Magnetoelectric Fracture Mechanics of Ferromagnetic Plates

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1. Introduction

Ferritic/martensitic steels are currently under study as candidate materials for first wall and structural materials applications for commercial fusion reactors(1). When a cracked magnetizable elastic plate is placed in an external magnetic field, the existence of the magnetic field may produce higher singular moments near the crack tip(2,3,4). In this paper, an experimental observation of the bending of a through-cracked ferromagnetic plate in a transverse magnetic field is first presented. A simple experimental procedure involving measurement of strain through strip gages with five strain sensors per strip gage, has been developed to determine the magnetic moment intensity factor. Next, the linear magneto-elastic problem for a ferromagnetic plate of finite length with a through crack under bending is analyzed. The classical plate bending theory of magnetoelasticity is applied. Numerical results are obtained for the bending moment intensity factors with different amplitudes of the magnetic field and compared with experimental data.

2. Magnetic moment intensity factor for a through-cracked ferromagnetic plate

2.1 Analysis procedure. We consider an elastic plate of thickness 2h, Poisson's ratio ν, and Young's modulus E containing a through crack. The crack length is assumed to be large in comparison with the plate thickness 2h. The middle plane bisecting the plate thickness as shown in Fig. 1 is usually taken as reference. The coordinate axes x and y are in the middle plane of the plate, the z axis is perpendicular to this plane and the coordinates r, θ are used to define the position of an element. In plate bending, the stresses near the crack tip are given by (4,5).

\[
\begin{align*}
\sigma_r &= -\frac{k_f}{(2\pi)^{1/2}} \left[ \cos\left(\frac{\theta}{2}\right) - \frac{3 + 5\nu}{7 + \nu} \cos\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
&\quad + 4\pi A\left[1 + \cos(2\theta)\right] + O\left(r^{1/2}\right)
\end{align*}
\]

\[
\begin{align*}
\sigma_\theta &= \frac{k_f}{(2\pi)^{1/2}} \left[ \cos\left(\frac{\theta}{2}\right) + \frac{5 + 3\nu}{7 + \nu} \cos\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
&\quad - 4\pi A\left[1 - \cos(2\theta)\right] + O\left(r^{1/2}\right)
\end{align*}
\]

\[
\begin{align*}
\sigma_{r\theta} &= \frac{k_f}{(2\pi)^{1/2}} \left[ \sin\left(\frac{\theta}{2}\right) + \frac{1 - \nu}{7 + \nu} \sin\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
&\quad + 4\pi A\sin(2\theta) + O\left(r^{1/2}\right)
\end{align*}
\]

where \(\sigma_r, \sigma_\theta, \) and \(\sigma_{r\theta}\) are the bending stresses, \(k_f\) is the stress intensity factor, and \(A\) is the constant. The constant \(A\) depends upon loading conditions; more specifically, either upon the boundary conditions at infinity in the case of an infinite sector, or upon those at some fixed radius when the plate has finite dimensions. Substituting Eq. (1) into the stress \(\sigma - \epsilon\) relations gives

\[
Ee_r = \sigma_r - \nu\sigma_\theta = -\frac{k_f}{(2\pi)^{1/2}} \left[ \cos\left(\frac{\theta}{2}\right) - \frac{3 + 5\nu}{7 + \nu} \cos\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
+ 4\pi A\left[1 + \cos(2\theta)\right] + O\left(r^{1/2}\right)
\]

\[
Ee_\theta = \sigma_\theta - \nu\sigma_r = \frac{k_f}{(2\pi)^{1/2}} \left[ \cos\left(\frac{\theta}{2}\right) + \frac{5 + 3\nu}{7 + \nu} \cos\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
- 4\pi A\left[1 - \cos(2\theta)\right] + O\left(r^{1/2}\right)
\]

\[
Ee_{r\theta} = \sigma_{r\theta} - \nu\sigma_r - \nu\sigma_\theta = \frac{k_f}{(2\pi)^{1/2}} \left[ \sin\left(\frac{\theta}{2}\right) + \frac{1 - \nu}{7 + \nu} \sin\left(\frac{\theta}{2}\right)^2 \right] \left(\frac{7 + \nu}{2(3 + \nu)2h}\right) \\
+ 4\pi A\sin(2\theta) + O\left(r^{1/2}\right)
\]

where \(e_r\) is the radial strain. Setting \(\theta = \pi/2\) and \(z = h\) gives

\[
a_0 Ee_r r^{1/2} = k_f + 8\pi a_0 A r^{1/2} + \cdots
\]

where

\[
a_0 = \frac{4\pi^{1/2}(3 + \nu)}{(5 - \nu)(1 + \nu)}
\]

