{bmatrix} E^r(\omega_k) & E^i(\omega_k) \end{bmatrix} \begin{bmatrix} E^r(\omega_k) \\ E^i(\omega_k) \end{bmatrix} \\ = \mathbf{x}^T \mathbf{H}^T \mathbf{H} \mathbf{x} + 2\mathbf{g}^T \mathbf{H} \mathbf{x} + \mathbf{g}^T \mathbf{g} \quad (16)$$

where

$$\mathbf{H} = \begin{bmatrix} \alpha(\omega_1) \\ \vdots \\ \alpha(\omega_N) \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \beta(\omega_1) \\ \vdots \\ \beta(\omega_N) \end{bmatrix}. \quad (17)$$

When $\mathbf{H}$ has full rank, $\mathbf{H}^T \mathbf{H}$ is a non-singular matrix. Thus, $J < \gamma$ becomes the LMI condition as the following by setting γ and using the Schur complement.

$$\begin{bmatrix} \gamma - 2\mathbf{g}^T \mathbf{H} \mathbf{x} - \mathbf{g}^T \mathbf{g} & \mathbf{x}^T \mathbf{H}^T \\ \mathbf{H} \mathbf{x} & \mathbf{I} \end{bmatrix} > 0 \quad (18)$$

The minimization problem of J is equivalent to a γ -minimization problem. The LMI condition of Eq.(18) is easy to solve as a convex problem.

3.3. Conditions of the ASPR property of $G_a(s)$

The sufficient condition of the ASPR property of $G_a(s)$ is described as the following (Mizumoto and Iwai, 2001).

- I) Relative degree of $G_a(s)$ is 0 or 1.
- II) $G_a(s)$ is a minimum phase system.
- III) Leading coefficient of $G_a(s)$ is positive.

The condition of I) is satisfied when the relative degree of the PFC is 1, therefore, Eq.(12) sets $n - 1$ to m .

The condition of II) is satisfied if and only if the polynomial in Eq.(19) is stable.

$$d_f(s)n_p(s) + d_p(s)n_f(s) = b_{a_{ma}}s^{m_a} + \dots + b_{a_0} \quad (19)$$

Coefficients $b_{a_0}, \dots, b_{a_{ma}}$ in Eq.(19) are affine for parameters a_i, b_i . In order to ensure the condition of II), the system with the denominator described in Eq.(19) is derived as follows,

$$G(s) = \frac{n(s)}{d_f(s)n_p(s) + d_p(s)n_f(s)}. \quad (20)$$

Where, $n(s)$ is an arbitrary polynomial such that $G(s)$ becomes proper. The system of Eq.(20) is transformed to the controllability canonical form as follows,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t), \quad (21)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & 1 \\ -\frac{b_{a_0}}{b_{a_{ma}}} & -\frac{b_{a_1}}{b_{a_{ma}}} & \dots & \dots & -\frac{b_{a_{ma-1}}}{b_{a_{ma}}} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{b_{a_{ma}}} \end{bmatrix}. \quad (22)$$

The system of Eq.(21) is stable if and only if there exists a matrix P which satisfies

$$PA + A^T P < 0, \quad P > 0. \quad (23)$$

By finding P and A in Eq.(23) at the same time, the condition of II) is guaranteed. In this case, Eq.(23) is the BMI condition for A and P . If A is a known constant matrix, Eq.(23) can be solved as LMI for P , on the other hand, if P is a known constant matrix, Eq.(23) can be solved as LMI for A , where, A in Eq.(22) includes the reciprocal of the leading coefficient, $b_{a_{m_a}}$, of $G_a(s)$. Equation (22) is not affine for design parameters because of the leading coefficient of the PFC included in $b_{a_{m_a}}$. Accordingly, in the conventional method, the leading coefficient of the PFC is a constant given preliminarily for Eq.(22) to be affine for design parameters.

The condition of III) is satisfied by setting preliminarily $b_{a_{m_a}}$ is positive.

From the above, the γ -minimization problem is solved with the following steps while the ASPR property of $G_a(s)$ is guaranteed. First, the stable matrix A is set, denoted by A_0 , of Eq.(22). Then, Eq.(23) is convex and can be solved as LMI for P . The obtained solution is assumed to be P_0 . Next, Eq.(23) with fixed P_0 is solved for A . Since Eq.(18) is the convex condition, the solution $A = A_1$, which does not increase γ , can be found. The iteration is repeated until γ becomes sufficiently small. However, the iteration does not guarantee the global optimality of γ .

