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Combined forced-convective and radiative heat transfer in cylindrical packed beds with constant wall heat flux

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Combined forced-convective and radiative heat transfer in cylindrical packed beds with constant wall heat flux was numerically studied. The zero-equation turbulence model proposed by the authors was incorporated into the governing equations. The macroscopic momentum equation considered the effects of turbulence and hydrodynamic dispersion, in addition to Darcy-Brinkman-Forchheimer flow resistances, while the effects of thermal radiation, turbulence and thermal dispersion were taken into account in the energy equation. With this model, numerical analysis of combined heat transfer in cylindrical packed beds with constant wall heat flux was made, and the results were compared with available experimental data. The agreement is acceptable, showing that proposed numerical model is satisfactory for predicting the characteristics of fluid flow and heat transfer in packed beds.

**Key Words:** Cylindrical Packed Bed, Combined Heat Transfer, Hydrodynamic and Thermal Dispersion, Constant Wall Heat Flux

**Introduction** Recently, the authors have proposed the zero-equation turbulence model (1) particularly considering the hydrodynamic dispersion and thermal radiation and have made numerical investigation for fluid flow and heat transfer in two-dimensional cylindrical packed beds with constant wall temperature to assess the validity of the proposed model. It has been concluded that when the flow is turbulent, the coupled hydrodynamic dispersion and turbulence term in the macroscopic momentum equation could not be disregarded. Moreover, it was found that the numerical analysis considering thermal radiation well predicts the experimental results. The present study is an extension of the previous work and aims at examining the ability of our proposed model in predicting coupled heat transfer in packed beds with constant wall heat flux.

**Governing equations** The assumptions for the numerical analysis are the same as those in previous study. The relevant macroscopic governing equations for steady-state fully-developed turbulent flow in cylindrical packed beds are as follows:

**Continuity equation:**
\[
\frac{\partial \langle \bar{u} \rangle}{\partial x} = 0 ,
\]  
(1)

**Momentum equation:**
\[
\frac{1}{\rho_f} \frac{d \langle \bar{p} \rangle}{dx} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \nu_f \left( 1 + \beta \gamma^h_f \phi \frac{d \langle \bar{u} \rangle}{dr} \right) \frac{\partial \langle \bar{u} \rangle}{dr} \right] - \phi \left[ \frac{\nu_f}{K} + C \langle \bar{u} \rangle \right] \langle \bar{u} \rangle .
\]  
(2)

Here \( \gamma^h_f (\phi) \) denotes the radial mixing function for heat and was given by
\[ \gamma^h_f (\phi) = 0.3519 (1 - \phi)^{2.3819} . \]

\( \beta \) in Eq. (2) is an effective Prandtl number for turbulence and dispersion, and a value of \( \beta \) was adopted as 0.9.

The energy equation:
\[
\rho_c \frac{d \langle \bar{T} \rangle}{dx} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r k_f \left( \gamma^r_f (\phi) + \gamma^c_f \frac{d \langle \bar{u} \rangle}{dr} \right) \frac{\partial \langle \bar{T} \rangle}{dr} \right] - \sigma_a (r) [4 \alpha T^4 (r) - G(r)] ,
\]  
(4)

where \( G \) is the incident radiation.

The boundary conditions are:
\[
r = r_0 : \frac{\partial u}{\partial r} = \frac{\partial T}{\partial r} = 0 ,
\]
\[
r = r_0 : u = 0 , \quad q_r = k_f \frac{\partial T}{\partial r} = H(T_u - T_m) , \quad \phi = 0 : T = T_0 .
\]  
(5)

**P** equations for the radial radiative heat flux and the incident radiation are
\[
\frac{\partial G}{\partial r} + 3 \beta^* (r)(1 - \omega^*) G(r) + 4 \alpha T^4 (r) = 0 ,
\]  
(6)

\[
\frac{\partial G}{\partial r} + 3 \beta^* (r)(1 - \omega^* \rho^*) q_r (r) = 0 .
\]  
(7)

The boundary conditions for these equations are:
\[
r = r_0 : \frac{\partial G}{\partial r} = 0 (or \quad q_r = 0) ,
\]
\[
r = r_0 : -(1 - \rho_s) \frac{G}{4} + (1 + \rho_s) \frac{q_r}{2} = - \epsilon_s \alpha T^4 ,
\]  
(8)

Equations (1) to (8) are combined with empirical relations for the permeability, inertial coefficient, effective thermal conductivity and radiative properties of packed spheres.

**Results and discussion** In order to examine the validity of our heat transfer model for constant wall heat flux condition, computations were carried out under the conditions corresponding to the experiments of Quinton and Sorrow (6) for air-glass bed and of Saitoh and Kamiuto (7) for air-steel bed. A detailed description of the solution
procedure can be found in our companion paper.

![Graph showing variation of local mean particle Nusselt number with experimental data of Quinton and Storrow](image)

**Fig. 1. Comparison of predicted local mean particle Nusselt number with experimental data of Quinton and Storrow**

Fig. 1 illustrates the local mean particle Nusselt numbers $N_{u_p}$ ($= N_{u_m} / 2 \Gamma$) averaged over the heated length of the bed against Reynolds number. The local mean Nusselt number was calculated for two limiting values of the heat flux $\Psi=0.078$ and $\Psi=1.01$ because values of $\Psi$ were not reported for each experimental run. The heated-wall emissivity $\varepsilon_w$ was assumed to be 0.07 for a nickel foil tube. As shown in Fig. 1, the agreement between numerical results and experimental ones is satisfactory.

The distribution of the local Nusselt number for an air-steel bed along the flow direction is shown in Fig. 2. The prediction (solid line) is in reasonable agreement with measurements (symbols).

![Graph showing variations in local Nusselt number along the flow direction](image)

**Fig. 2. Variations in local Nusselt number along the flow direction**

It should be noted that, in the presence of radiation, a thermally-developed state does not occur: Nusselt number increases after it has reached a minimum value. Sharp increase in the experimentally obtained Nusselt number is due to a decrease in the wall temperature near the end of the heating section because of the heat losses.

Fig. 3 shows the comparison of the minimum value of the local Nusselt number for air-steel beds at various Reynolds number. There exists a little discrepancy between numerical results and experimental ones: the prediction somewhat overestimates the Nusselt number. However, the deviation between them is 8-13% for the range of Reynolds under consideration.

![Graph showing minimum local Nusselt number at various Reynolds number](image)

**Fig. 3. Minimum local Nusselt number at various Reynolds number**

**Conclusions** Our previous numerical model for fluid flow and heat transfer in packed beds was applied to the case of constant wall heat flux condition. Comparison between numerical predictions and available experimental data shows the usefulness of the model.

**References**

(1) San San Yee and K. Kamiuto, Combined forced-convective and radiative heat transfer in cylindrical beds with constant wall temperature, Journal of porous media, in press.
