P54 片側縁き裂を有する強磁性帯板の磁気弾性解析・引張試験

Magnetoelastic Analysis and Tensile Testing of a Soft Ferromagnetic Strip with a Single-Edge Crack

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1. Introduction

In magnetomechanical devices such as fusion reactors, magneto hydro dynamics (MHD) structures, magnetically levitated vehicles, magnetic fields are automatically applied to surrounding components. Design and development of such structures require basic research on magnetic fracture mechanics¹⁾. Recently, Shindo et al.²⁾ confirmed theoretically and experimentally the fact that the applied magnetic field tends to intensify the stress intensity factor. Also, the excellent agreement between calculations and measurements of the stress intensity factor established the validity of the linear theory for magnetoelastic interactions in a cracked soft ferromagnetic material. This paper examines theoretically and experimentally the effect of magnetic fields on the fracture mechanics parameters of a soft ferromagnetic strip with a single-edge crack subjected to tensile load and uniform magnetic field.

2. Problem statement and basic equations

Consider a soft ferromagnetic isotropic linear elastic strip of width h which contains a single-edge crack of length a aligned with its plane normal to the free edge as shown in Fig. 1. Let x , y and z denote the Cartesian coordinates. The soft ferromagnetic strip occupies the region ($0 \leq x \leq h$, $|y| < \infty$). The strip is in the plane

Fig.1 A single-edge crack in a soft ferromagnetic elastic strip.

stress or plane strain state in the z -direction, and is subjected to a uniform normal stress $\sigma_{yy} = \sigma_0$ and a uniform magnetic field of magnetic induction $B_{0y} = B_0$. Because of symmetry, we need only to consider the upper half space region.

We consider small perturbations characterized by the displacement vector $\mathbf{u}$ produced in the strip. All magnetic quantities are divided into two parts, those in the rigid body state and those in the perturbation state as follows:

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b}, \quad \mathbf{M} = \mathbf{M}_0 + \mathbf{m}, \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{h} \quad (1)$$

where $\mathbf{B}$, $\mathbf{M}$ and $\mathbf{H}$ are the magnetic induction, magnetization and magnetic intensity vectors, respectively. Those quantities with the subscript 0 refer to the magnetic field in the undeformed state. The remaining ones are the corrections to account for the additional changes in magnetic field due to deformations. The magnetoelastic solution for the rigid body state can be written as

$$B_{0y} = B_0, \quad H_{0y} = \frac{B_0}{\mu_0 \mu_r}, \quad M_{0y} = \frac{\chi B_0}{\mu_0 \mu_r}; \quad (2)$$

$$B_{0y}^e = B_0 / \mu_r, \quad H_{0y}^e = \frac{B_0}{\mu_0 \mu_r}, \quad M_{0y}^e = 0; \quad (3)$$

$$B_{0y}^{ec} = B_0, \quad H_{0y}^{ec} = \frac{B_0}{\mu_0}, \quad M_{0y}^{ec} = 0; \quad (4)$$

where B_{0y} , H_{0y} and M_{0y} are, respectively, the y -components of $\mathbf{B}_0$, $\mathbf{H}_0$ and $\mathbf{M}_0$. The superscripts e ($x < 0, h < x < \infty, 0 < y < \infty$) and ec ($0 < x \leq a, y = 0$) stand for the components of the field quantity outside the strip. Note that $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$ is the magnetic permeability of the vacuum, $\mu_r = 1 + \chi$ is the specific magnetic permeability, and χ is the magnetic susceptibility.

For the magnetic field in the solid, it is assumed that

$$h_{x,x} + h_{y,y} = 0 \quad (5)$$

$$h_{x,y} - h_{y,x} = 0 \quad (6)$$

and

$$b_x = \mu_0 \mu_r h_x, \quad b_y = \mu_0 \mu_r h_y \quad (7)$$

$$m_x = \chi h_x, \quad m_y = \chi h_y \quad (8)$$

where the subscript comma implies a partial derivative with respect to the coordinate, and (h_x, h_y) , (b_x, b_y) and

(m_x, m_y) are the x and y -components of $\mathbf{h}$, $\mathbf{b}$ and $\mathbf{m}$. Eqs. (5) and (6) for the perturbed state are satisfied by introducing a magnetic potential ϕ such that

$$h_x = \phi_{,x}, \quad h_y = \phi_{,y} \quad (9)$$

$$\phi_{,xx} + \phi_{,yy} = 0 \quad (10)$$

By using a dipole model for the magnetization, we obtain the following equilibrium equations:

$$\left. \begin{aligned} t_{xx,x} + t_{yx,y} + \frac{\chi B_0 \phi_{,xy}}{\mu_r} &= 0 \\ t_{xy,x} + t_{yy,y} + \frac{\chi B_0 \phi_{,yy}}{\mu_r} &= 0 \end{aligned} \right\} \quad (11)$$

where t_{xx} , t_{yy} , $t_{xy} = t_{yx}$ is the components of magnetoelastic stresses. The two equations of equilibrium, Eqs. (11), may be expressed in terms of the displacement components u_x, u_y and magnetic potential ϕ by using the linearized constitutive equations, with the result

$$u_{x,xx} + u_{x,yy} + \left(\frac{\kappa}{\mu} + 1 \right) (u_{x,x} + u_{y,y})_x + \frac{2\chi B_0 \phi_{,xy}}{\mu \mu_r} = 0 \quad (12)$$

$$u_{y,xx} + u_{y,yy} + \left(\frac{\kappa}{\mu} + 1 \right) (u_{x,x} + u_{y,y})_y + \frac{2\chi B_0 \phi_{,yy}}{\mu \mu_r} = 0 \quad (13)$$

where $\kappa = \lambda$ for plane strain, and $\kappa = 2\lambda\mu/(\lambda + 2\mu)$ for plane stress, $\lambda = 2G\nu/(1 - 2\nu)$ and $\mu = G$ are the Lamé constants, $G = E/2(1 + \nu)$ is the modulus of rigidity, and E and ν are the Young's modulus and Poisson's ratio, respectively.

