Effects of Fiber Orientation Angles and Fluctuation on the Stiffness and Strength of Sliver-Based Green Composites

by Baosheng Ren*, Junji Noda** and Koichi Goda**

Natural fibers are usually used as reinforcement in green composites. Especially, the use of slivers of natural fibers is anticipated for increasing composites’ stiffness and strength. However, the slivers have various fiber orientation angles and often involve fiber fluctuation. This paper describes effects of fiber orientation angle and fluctuation on Young’s modulus and tensile strength of the so-called fully green composites. The composites were reinforced with slivers of high-strength natural fibers extracted from curaua plants. For this study, a fabrication method called ‘direct method’ was applied for obtaining sliver-based green composites with various fiber orientation angles and fluctuation. Then optical micrographs of composites with fiber fluctuation were obtained. After the micrographs were divided into many fine segments, the fiber orientation angle in each segment was measured. Results show that the tensile strength depends on autocorrelation coefficients expressing the degree of fluctuation in fiber orientation, as well as the fiber orientation angles. However, the Young’s modulus was dependent only on the angles. Furthermore, the Young’s modulus of the composites was predicted using classical lamination theory. In addition, a statistical concept was applied to an orthotropic analysis for prediction of the Young’s modulus. The predicted Young’s moduli showed better agreement with the experimental results, than that of the predicted values based on a definite orthotropic theory.

Key words: Green composites, Natural fibers, Young’s modulus, Tensile strength, Fiber orientation, Lamination theory, Orthotropic theory, Autocorrelation

1 Introduction

Polymer composites such as glass fiber reinforced plastics (GFRP) have played a dominant role in various applications because of their high mechanical stiffness and strength. However, the usage and disposal of these materials, especially GFRP, have come to cause a critical environmental problem. Environmental issues such as these are treated internationally with considerable attention. Consequently, composite industries are seeking more environmentally friendly materials for use with their products. Increasing interest surrounds biodegradable renewable composites reinforced with natural fibers such as jute, flax, ramie, kenaf, and curaua. Natural fibers are light and renewable. They are also an inexpensive high-specific-strength resource. The combination of interesting mechanical and physical properties together with their sustainable character has triggered various activities in the area of fully green composites. And therefore, using injection molding, green composites reinforced by short natural fibers have been applied for fabrication of some industrial products such as interior car parts.

However, long fibers are known to be more useful than short fibers to raise composites’ stiffness and strength. To take advantage of long fibers, one of the authors has evaluated mechanical properties of green composites reinforced by curaua fiber slivers through three kinds of fabrication methods. In general, the morphology of slivers is a wavy shape, so that fluctuation in the fiber orientation is observed in the green composites. According to the results of the reference, the composites’ stiffness and strength were improved by reducing their fluctuation in fiber orientation. However, a quantitative relation between the fluctuation and their mechanical properties has never been clarified in the field of sliver-based green composites, especially from stochastic aspects of the morphology of slivers.

In this study, therefore, effects of fiber orientation angles and fluctuation on the Young’s modulus and tensile strength of the sliver-based green composites were explored. Composites with fluctuation in fiber orientation were fabricated using compression molding and the axial tensile tests were conducted. The concept of autocorrelation was adopted as a stochastic parameter to express the morphology of the fluctuation, and a relation between the tensile strength and the parameter was explored. Furthermore, the Young’s modulus of the composites was analyzed based on classical lamination theory, which has never been applied for sliver-based green composites, and compared with the experimental results. Finally, we predicted the Young’s modulus through an orthotropic analysis based on a statistical dis-
tribution obtained from fiber orientation angles and compared with experimental results.

2 Experimental

2.1 Materials

The matrix used in this study was a cornstarch-based biodegradable resin (Randy CP-300; Miyoshi Oil and Fat Co. Ltd., Japan). The biodegradable resin is thermoplastic, has hydrophilic properties and is made from a blend of polycaprolactone (PCL) and cornstarch. Slivers of curaua fibers are shown in Fig. 1, supplied from POEMA of Para Federal University, Brazil, were used as reinforcement. Curaua fiber is a leaf fiber extracted from an Amazon forest plant that resembles a pineapple plant. This fiber has a low production cost and offers sufficient high tensile strength for practical applications. The chemical composition of curaua fiber is the following: lignin (7.5%), glucan (66.4%), xylan (11.6%), mannan (0.1%), and other materials.

