Analysis of Adaptive Sliding Mode Controller using Control Moment Gyros for Microsatellite
超小型衛星のためのコントロールモーメントジャイロを用いたアダプティブスライディングモードコントローラの解析

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Abstract
This paper analyzes an application of adaptation methodology in attitude control system for microsatellite. The control method is based on MPP (modified Rodrigues parameters) feedback control principle. The adequate adjustments of the magnitude of a discontinuous control are discussed here for the state trajectories are in and out of sliding surface. This control methodology is based on the equivalent control application and will reduce control action magnitude to the minimum possible value. The simulation results show the efficiency and feasibility of the proposed control method.

1. Introduction
Many researchers have focused on attitude control technology of large-angle maneuver for agile microsatellite. Since the rotational kinematics and attitude dynamics of spacecraft are strongly coupled nonlinear equations during the attitude control period and stabilization problems, the attitude control problem is an essentially complex nonlinear control issue. The attitude control problem considers how to reduce error of attitude represents and angular velocity to zero in a finite time. This problem is also equivalent to stabilization of error of attitude represents and angular velocity for an observation task over a required time.

Bilimoria\(^1\) and Fleming\(^2\) researched the minimum time rest-to-rest reorientation control problem of an inertial symmetric rigid spacecraft with independent three-axis control. The computed optimal controls and motion geometry can provide large torque to finish maneuver in less time corresponding to reorientations about a control axis. Creamer\(^3\) used a Bang-coast-Bang profile to conduct the large-angle optimal slew maneuver control for the Clementine spacecraft. A classical PID controller with quaternion feedback gains was chosen to ensure control accuracy. Junkins\(^4\) presented an optimal large-angle rotational maneuver method based on a relaxation process for an arbitrary specification. The optimal attitude maneuver method typically has some drawbacks like chattering effect, poor robust and adaptive capability, etc.

On the other hand, Junkins\(^5\) used Modified Rodrigues parameters to denote spacecraft kinematic equation and applied a nonlinear adaptive tracking control law to conduct near-minimum-time maneuvers. This method allowed presence of uncertainty in the inertia matrix and errors in the initial attitude determination system. But parameter convergence was not guaranteed in general even though this adaptive tracking control system was demonstrated its feasibility by computer simulations. Abdelrahman\(^6\) showed an attitude maneuver algorithms based on combination of adaptive nonlinear control and neuro control for Space Station. This method is suitable for varying inertia characteristics spacecraft using a memory filter. However, stability and performance robustness of these techniques and the memory filter need to do much more examination.

For nonlinear system of feedback control structure, a kind of integral back-stepping robust controller\(^7\) is very suitable for spacecraft with large angle maneuver. Kristiansen\(^8\) presented a linear back-stepping controller for attitude tracking task using reaction wheel and thruster as actuator. Kim\(^9\) purposed a nonlinear back-stepping controller in tracking maneuver. A nonlinear tracking function was applied to overcome the problem of control input and convergence introduced by linear back-stepping design. However, the nonlinear controller has many parameters to be tuned. This process is too complicated to apply for project implementation.

Recently, sliding mode control is a type of closed loop methodology of VSC (Variable Structure Control), which has been considered and applied for use in an attitude control system by many researchers. This control method ensures the precise and stable pointing accuracy during the slew maneuvering process. S. R. Vadali was the first to apply the principles of variable-structure control theory to the large angle maneuver problem\(^10\). The demand torque was generated by jet thrusters. However, the stability of the ideal kinematics cannot be prescribed independently for each orientation parameter. The following improvement was motivated by Dwyer and A. W. Thomas\(^11\). They used Cayley-Rodrigues sliding mode controllers for attitude reorientation and detumbling maneuvers. This controller drove the errors to zero successfully using a linear model.
and a linear sliding surface. B. B. Goeres\textsuperscript{(3)} derived a general attitude representation sliding mode controller. Assumption for this type of controller is that the angular velocity error is equal to the derivation of attitude error. However, the approximation cannot work well if attitude errors are large even if the simulation results shown that it was successfully tracked a specified ground station.