The boundary conditions in the perturbation state are

$$h_x^{ec}(x, 0) - h_x(x, 0) = -(\chi B_0 / \mu_0 \mu_r) u_{y,x}(x, 0) \quad (0 \leq x < a) \quad (14)$$

$$\begin{aligned} \phi(x, 0) &= 0 & (a \leq x \leq h) \\ b_y^{ec}(x, 0) - b_y(x, 0) &= 0 & (0 \leq x < a) \\ \phi^{ec}(x, 0) &= 0 & (0 \leq x < a) \end{aligned} \quad (15)$$

$$\sigma_{yx}(x, 0) = -(\chi B_0 / \mu_r) h_x(x, 0) \quad (0 \leq x \leq h) \quad (16)$$

$$\begin{aligned} \sigma_{yy}(x, 0) &= (\chi B_0 / \mu_r) \{(\chi - 2)h_y(x, 0) - B_0 / \mu_0 \mu_r\} & (0 \leq x < a) \\ u_y(x, 0) &= 0 & (a \leq x \leq h) \end{aligned} \quad (17)$$

$$h_y^e(0, y) - h_y(0, y) = 0 \quad (0 \leq y < \infty) \quad (18)$$

$$h_x^e(0, y) - \mu_r h_x(0, y) = -(\chi B_0 / \mu_0 \mu_r) u_{x,y}(0, y) \quad (0 \leq y < \infty) \quad (19)$$

$$\sigma_{xx}(0, y) = 0 \quad (0 \leq y < \infty) \quad (20)$$

$$\sigma_{xy}(0, y) = -(\chi B_0 / \mu_r) h_x(0, y) \quad (0 \leq y < \infty) \quad (21)$$

$$h_y^e(h, y) - h_y(h, y) = 0 \quad (0 \leq y < \infty) \quad (22)$$

$$h_x^e(h, y) - \mu_r h_x(h, y) = -(\chi B_0 / \mu_0 \mu_r) u_{x,y}(h, y) \quad (0 \leq y < \infty) \quad (23)$$

$$\sigma_{xx}(h, y) = 0 \quad (0 \leq y < \infty) \quad (24)$$

$$\sigma_{xy}(h, y) = -(\chi B_0 / \mu_r) h_x(h, y) \quad (0 \leq y < \infty) \quad (25)$$

where σ_{xx} , σ_{yy} , $\sigma_{xy} = \sigma_{yx}$ are the elastic stress components.

Integral transform techniques are used to formulate the problem as a singular integral equation. The magnetic stress intensity factor is obtained as

$$k_{h1} = \lim_{x \rightarrow a^+} \{2(x - a)\}^{1/2} \{t_{yy}(x, 0) + t_{yy}^M(x, 0)\} \quad (26)$$

where t_{yy}^M is the Maxwell stress component, and

$$t_{yy} = \sigma_{yy} + \frac{\chi B_0^2}{\mu_0 \mu_r^2} + \frac{2\chi B_0 \phi_{,y}}{\mu_r} \quad (27)$$

$$t_{yy}^M = \frac{(1 + 2\chi)B_0 \phi_{,y}}{\mu_r} + \frac{(1 + 2\chi)B_0^2}{2\mu_0 \mu_r^2} \quad (28)$$

3. Experimental procedure

Three nickel-iron soft magnetic materials (TMC-V: $E = 182$ GPa, $\nu = 0.146$, $\mu_r = 27900$; TMH-B: $E = 203$ GPa, $\nu = 0.279$, $\mu_r = 10690$; TMB: $E = 146$ GPa, $\nu = 0.228$, $\mu_r = 9030$) developed by NEC/Tokin Co. Ltd. were selected. The specimen geometry was a plate specimen containing a single-edge crack. The edge-cracked specimens have a length of 140 mm, a thickness of 1 mm, and a width, h , of 40 mm. A five element strip gage was installed along the 90-deg line and the center point of the element closest to the crack tip was 2 mm.

Tensile load and a magnetic field were simultaneously applied to edge-cracked plate specimens at room temperature. A 10 T cryocooler-cooled superconducting magnet with a 100 mm diameter working bore was used to create a static uniform magnetic field. The specimens were loaded by $P = 29.4$ N load that consisted of weight.

4. Results and discussion

The calculated results of the normalized stress intensity factor $k_{h1}/\sigma_0 a^{1/2}$ versus strip-width to crack-length ratio h/a are plotted for TMC-V together with the experimental data in Fig. 2 for several values of magnetic field B_0 . The trend is sufficiently similar between the theoretical results and experiment. Similar results were observed for TMH-B and TMB.

Our next plan is to perform fatigue crack growth rate tests using single-edge-crack tension specimens in a uniform magnetic field. The ideal plasticity strip model can be used in conjunction with the accumulated plastic displacement criterion determining crack growth. This approach will be applied to derive the cyclic crack growth rate of a crack in a soft ferromagnetic material. Work in this area is currently being pursued.

Fig. 2 Comparison of the theoretical and experimental stress intensity factors.

Reference

- Shindo, Y., Sekiya, D., Narita, F., Horiguchi, K., *Acta Mater.* **52**, (2004), pp. 4677-4684.