Table 1 presents some physical and mechanical properties of these materials.

2.2 Fabrication and tensile tests

To fabricate fully green composites, two methods were prepared: the prepreg sheet method (PS) and the direct method (DM). The latter method is more appropriate for mass production than the former, but the products fabricated using DM include a large fluctuation in fiber orientation. In this study, a hot press molding machine was used for fabrication of the composites. For PS, the sliver was first combed carefully to form unidirectionally oriented fibers. The emulsion type resin was applied to them using a small brush. Next, thin prepreg sheets were obtained by pressing the resin-pasted sliver. Finally, the fabricated prepreg sheets were cut to the desired dimensions. Later, a set of five sheets, each with identical fiber orientation, was inserted in the mold and pressed using 3.27 MPa at 150°C for 1 hr. For DM, an as-supplied sliver was placed on a metallic plate without combing. Then the resin was directly applied to the sliver. Finally, the resin-pasted pre-forms were dried at room temperature for 24 hrs. The two pre-forms were inserted into the metallic mold and pressed using identical conditions. For both PS and DM, the resultant composite thickness was about 1 mm, and its length and width were 100 mm and 15 mm, respectively. Thus, the composites used in this study were a relatively thin plate. Then, the fiber volume fraction of all fabricated composites was calculated using the following Eq.

\[
V_f = 1 - \frac{W - W_f}{\rho_m V}
\]

Where, \(W\) is the fabricated composite weight, \(W_f\) is the curaua fiber weight in the composite, \(V\) is for the fabricated composite volume, and \(\rho_m\) is the biodegradable resin density. The average volume fraction of the fabricated composites obtained here was 69.7% for DM. The variation in volume fraction was almost constant, as shown in Table 2.

Axial tensile tests were carried out for all fabricated composites. To avoid stress concentration, before the experiment, aluminum thin plates with edges angled at 45° were attached with epoxy adhesive on both ends of all composites. As a result, the tensile specimen was 50 mm gage length. A strain gage was fixed at the center of each specimen for measuring uniaxial strain. An Instron-type testing machine (Autograph IS-500; Shimadzu Co.) was used for tensile test, and cross-head speed of the testing machine was 1 mm/min. From the obtained stress-strain diagram, it was obvious that the composites treated here behave linearly up to around 1.0% strain. Thus, Young’s modulus was calculated in the range of 0.2 to 0.5% strain for each specimen.

2.3 Angle measurements

Figure 2 shows the measurement method used for the fiber orientation angles in the sliver-based composites. The specimen photograph on the gage length was divided into a “unit composite” with short length of \(\Delta x\), as shown in Fig. 2 (a). One unit composite can also be divided into short width of \(\Delta y\) along the transverse direction, called “segments,” as shown in Fig. 2 (b). Fiber orientation angles of each segment were measured from the obtained optical micrograph using image analysis software (Asahi Kasei Co.). The angle often depends on the size of segment. It was difficult to decide an exact angle on segment sizes more than 1.0 mm × 1.0 mm, because of

<table>
<thead>
<tr>
<th></th>
<th>Density (Mg/m³)</th>
<th>c.s.a.* (μm²)</th>
<th>Tensile strength (MPa)</th>
<th>Fracture strain (%)</th>
<th>Young’s modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
<td>1.16</td>
<td>-</td>
<td>10.6</td>
<td>6.50</td>
<td>0.531</td>
</tr>
<tr>
<td>Curaua fiber</td>
<td>1.38</td>
<td>5267</td>
<td>913</td>
<td>3.90</td>
<td>30.0</td>
</tr>
</tbody>
</table>

* Cross-sectional area. In this table, average c.s.a. of a curaua fiber is shown because its shape is not circle. In case the shape is a circle, the diameter is given as 82 μm.
the fiber waviness, while the angle was relatively easy to be decided in the case of equal to or less than 1.0mm × 1.0mm. As shown in Fig. 2 (b), however, all segments are not necessarily observed as a single orientation at 1.0mm × 1.0mm, but different orientations were sometimes observed in the same segment. In this case, the segment area more than 70% is occupied by an identical angle \( \theta \), as shown in the shaded portion, and may be represented by this angle. In this study, 70% of the segment area was defined as a base area for measuring the angle, and in the case of less than 70%, the average of \( \theta \) and \( \theta' \) was used as a representative angle. The segment size was thus set as 1.0mm × 1.0mm. This model comprises 750 segments, i.e., 50 segments along the x-axis and 15 segments along the y-axis.