For normal sliding mode, there are two main problems exists: chattering effect and high frequency switch action. Actually, the amplitude of chattering effect is proportional to the magnitude value of a discontinuous control. These two phenomena can be handled simultaneously if the magnitude value is reduced to a proper level controlled by the definitions of sliding mode to exist. The design requires the knowledge of the bound on the uncertainties, which could be, from a practical point of view, a hard task: it often follows that this bound is overestimated, which yields excessive gain. Then, the main drawback of the sliding mode control, the well-known chattering phenomenon, is important and could damage actuators and system. Practically, we have two ways to reduce the chattering effect. First, a boundary layer was used to tune the control gains. Second, higher order controller is used. Both methods are determined on the boundary of disturbance. For adaptive sliding mode control method, the control gain for attitude control is chosen to be as small as possible while is sufficient to control uncertainties and perturbations at the same time.

The basic principle of the adaptive control approach is modifying the attitude control law to cope with the parameters of the system which is slowly time-varying or environmental changing furthermore. This paper proposed a method that guarantees the pointing accuracy and convergence speed for large angle maneuver.

2. Problem Statement

2.1 Attitude Dynamics with Control Momentum Gyros

The satellite modeled here is assumed to be a rigid body with actuators acting along three mutually perpendicular axes (body axes). The attitude dynamic equation of satellite’s motion is described in the following expression\textsuperscript{(3)}

\[ \dot{J}\omega = -\omega \times J\omega + T_c + T_d \]  

(1)

where \( J \) is the inertial matrix of satellite, \( \omega \) is the angular velocity, \( T_c \) is the control torque that is generated by micro control moment gyros and \( T_d \) is the disturbance torque. Equation 1 describes the general attitude control motion under ideal circumstances. In practice, it is a significant challenge to design an ACS using CMGs because of the actuator saturation limits, torque-generated error and internal singularity problem. The desired CMG momentum rate is often denoted as

\[ \bar{T}_c = -h - \omega \times h \]  

(2)

where \( \bar{h} \) is the CMG angular momentum rate, which in general, is a function of CMG gimbal angles \( \delta \). This paper discusses the pyramid model of four single-gimbal CMGs with a skew angle of \( \beta \). The illustration of CMGs pyramid configuration is shown in Fig. 1.

![Illustration of the CMG Pyramid Configuration](image)

The total CMG momentum can be expressed in matrix form as

\[
h = h_1(\delta_1) + h_2(\delta_2) + h_3(\delta_3) + h_4(\delta_4)
\]  

\[
= \begin{bmatrix}
-\cos \beta \sin \delta_1 \\
\cos \delta_1 \\
\sin \beta \sin \delta_1 \\
-\cos \delta_2 \\
-\cos \beta \sin \delta_2 \\
\sin \delta_2 \\
-\cos \delta_3 \\
\cos \beta \sin \delta_3 \\
\sin \delta_3 \\
-\cos \delta_4 \\
\cos \beta \sin \delta_4 \\
\sin \delta_4
\end{bmatrix} 
\]  

(3)

where \( h_i (i=1,2,3,4) \) and \( \delta_i (i=1,2,3,4) \) are the angular momentum and gimbal angle of each CMG, respectively. Additionally, we have the assumption that the measured attitude parameters are well estimated for attitude maneuver, as described in Section 3.