3.4. Problem of the conventional method

By using the LMI condition of Eq.(23), stability of $G_f(s)$ can be guaranteed. Equation (22) is affine for parameters because the leading coefficient of the denominator of $G_f(s)$ is 1. Thus, it does not be required that the parameter is given preliminarily. As for the minimum phase property of $G_a(s)$, the parameter is given preliminarily because A of Eq.(22) included the reciprocal of the leading coefficient, $b_{a_{m_a}}$, of Eq.(19). However, when the suitable value of the leading coefficient cannot be given preliminarily, all parameters of the PFC are required to be designed. Therefore, the method to overcome this problem about the minimum phase property of $G_a(s)$ is proposed in the following chapter.

4. Proposed method

4.1. Stability theorem of descriptor system

A descriptor system is introduced to make all parameters of the PFC design parameters, when the stability theorem of the descriptor system is introduced. The system of Eq.(20) is transformed to the following descriptor system (Shiotsuki, 2011).

$$E_D \dot{x}_D(t) = A_D x_D(t) + b_D u_D(t) \quad (24)$$

$$E_D = \begin{bmatrix} I_{m_a} & 0_{m_a \times 1} \\ 0_{1 \times m_a} & 0 \end{bmatrix}, \quad A_D = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 \\ b_{a_0} & b_{a_1} & \cdots & \cdots & b_{a_{m_a}} \end{bmatrix}, \quad b_D = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix} \quad (25)$$

Where, A_D is affine for parameters in Eq.(25). The stability of this descriptor system is defined as the following.

Definition (Uezato and Ikeda, 1998), (Katayama, 1999) It is assumed that $b_{a_{m_a}} \neq 0$. If the real part of all exponential modes is negative, the system of Eq.(24) is stable. Where, exponential modes are roots of $\det(sE_D - A_D) = 0$.

When $b_{a_{m_a}} \neq 0$, Eq.(24) is reduced to the state equation of Eq.(21). In addition, exponential modes are equal to modes of the state equation (Shiotsuki, 2011). For this reason, the stability theorem of this descriptor system is employed to ensure the minimum phase property of $G_a(s)$. Where, V and U are $(m_a + 1) \times 1$ vector constituted by form bases of $\text{Null}E_D$ and the one by form bases of $\text{Null}E_D^T$, respectively.

Theorem (Uezato and Ikeda, 1998) The system of Eq.(24) is stable if and only if there exist a $P > 0 \in R^{(m_a+1) \times (m_a+1)}$ and $S \neq 0$ which satisfy

$$A_D(P E_D^T + V S U^T) + (E_D P + U S^T V^T) A_D^T < 0. \quad (26)$$

Proof Refer to the reference (Uezato and Ikeda, 1998).

Variables are A_D , P and S in Eq.(26). If A_D is a known constant matrix, Eq.(26) can be solved as LMI for P and S . On the other hand, if P and S are known constants, Eq.(26) can be solved as LMI for A_D . Owing to this, Eq.(26) can be solved as LMI using the iterative method.

4.2. Procedure of PFC design

Step1 The desired model, $G_r(s)$, is specified.

Step2 The stable PFC, $G_f(s)$, which provides the ASPR property of the augmented system is specified.

Step3 The denominator polynomial, $d(j\omega)$, of the weighting function $M(j\omega)$ is specified. Specify the reference frequency ω_k . H and g of Eq.(17) are derived using ω_k . E_D of Eq.(25), and V and U of Eq.(26) are derived.

Step4 By using $G_f(s)$ obtained in Step2, the matrix A of Eq.(24), A_D , about the numerator polynomial of $G_a(s)$ is specified. ϵ -minimizing solutions P_D and S_D are derived under the conditions of

$$\begin{aligned} A_D(P_D E_D^T + V S_D U^T) + (E_D P_D + U S_D^T V^T) A_D^T &< 0, \\ P_D > 0, \quad P_D - \epsilon I &< 0. \end{aligned} \quad (27)$$

Where, ϵ is a positive constant value. (Remark)

Step5 By using $G_f(s)$ obtained in Step2, the matrix A of Eq.(21), A_f , about the denominator polynomial of $G_f(s)$ is specified. The ϵ -minimizing solution P_f is derived under the conditions of

$$\begin{aligned} P_f A_f + A_f^T P_f &< 0, \\ P_f > 0, \quad P_f - \epsilon I &< 0. \end{aligned} \quad (28)$$

Where, ϵ is a positive constant value. (Remark)

Step6 By using P_D, S_D and P_f obtained in Step4 and 5, the γ -minimizing solution x is derived under LMI conditions. Here, variables are x, A_D, A_f, b_{m_a} . The condition of III) for the ASPR property is guaranteed by Eq.(32).

$$\begin{bmatrix} \gamma - 2g^T Hx - g^T g & x^T H^T \\ Hx & I \end{bmatrix} > 0 \quad (29)$$

$$A_D(P_D E_D^T + V S_D U^T) + (E_D P_D + U S_D^T V^T) A_D^T < 0 \quad (30)$$

$$P_f A_f + A_f^T P_f < 0 \quad (31)$$

$$b_{m_a} > 0 \quad (32)$$

Step7 Step4 to 6 are repeated until the value of γ becomes sufficiently small. Here, $G_f(s)$ used in Step4 and 5 is an updated solution obtained in Step6.