### 3 Results and Discussion

#### 3.1 Angle measurement and stochastic properties of fiber orientation

The frequencies of two representative angle distributions are shown in Fig. 3. As shown in this figure, measured angles tend to be distributed around zero degree, but one distribution denoted as Sample 1 is narrower in width than another distribution denoted as Sample 2. Such distribution patterns may be regarded as the nature of a curaua sliver. On the other hand, as mentioned later, anisotropy of fibrous composites is decided through absolute values of the angles, irrespective of positive or negative angles. Thus, measured fiber orientation angles are hereinafter shown in absolute value.

In Table 2, the results of angle measurements of both sides of the specimens are presented. In this table, side A shows results of measured angles on one surface of each specimen. Side B shows results for the reverse surface. On side A, the smaller value in the average angle is shown. Hereinafter, it is denoted as DM1-A for DM composite of the specimen no.1 and side A. Results...
show that the angles vary from 7.5° to 21.5° in average, and from 4.52° to 11.0° in standard deviation. In addition, a less average angle specimen tends to indicate a smaller standard deviation because the fiber orientation with less average angle provides a smaller amplitude if the flow of slivers is regarded as a wavy shape. Samples 1 and 2 shown in Fig. 3 correspond to DM2-A and DM7-B, respectively. The angles of DM2-A and DM7-B are 9.30° and 18.6° in average, respectively, and typical angle distributions of sides A and B. In this study, furthermore, we consider a stochastic aspect related to the morphology of fiber orientation in the specimens as well as the angle distribution. Figures 4 (a) and 4 (b) show contour maps of angle distributions for DM2-A and DM7-B, respectively. Comparison of these two maps shows a lighter aspect in Fig. 4 (a) because DM2-A has many small angles. It is noteworthy in Fig. 4 (a) that more white areas (smaller angles) are observed at the left and upper parts, while more gray areas (larger angles) are observed at the right and lower parts. In contrast, black areas are dispersed at every part in Fig. 4 (b). In other words, similar angle distributions continue successively along the longitudinal direction in the former, but not in the latter. Therefore, to clarify the difference between morphologies of the two samples quantitatively, we adopt correlation coefficients between unit composites. For example, the correlation coefficient between i-th and j-th unit composites, \( \rho_{ij} \), is given as:

\[
\rho_{ij} = \frac{\text{Cov}(\theta_i, \theta_j)}{\sqrt{\text{Var}(\theta_i) \cdot \text{Var}(\theta_j)}}
\]

Where, \( \theta_i \) and \( \theta_j \) are measured angles on i-th and j-th unit composites. \( \text{Cov} (\cdot) \) is the autocovariance between two unit composites, and \( \text{Var} (\cdot) \) is the variance on a unit composite. This coefficient is denoted as autocorrelation coefficient (a.c.c.). In this study, on the basis of the angle distribution in 25th unit composite at the center of the tensile specimen, a.c.c. was calculated as \( \rho_{ij}, (i = 25, j = 1, 2, ..., 50) \), for each specimen. While calculation a.c.c. along y-axis is necessary for biaxial or transverse tensile test, in this study we explored only a.c.c. along x-axis because of uni-axial tensile test along x-axis. The calculation results of a.c.c. for Figs. 4 (a) and 4 (b) are shown in Figs. 5 (a) and 5 (b), respectively. In Fig. 5 (a), relatively large coefficients around 0.5 are observed, while less coefficients around zero are found in Fig. 5 (b). As mentioned earlier, this indicates that the fibers in the former flow along the longitudinal direction more smoothly than those in the latter. The former has less fluctuation in fiber orientation. In this study, a.c.c. values of all samples were calculated and their averages are additionally shown in Table 2. Results show that the average a.c.c. (denoted as a.a.c.c.) is 0.04 to 0.37.

