A3 Singularity Avoidance for Control Moment Gyro Systems Using State Feedback Control Law

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This paper is on singularity avoidance on of the satellite using Control Moment Gyros as attitude control device. The Control Moment Gyros has very high ability for the space structures to control its attitude, but there remains the problem of singularity in the system. The singularity avoidance control law using state feedback is introduced and examined in this paper. The state feedback control law uses idea from optimal control theory and traditional laws to control the system. The control law shows ability to avoid the singularity with online feedback and keeps the designated constraint of the control.

Key Words: Control Moment Gyros, Singularity Avoidance, Optimal Control

INTRODUCTION

The Control Moment Gyros (CMGs) has been one of the feasible attitude controllers for satellite in space environment for their high maneuverability and low energy consumption. The CMGs have ability to produce some hundred times higher torque than that of the reaction wheels. The CMGs are installed on large space structures like ISS and small agile satellites like Peiades-HR. The ability of the CMG will enhance usage of space environment in near future.

GMGs have varieties of configurations. Some are lined up in parallel rows and some are in square shape. One of the most popular configurations of the CMG is pyramid type 4 single gimbal (SG) – CMGs as in figure 1. This type of the CMG system is used as basic configuration and studied well by many researchers[1][4].

Although the ability of the CMGs are extremely high, there is some problems that is related to singularity. The singular problem appears where the control law has 0 in the denominator which makes the control input diverge. This problem occurs in almost all occasions using the 4 SG-CMG systems and requires singular avoidance control system.

As the singular avoidance control law, many researchers have been studying since 1970s and still lots of control laws are designed[5][9]. The designated control laws has ability to avoid the singular points, but basically uses the control law itself to make different trajectory. This procedure still has problems for keeping constraints and chattering in some control laws.

This paper introduces state feedback control law designed based on the ideas from optimal control theory and avoids singular points using the reference state feedback.

SIMULATION MODEL AND SINGULARITY PROBLEM

This section explains the fundamental principles of the satellite with 4 SG-CMGs in pyramid configuration and its singularity problem considered in this paper.

Satellite and CMG Model

Figure 1 shows the schematic image of satellite with pyramid configured CMGs. The satellite consists of 4 single gimbal for control in squared configuration in bottom of the pyramid. This is commonly called as pyramid type 4 SG-CMG system and most of actual existing CMGs has this configuration.

The system of this pyramid type 4 SG-CMG can be written as the following equations. First, the angular momentum of the CMG is written as,
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The derivative of the angular momentum is,

$$H_{CMG} = \dot{H}_0 \mathbf{A}(\mathbf{x}) = H_0 \mathbf{A}(\mathbf{x}) \mathbf{u}$$

(2)

and,

$$\mathbf{A} = H_0 = \begin{bmatrix}
-\cos \gamma \cos \alpha_1 & \sin \alpha_1 & \cos \gamma \cos \alpha_2 & -\sin \alpha_2 \\
-\sin \alpha_1 & -\cos \gamma \cos \alpha_1 & \sin \alpha_1 & \cos \gamma \cos \alpha_2 \\
\sin \gamma \cos \alpha_1 & \sin \gamma \cos \alpha_2 & \sin \gamma \cos \alpha_3 & \sin \gamma \cos \alpha_4 \\
\sin \gamma \cos \alpha_3 & \sin \gamma \cos \alpha_4 & \sin \gamma \cos \alpha_1 & \sin \gamma \cos \alpha_2
\end{bmatrix}$$

(3)

The equation shows that the state equation of the CMG can be written as the simple product of the nonlinear matrix and the input. This system is widely known as the symmetric affine systems.

The Euler equations of the satellite with the CMG cluster can be written as,

$$\frac{d\mathbf{H}_b}{dt} + \omega \times \mathbf{H}_b = 0$$

(4)

where,

$$\mathbf{H}_b = I_0 + H_{CMG}$$

(5)

As whole system, the constraint is given as,

$$\mathbf{A}(\mathbf{x}) \mathbf{u} - \mathbf{S}(\mathbf{x}, t) = 0$$

(6)

where,

$$\mathbf{S} = \mathbf{H}_{CMG} = -I_0 - \omega \times (I_0 + H_{CMG})$$

(7)

Here, the first term of the equation (6) is the required torque for the system which will be the input for the system.