Remark: Equations (27) and (28) are equivalent to conditions such that maximum singular values of P_D and P_f reduce.

5. Numerical simulations

5.1. Plant

In order to evaluate the performance of the proposed method, the following stable continuous-time LTI SISO plant is considered,

$$G_p(s) = \frac{81.02(s^2 + 0.0009255s + 137.9)}{(s^2 + 17.38s + 416.6)(s^2 + 6.926s + 26.81)}. \quad (33)$$

Figure 4 shows the frequency response of $G_p(s)$. $G_p(s)$ is not ASPR because the relative degree is 2. In addition, $G_p(s)$ includes the anti-resonance mode. The design of the PFC for $G_p(s)$ is shown in the following chapter.

5.2. PFC design

5.2.1. Design by the proposed method

The desired model is set as follows,

$$G_r(s) = \frac{6.371s^2 + 416.3s + 1006}{s^3 + 24.12s^2 + 211.9s + 828.4}. \quad (34)$$

Figure 5 shows the frequency response of $G_r(s)$. The broken line and the solid line show $G_p(s)$ and $G_r(s)$, respectively. $G_r(s)$ has large gain at the anti-resonance frequency of $G_p(s)$. The initial value of the PFC was specified as

$$G_{f0}(s) = \frac{n_{f0}(s)}{d_{f0}(s)} = \frac{20(s^2 + 2 \cdot 20\pi s + (20\pi)^2)}{(s + 10)(s^2 + 2 \cdot 40\pi s + (40\pi)^2)}. \quad (35)$$

Fig. 4 Bode plot of $G_p(s)$ Fig. 5 Bode plot of $G_p(s)$ and $G_r(s)$ Fig. 6 Bode plot of $G_r(s)$ and $G_a(s)$ Fig. 7 The trajectory of γ

The denominator polynomial of $M(j\omega)$ was specified as $d(j\omega) = d_{f0}(j\omega)$. Reference frequencies, ω_k , were specified as the following 9 points; 1×10^{-4} , 1×10^{-3} , 1×10^{-2} , 0.5, 1, 2, 1×10^2 , 1×10^3 , 1×10^4 Hz. Step4-6 was iterated 800 times. The PFC was obtained as follows,

$$G_f(s) = \frac{6.381s^2 + 359.5s + 248.2}{s^3 + 22.6s^2 + 166s + 772.3}. \quad (36)$$

Figure 6 shows the frequency response of $G_a(s)$ which is constituted using the PFC of Eq.(36). The broken line and the solid line show $G_a(s)$ and $G_r(s)$, respectively. According to Fig.6, the frequency response of $G_a(s)$ was matched well to that of $G_r(s)$. The convergence trajectory of γ is shown in Fig.7. γ reduced by the iteration.

5.2.2. Comparison with the conventional method

In this section, the performance between the proposed method and the conventional method is compared. In the conventional method (Tanemura, et al., 2013), the leading coefficient is given preliminarily in order to ensure the minimum phase property of the augmented system. The decision of the leading coefficient is as follows: The leading coefficient of the PFC is determined such that the leading coefficient of $G_a(s)$ becomes equal to that of $G_r(s)$. Owing to this, the condition III) of the ASPR condition is satisfied. In addition, the frequency response of $G_a(s)$ can fit that of $G_r(s)$ in the high frequency band because the band depends on the leading coefficient. The PFC was designed in these two cases.

$$\text{Case1: } b_{n-1} = 6.371 \quad (37)$$

$$\text{Case2: } b_{n-1} = 10 \quad (38)$$

Where, Case1 is that the leading coefficient appropriately is given, and Case2 is that the decision procedure of the leading coefficient is not known. An initial value of the PFC was described as follows,

$$G_{f0}(s) = \frac{b_{n-1}(s^2 + 2 \cdot 20\pi s + (20\pi)^2)}{(s + 10)(s^2 + 2 \cdot 40\pi s + (40\pi)^2)}. \quad (39)$$