2.2 Attitude Representation

The modified Rodrigues parameters can be used to provide effective attitude representation for attitude control and can be transformed directly from the classical Rodrigues parameters. In this paper, the MRP vector \( \mathbf{\sigma} \) is defined in terms of the principle rotation elements \( (\Phi, \varepsilon) \)

\[ \mathbf{\sigma} = \tan \frac{\Phi}{4} \mathbf{e} \]  

(4)

From Eq. (4), we can find that the MRP vector has a mathematical singularity at a rotation angle of \( \pm 360^\circ \). Because \( |\mathbf{\sigma}| \) has the expression of \( |\mathbf{\sigma}| = \sqrt{\mathbf{\sigma}^T \mathbf{\sigma}} \), it is clear that the following apply:

\[ |\mathbf{\sigma}| \leq 1 \quad \text{if} \quad \Phi \leq 180^\circ \]

\[ |\mathbf{\sigma}| \geq 1 \quad \text{if} \quad \Phi \geq 180^\circ \]

(5)

Different attitude representations have the inherent relationships, as discussed in reference 10. Given two MRP vectors \( \mathbf{\sigma}_a \) and \( \mathbf{\sigma}_b \), we are able to express the attitude error vector \( \Delta \mathbf{\sigma} \) from \( \mathbf{\sigma}_a \) to \( \mathbf{\sigma}_b \) as

\[ \Delta \mathbf{\sigma} = \frac{1}{1 + |\mathbf{\sigma}_b|^2} (|\mathbf{\sigma}_b|^2 |\mathbf{\sigma}_a| - 2 \mathbf{\sigma}_b \times \mathbf{\sigma}_a) \]  

(6)

where \( \mathbf{\sigma}_b \times \) is the skew symmetric matrix that has a normal form in the following expression

\[ \mathbf{\sigma}_b \times = \begin{bmatrix}
0 & -\sigma_{a3} & \sigma_{a2} \\
\sigma_{a3} & 0 & -\sigma_{a1} \\
-\sigma_{a2} & \sigma_{a1} & 0
\end{bmatrix} \]  

(7)

where \( (\sigma_{a1}, \sigma_{a2}, \sigma_{a3}) \) defines the MRP vector \( \mathbf{\sigma}_a \) in terms of the Euler parameters. Eq. (6) appears to be more
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complicated than the other attitude represents counterparts. However, it provides a numerically efficient method to compute the composition of two MRP vectors. The MRP kinematic differential equation in vector form can be expressed as

\[
\dot{\sigma} = \frac{1}{4} \left[ l_3 \sigma^2 + 2r x + 2r r^2 \right]
\]

or

\[
\dot{\sigma} = \frac{1}{4} \left[ B(\sigma) \right] \dot{\omega}
\]

where \(B(\sigma) = \left[ l_3 \sigma^2 + 2r x + 2r r^2 \right]. I \) is the identity matrix. The matrix formulation of kinematic equation is

\[
\sigma = \begin{bmatrix}
1 - \sigma^2 + 2\sigma_1^2 \\
2(\sigma_1 \sigma_2 - \sigma_3) \\
2(\sigma_2 \sigma_3 - \sigma_1)
\end{bmatrix}
\]

or

\[
\omega = \frac{4}{(1 + \sigma^2)^2} \left[ l_3 - 2(\sigma \sigma^2 - \sigma^2 I_3) + 2(2r r^2) \right]
\]

These equations are used in the following part of the adaptive attitude feedback control algorithm.

2.3 Error System

Let \(\Delta \omega\) represent the angular velocity tracking error described in the body frame with respect to the inertial frame. \(\Delta \omega\) satisfies the following equation

\[
\Delta \omega = \dot{\omega} - R \dot{\omega}_i
\]

where \(\omega_i\) is the target angular velocity with respect to the inertial frame and \(R\) is the corresponding rotation matrix from the inertial frame to the body frame. The attitude control problem considers how to reduce \(\Delta \sigma\) and \(\Delta \omega\) to zero in a finite time. This problem is also equivalent to stabilization of \(\Delta \sigma\) and \(\Delta \omega\) for an observation task over a required time.