Furthermore, for the attitude control around non euler axis, the attitude angle and angular velocity of the satellite will be shown in quaternion as shown in the following equations.

$$\omega = \begin{bmatrix}
q_4 & q_3 & -q_2 & -q_1 \\
-q_3 & q_4 & q_1 & -q_2 \\
q_2 & -q_1 & q_4 & q_3 \\
q_1 & q_2 & -q_3 & q_4
\end{bmatrix}$$

(8)

Basic Control Law

The basic control law for the CMG systems is the pseudo inverse matrix of equation (6) which gives least square of the input following the constraint, that is,

$$\min(\mathbf{u}^T \mathbf{u})$$

(10)

under the condition of,

$$\mathbf{A} \mathbf{u} - \mathbf{S} = 0$$

(11)

which gives the basic control law for the CMG systems as,

$$\mathbf{u} = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \mathbf{S}$$

(12)

This is the main control law for the CMG systems under designated constraint for the satellite motion.

CONTROL LAW AND SIMULATION RESULTS

This section explains the singular problem and the control law to avoid this singular point using state feedback law with simulation results.

Singularity Problem

The singularity of the CMG systems is well studied among many researchers throughout the world. This shows how important the CMG system itself is and the criticalness of the singular points. The problem of the singularity occurs where input equation (12) cannot obtain the result, that is,

$$\det(\mathbf{A} \mathbf{A}^T) = 0$$

(13)

which leads the input diverges to infinity. As for the basic avoidance for this problem, the singularity-robust (SR) steering logic is introduced to avoid the singularity. This method inserts certain value in the inverse matrix in equation (12), and avoids the singular point. The insert value is inspected in variety of ways but there lefts the problem that the constraint of the required torque cannot be preserved through this method.
Singularity Avoidance with State Feedback Control Law

One of feasible way to control the CMG system is using so called null-motion control,

\[ u = \left[ I - A \left( AA^T \right)^{-1} \right] (x_{r} - x) \]  \hspace{1cm} (14)

To verify this equation, the matrix A is premultiplied as,

\[ Au = \left[ A - AA^T \left( AA^T \right)^{-1} \right] (x_{r} - x) = 0 \]  \hspace{1cm} (15)

which means the motion is derived with torque constraint of 0, that is null-motion. This input is known to have a Lyapunov function of,

\[ V = \frac{1}{2} (x_{r} - x)^T (x_{r} - x) \]  \hspace{1cm} (16)

and, \( \dot{V} \) is always locally negative from the characteristic of matrix in equation (14). This null-motion control is designed to change the configuration of the gimbal angles during the satellite is under stable state.

From these two control theories, the state feedback control law for singular avoidance is designed. First let the problem be,

\[ \min (u^T u + \rho (x - x_r)^T u) \]  \hspace{1cm} (17)

under the condition of,

\[ Au - S = 0 \]  \hspace{1cm} (18)

The second factor of equation (17) is the time derivative of the Lyapunov function and has stability to converge when it is negative. To design the control law, the Lagrange multiplier is brought in as following,

\[ L = \frac{1}{2} u^T u + \rho (x - x_r)^T u + \lambda^T (Au - S) \]  \hspace{1cm} (19)

The necessary condition of minimizing (17) can be calculated by following conditions.

\[ \frac{\partial L}{\partial u} = \frac{\partial L}{\partial x} = 0 \]  \hspace{1cm} (20)

Solving these equations, the input \( u \) can be calculated as follows (see Appendix).

\[ u = A^T \left( AA^T \right)^{-1} S + \rho \left[ I - A \left( AA^T \right)^{-1} \right] (x - x_r) \]  \hspace{1cm} (21)

This control input is combination of the pseudo inverse input of the torque constraint and null-motion control. They are originally designed to be used differently, but by combining the criteria into one law, the similar control input can be derived with different procedure. Since the latter half of the input is known to be stable, the input can be used anytime except the case where the A matrix meets the condition of singularity as equation (13).

To control the satellite avoiding the singularity, the \( x \), of the input (21) can be used as reference trajectory. Supposing that the singular points are unknown, the ideal control will be online feedback control that avoids the singularity when the state comes close to singular point. So in this study, the reference state \( x_r \), is treated as feedback for avoiding the singular point, using determinant of \( AA^T \).