3.2 Effects of fiber orientation angle and fluctuation on Young’s modulus and tensile strength

Tensile test results of sliver-based green composites fabricated by PS and DM are added to Table 2. For PS, only average values are shown because the fibers in PS were quite unidirectionally oriented. The variations in stiffness and strength were reasonably small. The tensile strength of 327MPa obtained in PS composites is almost comparable to the level of aluminum alloy, and Young’s
modulus is almost comparable to the level of GFRP. On the other hand, results showed that the average tensile strength of DM composites were 31% lower than that of PS composites, and the average Young’s modulus was 37% lower than that of PS composites. However, the maximum values of the stiffness and strength are approximately 80% of the levels of PS composites. The fracture surfaces of DM composites showed on the whole a large-scale uneven aspect with plural shear fractures. Thus, it was difficult to relate the measured fiber orientation angles with the fracture path.

In general, it is known that fiber orientation angles are deeply related with mechanical properties of unidirectional fibrous composites. Furthermore, we consider a.c.c., as discussed above might be related with those of the composites. Figures 6 (a) and 6 (b) show effects of $\bar{\theta}$ and a.a.c.c. on the tensile strength, respectively. The averages of both sides A and B of one specimen are shown for $\bar{\theta}$ and a.a.c.c. The straight line in the figures indicates an estimation using the least squares method. It is proved that, although the tensile strength decreases slightly concomitantly with increasing $\bar{\theta}$, it increases clearly with increasing a.a.c.c. The correlation coefficient between a.c.c. and strength was 0.843, while its coefficient between $\bar{\theta}$ and strength was only $-0.237$. Thus, DM composites are expected to increase in strength by a further increase in a.a.c.c. As easily guessed, a.a.c.c. might be varied depending on the position of a base unit composite, which was set as $i = 25$ in this study. To explore the effect of the position of unit composites on the correlation coefficient between a.a.c.c. and strength, calculations for $i = 5, 10, 15, \text{and } 20$ were additionally carried out. Table 3 shows the results. It is obvious from this table that the coefficients are all positive, and larger than 0.237, the absolute value of the coefficient between $\bar{\theta}$ and strength.

In general, the tensile strength of unidirectional fibrous composites is controlled by the fiber orientation angle. However, in case the composites have fluctuation in fiber orientation, as discussed in this study, its effect is not so significant. On the other hand, it was newly found that the magnitude of a.a.c.c. was sensitive to change in the strength level. On the other hand, the correlation coefficient between a.a.c.c. and $\bar{\theta}$ was calculated as 0.125. This means that the specimens with less fiber orientation angle do not necessarily have larger a.a.c.c. Consequently, because a smaller a.a.c.c. provides more irregular fiber fluctuation, it is considered that such fluctuated parts induce an extra stress concentration in the specimen, thereby decreasing the tensile strength. It is concluded from the above that the autocorrelation coefficient, a.c.c. is possibly useful as a new parameter for determining the tensile strength of sliver-based green composites.

Figures 7 (a) and 7 (b) depict effects of $\bar{\theta}$ and a.a.c.c. on the Young’s modulus, respectively. The line is similarly drawn using the least squares method. The Young’s modulus is also shown as a function of $\bar{\theta}$ and a.a.c.c. in Figures 7 (a) and 7 (b), respectively.
modulus is observed to increase almost linearly with decreasing $\theta$. The estimated line also approaches the value of PS at $\theta = 0^\circ$. The correlation coefficient between the modulus and $\bar{\theta}$ was also -0.954. On the other hand, little relation exists between the modulus and a.a.c.c. In this case also the effect of the position of unit composites on the correlation coefficient between a.a.c.c. and strength was explored for $i = 5, 10, 15, \text{and } 20$. The results are added to Table 3. It is proved that all the coefficients are near to zero. In other words, the composites’ stiffness is determined only through fiber orientation angles, and it is insensitive to slivers’ morphology such as fluctuation in fiber orientation. Since the Young’s modulus of sliver-based green composites depends on the fiber orientation angles, it might be discussed from the viewpoint of orthotropic elasticity in the same method as the field of unidirectional fibrous composites.