Reference frequencies, ω_k , were specified as the following 8 points; 1×10^{-4} , 1×10^{-3} , 1×10^{-2} , 0.5, 1, 2, 10, 1×10^2 Hz. Step4-6 was iterated 10 times. PFC were obtained as follows.

$$\text{Case1: } G_f(s) = \frac{6.371s^2 + 289.3s + 83.17}{s^3 + 17.2s^2 + 147.6s + 483.2} \quad (40)$$

$$\text{Case2: } G_f(s) = \frac{10s^2 + 161.8s + 10.59}{s^3 + 7.001s^2 + 96.51s + 61.55} \quad (41)$$

Fig. 8 Bode plot of $G_r(s)$ and $G_a(s)$ of Case1 and Case2

Fig. 9 The trajectory of γ of Case1 and Case2

Table 1 Comparison of the performance

	Proposed method	Case1	Case2
Number of times up to convergence	800	3	3
Fit rate[%]	96.0	98.8	82.0

Figure 8 shows the frequency responses of $G_a(s)$ which are constituted using the PFC of Eq.(40) and (41). The solid line, the broken line and the dotted line show $G_r(s)$, $G_a(s)$ (Case1) and $G_a(s)$ (Case2), respectively. Case1 achieved the sufficient fitting of the frequency response, however, Case2 was insufficient. The convergence trajectories of γ are shown in Fig.9. The broken line and the dotted line show Case1 and Case2, respectively. The approximate number of times up to convergence and the fit rates (Adachi, 2009) in the proposed method, Case1 and Case2, are shown in Table1. The proposed method required much more iteration because the degree freedom of designing increased by the leading coefficient. The performance of the proposed method was nearly equal to that of the Case1 in regard to the fit rate. By contrast, the fit rate decreased when the leading coefficient was inappropriately as Case2. Accordingly, the proposed method, although it required much iteration, gave good matching without considering the leading coefficient.

5.3. Simulation conditions

The vibration suppression effect of the PFC designed by the proposed method is verified by numerical simulations. The conventional method was assumed that SAC was applied by using the PFC of Eq.(42) which was designed without considering the anti-resonance. The proposed method was assumed that SAC was applied by using the PFC of Eq.(36) which was designed by LMI/BMI. It was assumed that the reference input, $u_m(t)$, is the square wave of which the amplitude and period are 1 and 50 s, respectively. SAC controllers were used the followings.

$$\begin{aligned}
 G_f(s) &= \frac{0.001}{0.1989s + 1} \\
 G_m(s) &= \frac{\omega_n^2}{s^2 + 2 \cdot 0.707\omega_n s + \omega_n^2}, \quad \omega_n = 0.8 \cdot 2\pi \\
 \Gamma_P &= \text{diag} \{ 10, \quad 1, \quad 1, \quad 0.1 \} \\
 \Gamma_I &= \text{diag} \{ 100, \quad 1, \quad 1, \quad 1 \} \\
 \sigma(t) &= \frac{e_a^2(t)}{1 + e_a^2(t)} \sigma_1 + \sigma_2, \quad \sigma_1 = 1 \times 10^{-2}, \sigma_2 = 1 \times 10^{-5} \\
 k_I(0) &= [-20, \quad 0, \quad 0, \quad 1]^T
 \end{aligned} \tag{42}$$

An approximate differentiator was used as the derivative gain of the PD controller.

5.4. Simulation results

The time responses of the control input, $u_p(t)$, and the control output, $y_p(t)$, are shown in Fig.10 and 11. The time responses from 0.5 s to 4 s are shown in Fig.12 and 13. The dotted line and the solid line show the conventional method and the proposed method, respectively. The proposed method suppresses the vibration of the input in comparison with the conventional method. In consequence, the vibratory output doesn't appear in the proposed method. The simulation results show the effectiveness of the proposed method for suppression of vibration by the effect of the proposed PFC design.

Fig. 10 Time response of $u_p(t)$ Fig. 11 Time response of $y_p(t)$ Fig. 12 Time response of $u_p(t)(0.5-4\text{ s})$ Fig. 13 Time response of $y_p(t)(0.5-4\text{ s})$

6. Conclusion

It is shown that the conventional method (Tanemura, et al., 2013) cannot treat the leading coefficient of the PFC as the design parameter in order to ensure the minimum phase property of the augmented system. In this paper, the PFC design method using the stability theorem of descriptor systems to ensure minimum phase property is proposed. Owing to this, it is shown that all parameters of the PFC can be treated as design parameters. The numerical simulations show that the PFC designed by the proposed method suppresses the vibratory input effectively. Good results are also obtained by experiments. They will be shown at the presentation.

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