Simulation Results

The simulation results are shown in this section. First the configuration of the satellite used in this study is as follows.

\[ I = \begin{bmatrix} 21400 & 0 & 0 \\ 0 & 20100 & 0 \\ 0 & 0 & 5000 \end{bmatrix} \]  \hspace{1cm} (22)

\[ H_0 = 1000 \]

\[ \gamma = 53.13 \]  \hspace{1cm} (23)

The initial condition of the state variable is,

\[ x(0) = [-50 \ 50 \ -50 \ 50]^T \]  \hspace{1cm} (24)

and the constraint of the system is,

\[ \begin{bmatrix} \theta \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} 6.0 \sin \left( \frac{\pi}{30} k \right) \\ 6.0 \sin \left( \frac{\pi}{30} k \right) \\ 0 \end{bmatrix} \]  \hspace{1cm} (25)

The following is example of the control without state feedback. The input is given by equation (12) with only pseudo inverse matrix. Figure 2 is the time history of gimbal angle through the control by basic control law. The figure

Fig.2 Gimbal angle (without feedback control)
shows that the system drops into singular condition around 5 sec after the control has started. Figure 3 is time history of determinant as of equation (13), clearly shows that the singular condition is caused by the divergence of the control.

In figure 4, the time history of the attitude angle of the satellite is shown. From the constraint, the $\theta$ and $\phi$ should be the same, and $\psi$ should be 0 throughout the control. After 5 sec when the input diverges, the attitude angle clearly moves out of the constraint. This problem is one of the major problems that appear in CMG systems.

To avoid this singularity problem, the state feedback control law is brought in. The system uses control law designed as equation (21). For the reference $x_r$, the following law is brought in.

$$x_r = \frac{x_{r0}}{\det(\mathbf{AA}^T)}$$

and,

$$x_{r0} = [-10 \ 10 \ -10 \ 10]^T$$

This reference state is given to move the gimbal angles away from the set of the points where the singular points are.

Using this control law, figure 5 shows the time history of the gimbal angle with feedback control law. At the beginning, the gimbals move very quickly to certain state and then stabilizes after about 0.5 sec. Figure 6 is time history of the determinant in this case. We can see the drop of the value in very beginning of the control then stabilizes up after the drop. The control law seems successfully avoid the singular point by using the newly designed state feedback control law. The singularity that appeared in the beginning of the control seems to be caused by the state feedback control law. Although the
gimbal angles got closer to the singular points, they succeeded to avoid the singularity and continued the control. Figure 7 is time history of the satellite attitude angle, which shows that $\theta$ and $\phi$ are the same and $\psi$ is kept as 0.

To compare 2 results, the satellite attitude angle after 30 sec is shown in figure 8 the bold lines are with feedback control and thin lines are without feedback control. After 30 sec, all angles are designed to come back to 0.0 deg. Comparing these 2 cases, all of the bold lines are closer to 0 and shows that they have ability to keep the constraint. Especially in the yaw angle which should be kept 0 through the control, the case with feedback has 5 times or better result than the original control law.

CONCLUSION

The state feedback control law for singularity avoidance of satellites with CMG system is introduced and examined. The control law has shown feasible avoidance ability to control the gimbal angles out of the singularity with keeping the designated constraints with simple feedback control theory. The control law is designed using the idea of optimal control theory which matches with the traditional inverse kinematics theory.

The control law still needs improvement in cases where the singular avoidance is not needed. Also there is some revisions needs to be made for the direction of avoidance. Thought these problems are still needed to be solved, these points can be used for the allowance for improvement in the control. The base of the control shows feasible ability for more improvements to appear using this control law.

NOMENCLATURE

- $H_{CMG}$: total CMG angular momentum vector
- $H_{B}$: constant magnitude of the angular momentum of each CMG
- $\gamma$: pyramid skew angle
- $\alpha$: gimbal angle
- $A$: Jacobian matrix

REFERENCES

APPENDIX

From equation (20), the differential equations are,

\[
\frac{\partial L}{\partial u} = u^T + \rho (x - x_0)^T + \lambda^T A = 0 \tag{A1}
\]

and,

\[
\frac{\partial L}{\partial \lambda} = Au - S = 0 \tag{A2}
\]

From equation (A1),

\[
u = -\rho (x - x_0) - A^T \lambda \tag{A3}\]

Substitute \(u\) into equation (A2),

\[
\rho A(x - x_0) + AA^T \lambda + S = 0 \tag{A4}
\]

\[
\lambda = -\rho (AA^T)^{-1} A(x - x_0) - (AA^T)^{-1} S \tag{A5}
\]

Substitute \(\lambda\) into equation (A3),

\[
u = -\rho (x - x_0) + \rho A^T (AA^T)^{-1} A(x - x_0) + A^T (AA^T)^{-1} S \tag{A6}
\]

which is equal to equation (21).