2.4 Actuator Restrictions

The above equations describe the general attitude control motions under circumstances. In practice, the situation is further complicated when CMGs are adopted as control actuators because of their internal singularity problem. First, gimbal rate limits will cause state dependent and time-varying control input saturation limits. Additionally, by taking the CMG gimbal friction as un-modeled disturbances, it is assumed that the control signal strictly dominates the unknown disturbance. Second, we use the singular robust (SR) inverse methods that permit error in the output to get gimbal rate command. However, in the case of microsatellites like TSUBAME, the minimum driving step can only achieve 0.1 deg/s due to mechanical driving restrictions of the gimbal motor. Thus, the corresponding minimum torque generated by the CMGs \(h = 0.0527 \text{ Nm}\) is about \(1 \times 10^{-4} \text{ Nm}\), while the maximum magnitude of environmental disturbance is about \(5 \times 10^{-6} \text{ Nm}\). Consequently, control overshooting has an un-ignoreable impact during the stabilization phase. Here, we assumed that control torque errors introduced by steering logic and steering resolution have an upper boundary as \(|\Delta T_c| \leq e_{\text{max}}\).

3. Design of Adaptive Sliding Mode Controller

Define the sliding surface as

\[
s = K_p \Delta \omega + K_i \int_0^t \omega \text{d}t
\]

where \(K_p\) and \(K_i\) are \(3 \times 3\) positive constant matrix and a symmetric positive-definite constant matrix, respectively. If the system remains on the non-linear sliding manifold \(s = 0\), the system will track the desired state. The motion of closed-loop system using the sliding mode control law is composed of 2 modes. The first mode is a reaching mode where the states beginning from arbitrary states are attracted to the sliding surface. In the second mode, the states slide along the surface and the state error converges to zero. Hence, once the sliding surface has been chosen, a controller should be designed to make the sliding surface be an attractive surface. The differential of the sliding variable is described as

\[
\dot{s} = K_p (\omega \times \omega) + K_i \int_0^t \omega \text{d}t
\]

The feedback control torque of CMGs can be obtained as

\[
T_{\text{CMG}} = J(K_p (-J^{-1} \omega \times J \omega) + J^{-1} \omega \text{d}t + R \dot{\omega}_i
\]

where the description of \(T_{\text{CMG}}\) is control torque of CMGs. \(T_{A-SMC}\) is the adaptive compensation part which is formulated on a saturation function

\[
T_{A-SMC} = -y \Delta \sigma \text{sgn}(s)
\]

where \(y\) is a boundary thickness. And higher \(y\) value will make control emphasis from \(\Delta \omega\) to \(\Delta \sigma\). The upper bound of parameter is defined as

\[
y = \Omega = \{y: 0 < y_{\text{min}} \leq y \leq y_{\text{max}}\}
\]

Eq. 15 is also an important compensation part for the whole control system. Therefore, controller can be rewritten as

\[
T_c = T_{\text{CMG}} + T_{A-SMC}
\]

Choosing the Lyapunov candidate as

\[
V = 2K_p (\log(1 + \omega^2 \Delta \omega)) +
\]

the first derivative of \(V\) is then given by

\[
\dot{V} = -\Delta \omega^2 K_p \Delta \omega - \Delta \omega^2 K_i \int_0^t \omega \text{d}t +
\]

Then we can get expression as follows

\[
\dot{V} \leq -\Delta \omega^2 K_p \Delta \omega - \gamma \Delta \sigma \|\|
\]

From Eq. 18, it can be stated that the equilibrium state is stable. Finally, it can be shown that the error state \(\Delta \omega\) tends to zero as time \(t\) goes to \(\infty\) by LaSalle-Yoshizawa.
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Fig. 2 shows the attitude control scheme with the proposed algorithms and steering logic of the CMGs. The Global SR steering logic is adopted for singularity avoidance.

4. Numerical Simulation

TSUBAME is a fourth satellite developed in the Laboratory for Space Systems (LSS) at Tokyo Institute of Technology and Institute of Space and Astronautical Science (ISAS) in Japan Aero Exploration Agency. The 50kg TSUBAME is a demonstration microsatellite for Earth and astronomical observation technology and has successfully launched on 6th, November this year. One important task for this satellite is Science (GRB) Observation mode; it requires that satellite can perform a maneuvering up to 90 degrees in 15 second. This is because GRB abruptly occurs in the undefined direction and disappears in a short time. Therefore, this is the main reason why a high-speed position changing technology using the CMG is requested. This challenge makes attitude determination and control system (ADCS) becomes one of the key technologies of TSUBAME. Main aspects of the ADCS are new proposed attitude control and attitude determination algorithms, a multiplicity of sensors and actuators due to redundancy and/or power saving, and a health monitoring system to reduce the possibility of system failures.