From Eq. (3), a plot of \(a_0 Ee_r r^{1/2}\) versus \(r^{1/2}\) is linear for small values of \(r\) and the intercept at \(r = 0\), at the crack front, gives the stress intensity factor \(k_f\). Since the moments \(M_r, M_\theta\) and \(M_{r\theta}\) are related to the stresses \(\sigma_r, \sigma_\theta\) and \(\sigma_{r\theta}\) by the relations

\[
M_r = \frac{2h^3}{3\pi} \sigma_r, \quad M_\theta = \frac{2h^3}{3\pi} \sigma_\theta, \quad M_{r\theta} = \frac{2h^3}{3\pi} \sigma_{r\theta}
\]

the moment intensity factor may be obtained by multiplying \(k_f\) by the factor \(2h^3/3\pi\).
2.2 Experimental procedure. Ferritic stainless steel SUS430 was used as specimen material. The Young's modulus $E$, Poisson's ratio $\nu$, and specific magnetic permeability $\mu_r$ obtained for the SUS430 tested are listed in Table 1. The specimen geometry used was a single through-cracked specimen (length $l = 100$ mm, thickness $2h = 2$ mm, width $W = 40$ mm, and crack length $2a = 18$ mm), as shown in Fig. 2. Figure 3 depicts the setup for the experiment. Experiments were conducted in the bore of a superconducting magnet (SM-3, High Field Laboratory for Superconducting Materials, Institute for Materials Research, Tohoku University) at room temperature. The fixed-end through-cracked specimen was bent by a normal load $P$ applied at the center of the plate and was permeated by a uniform static magnetic field of magnetic induction $B_0$ normal to the plate surface. A five-element strip gage was installed along the $\theta=90$-deg line and the center point of the element closest to the crack tip was 2 mm. The strains were recorded as a function of magnetic field.

2.3 Experimental results and discussion. Figure 4 shows the normalized moment intensity factor $K_{1M}/K_{10}$ versus magnetic field $B_0$, where $K_{1M}$ and $K_{10}$ are the moment intensity factors with and without magnetic field. The moment intensity factor is found to increase with increasing magnetic field. No dependence of $K_{1M}/K_{10}$ values on load $P$ is observed.

3. Magnetoelastic analysis of a soft ferromagnetic plate with a through crack under bending

We consider a soft ferromagnetic elastic plate of length $l$, thickness $2h$, Poisson's ratio $\nu$, Young's modulus $E$, and flexural rigidity $D = 2EH^3/(3(1-\nu))$ having a through crack of length $2a$ as shown in Fig. 5. The coordinate axes $x$ and $y$ are in the middle plane of the plate and the $z$-axis is perpendicular to this plane. The fixed-end through-cracked plate is permeated by a static uniform magnetic field of magnetic induction $B_0$ normal to the plate surface, and is bent by the force per unit length $P$ distributed uniformly along the $x$-axis. The uniformly distributed load $P$ corresponds to the experimentally applied load $P$.

Fourier transform method is employed and the result is expressed in terms of a Fredholm integral equation of the second kind. The moment intensity factor is defined by

$$K_{1M} = \lim_{x \to a^+} \frac{1}{2\pi(x-a)}^{1/2} M_{yy}$$

where $M_{yy}$ is the bending moment. The moment intensity factor for bending stress loading is computed. Theoretical predictions are compared with experimental data and good agreement is observed.

Fig. 2 Specimen configuration

Fig. 3 Schematic diagram of the test system

Fig. 4 Bending moment intensity factor $K_{1M}/K_{10}$ versus magnetic field $B_0$

Fig. 5 A through-cracked soft ferromagnetic plate

References


Table 1 Mechanical and magnetic properties of specimen

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<tr>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\mu_r$</th>
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<td>162.3</td>
<td>0.294</td>
<td>122.9</td>
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