### 3.3 Estimation of Young’s modulus

According to Hisao & Daniel,14) the morphology of the carbon fiber orientation was modeled as a wavy shape; the composite stiffness was analyzed using their proposed model. According to this model, the composite was divided similarly into unit composites and segments, as shown in Fig. 2. Furthermore, classical lamination theory was applied by regarding the unit composite structure as a laminate. In this study also, the stiffness of unit composite structure was analyzed in the same way, but the model used in this study is different in a point that transformed reduced stiffness matrix used in the analysis is given through the experimentally obtained angle on each segment. The detailed analytical method is described in Appendix.

Young’s moduli of the composites were analyzed from the lamination model mentioned in Appendix using the following constants.

\[ E_1 = 36.0 \text{GPa}, \quad E_2 = 3.57 \text{GPa}, \quad G_{12} = 1.78 \text{ or } 2.85 \text{GPa}, \quad \nu_{12} = 0.40 \]

Therein, $E_1$ was the value obtained from PS composites, $E_2$ was calculated using the rule of mixture; in the case of $G_{12}$, this was selected as 0.5–0.8 times\(^{12,14}\) lower than $E_2$. The value of $\nu_{12}$ is often used for unidirectional fiber composites. The analytical results are added in Table 2. Results show that the Young’s moduli are quite higher than those obtained in the experiments in all cases, meaning that usual lamination theory cannot well predict the Young’s modulus of sliver-based green composites. It might be considered that classical lamination theory was strongly restricted in in-plane deformation.\(^{15}\)

Therefore, we propose a simple concept for estimation of Young’s modulus of the green composites. As shown in Figs. 7 (a) and 7 (b), the Young’s modulus of sliver-based green composites were determined solely using the fiber orientation angle, independently of the degree of fluctuation in fiber orientation. Therefore, a simple orthotropic theory might be applied, given as:

\[
\frac{1}{E_0} = \cos^4 \theta \sin^3 \theta + \sin^4 \theta E_1 + \left( \frac{1}{E_1} - \frac{2\nu_{12}}{E_1} \right) \cos^2 \theta \sin \theta
\]

Where, $E_0$ is the Young’s modulus of an orthotropic body with angle $\theta$. Young’s modulus of each segment varies depending on angle $\theta$, of which the distribution was measured preliminarily for each specimen, as seen in Table 2. An expected Young’s modulus $\bar{E}$ might be defined as shown below.

\[
\bar{E} = \left[ E_0 f(\theta) d\theta \right] = \sum E_i f_i
\]

Therein, $f(\theta)$ is a fiber angle distribution function on each composite specimen. Actually, $f_i$ is the relative frequency measured at every $3^\circ$ angle interval. For analyzing $E_0$, a representative angle in the interval, e.g. 1.5° at the interval of 0° to 3°, was substituted into Eq.(3). Frequencies of the fiber orientation angle for DM2-A and DM7-B are shown representatively in Fig. 3. In this study, measured negative angles were all converted to absolute values. If absolute value of relative frequencies are substituted for Eq. (4), then the $\bar{E}$ will be obtained.

Figure 8 presents the relation between the expected Young’s modulus $\bar{E}$ analyzed from Eq. (4) and the fiber orientation angle. Each plot shows the average of both sides A and B. Figures 8 (a) and 8 (b) show the cases of $G_{12} = 1.78$ and 2.85GPa, respectively. Both results show good agreement with experimental values. For comparison, the analytical lines without angle distribution are presented in Fig. 8. The analytical line also shows good agreement with the experimental values in the case of $G_{12} = 2.85$ GPa in Fig. 8 (b), but are lowered at $G_{12} = 1.78$ GPa in Fig. 8 (a) because of shear modulus sensitivity. Therefore, we conclude that the proposed statistical estimation method using Eq. (4) is quite effective for prediction of Young’s modulus of sliver-based green composites.

### 4 Conclusions

This paper deals with effects of fiber orientation angles and fluctuation on the stiffness and strength of sliver-based green composites. The optical micrographs of the composites were divided into many fine segments. Then the fiber orientation angle was measured for each segment. Furthermore, the relation between tensile

<table>
<thead>
<tr>
<th>Position of base unit composite</th>
<th>5th</th>
<th>10th</th>
<th>15th</th>
<th>20th</th>
<th>25th</th>
</tr>
</thead>
<tbody>
<tr>
<td>For tensile strength</td>
<td>0.534</td>
<td>0.337</td>
<td>0.585</td>
<td>0.849</td>
<td>0.843</td>
</tr>
<tr>
<td>For Young’s modulus</td>
<td>-0.020</td>
<td>-0.128</td>
<td>-0.041</td>
<td>0.019</td>
<td>-0.112</td>
</tr>
</tbody>
</table>
Effects of Fiber Orientation Angles and Fluctuation on the Stiffness and Strength of Sliver-Based Green Composites

Test results and measured fiber orientation angles were discussed. The results are the following.