The general feasibility of the proposed control algorithm is demonstrated through a slew maneuvering of attitude control from Euler angles about [Roll, Pitch, Yaw]= [0, 0, 0] to [0, 0, 90]. Non-zero initial angular velocities are assumed to bring an initial angular momentum for practical consideration. Table 1 gives the main simulation parameters of the control system. Both gravity gradient torque and residual magnetic torque are considered as environmental disturbance.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia matrix, $kg \cdot m^2$ (Main Body)</td>
<td>$[1.82249024-4.39508890e-04-1.9831016e-03; -4.39508890e-04 1.8605438-7.7562381e-03; -1.9831016e-03-7.7562381e-03 1.9468517;]$</td>
</tr>
<tr>
<td>Skew-angle, deg</td>
<td>54.75</td>
</tr>
<tr>
<td>Gimbal rate (MAX), rad/s</td>
<td>±1</td>
</tr>
<tr>
<td>Gimbal rate acceleration (MAX), rad/s²</td>
<td>1</td>
</tr>
<tr>
<td>Gimbal control step, s</td>
<td>0.1</td>
</tr>
<tr>
<td>Residual magnetic disturbance, $A \cdot m^2$</td>
<td>[-0.07481;-0.17965; -0.04479]</td>
</tr>
<tr>
<td>Initial attitude Euler angle, deg</td>
<td>[0, 0, 0]</td>
</tr>
<tr>
<td>Initial angular velocity, deg/s</td>
<td>[0, 0, 0] or [0.03,0.03,0.03]</td>
</tr>
<tr>
<td>Initial gimbal angle, deg</td>
<td>[0, 0, 0, 0]</td>
</tr>
<tr>
<td>Initial gimbal rate, rad/s</td>
<td>[0, 0, 0, 0]</td>
</tr>
<tr>
<td>Saturation Limit of Control Output, Nm</td>
<td>±[0.05,0.05,0.05]</td>
</tr>
<tr>
<td>Sliding variables</td>
<td>$K_p = diag[0.9,0.9,0.55], K_i = diag[0.02,0.02,0.27]$</td>
</tr>
<tr>
<td>Adaptive control gain</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Figure 3 and Figure 4 show the comparing results of different control methods. Adaptive sliding mode controller has the most fast convergence speed the chattering effect has controlled well. The rapid attitude maneuvering was finished in about 11 sec. The PD control shows relative stable angular velocity during maneuver but with a slower convergence speed. Since the saturation limit of control output of CMGs was set to 0.05 Nm for each axis, from near 3 s to 8 s, the system is near singularity and control torque output are around [0 0 0] Nm for each axis. Each CMG’s gimbal is rotated fast during this period but not to gimbal rate maximum value. After 8 s, the system is get away from singularity and control torque are increased. The system did not trapped into singularity although the gimbal rotated very fast during the attitude control from Fig. 7.
The second simulation case has changed the initial angular velocity to \([0.3 \ 0.3 \ 0.3]\) deg/s for microsatellite, and the target Euler angle is \([40, 60, 90]\) degree. The simulation results are shown from Fig.8 to Fig.14. The system needs relative long time to finish the maneuver (about 45 s) to reach same accuracy 0.5 degree. During the attitude control period, CMGs system is near singularity from 5 s to almost 40 s. The working condition of gimbal can be obtained from Fig.10 gimbal rate and Fig.11 gimbal angle.
global stability of the overall system was discussed and the simulation results comparing with classical sliding mode control and PD control are given.

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Reference