1) The results of tensile tests show that the tensile strength is dependent upon autocorrelation coefficients (a.c.c.), which express the degree of fluctuation in the fiber orientation of the composites, rather than fiber orientation angle ($-\theta$). The Young's modulus was strongly dependent on the angle.

2) Therefore, Young's modulus and tensile strength of the composites can be improved by changing the morphology of slivers to lower $-\theta$ and higher a.c.c., respectively.

3) The Young's modulus of the composites was predicted using classical lamination theory. The prediction overestimates the modulus rather than experimental values.

4) The Young's modulus of the composites was estimated by combining the orthotropic theory with the statistical distribution of fiber orientation angles. Results showed that the estimated Young's moduli had good agreement with the experimental results.

References


Appendix

Unit composites obtained in this study exhibit different stiffness because of their different laminated structure. Therefore, strains occurring in the unit composites are also different, but bearing loads on the unit composites must be all the same. When an identical load is given to each unit composite as a boundary condition, the whole composite strain can be estimated as an average of strains occurring in all unit composites. The relation between the stress components \(\{\sigma\}\) and the strain components \(\{\varepsilon\}\) of a segment are given as

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_6
\end{bmatrix}
\]  

(A1)

where \(Q_{ij}\) is the transformed reduced stiffness matrix as shown below.

\[
\begin{align*}
Q_{11} &= l^2Q_{11} + 2l^2m^2(Q_{12} + 2Q_{66}) + m^4Q_{22} \\
Q_{22} &= m^4Q_{11} + 2l^2m^2(Q_{12} + 2Q_{66}) + l^4Q_{22} \\
Q_{16} &= l^2m^2(Q_{11} + 2Q_{12} - 2Q_{66}) + l^4m^2Q_{66} \\
Q_{12} &= l^2m^2(Q_{11} + 2Q_{12} - 4Q_{66}) + l^4m^2Q_{12} \\
Q_{66} &= -l^4m(2Q_{66} - Q_{11} + Q_{12}) + l^4m(Q_{66} - 2Q_{22} + Q_{12}) \\
Q_{26} &= -l^4m(2Q_{66} - Q_{11} + Q_{12}) + l^4m(Q_{66} - 2Q_{22} + Q_{12})
\end{align*}
\]  

(A2)

Therein, \(l = \cos \theta\), \(m = \sin \theta\), and the fiber orientation angle \(\theta\) are given from the positive rotation of principal material axes \(1-2\) along the fiber-axis from arbitrary \(x-y\) axes; \(Q_{ij}\) is the reduced stiffness matrix, given as follows.

\[
\begin{align*}
Q_{11} &= \frac{E_i}{1 - \nu_{12} \nu_{21}} \\
Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12} \nu_{21}} = -\frac{\nu_{21}E_1}{1 - \nu_{12} \nu_{21}} \\
Q_{22} &= \frac{E_2}{1 - \nu_{12} \nu_{21}} \\
Q_{66} &= \frac{E_6}{1 - \nu_{12} \nu_{21}}
\end{align*}
\]  

(A3)

where \(E_i\), \(G_{ij}\), and \(\nu_{ij}\) (\(i, j = 1, 2\)) are the Young's modulus, the shear modulus, and the Poisson's ratio, respectively.

The laminated structure of a unit composite is given from coordinate system, as shown in Fig. A1. In this structure, segments with various fiber orientation angles are piled up along \(y\)-axis to simulate the structure of sliver-based composites. And unit composites consisting of the segments are lined up in series along \(x\)-axis. Thus, the coordinate system is different from that of classical laminate theory. In-plane forces per unit width, called stress resultants, acting on a unit composite, are estimated by integrating stress components of the first to \(n\)-th lamina, as shown below.

\[
(N_x, N_y) = \sum_{k=1}^{n} \left[ \begin{array}{c} \sigma_{x}^{(k)} \\ \sigma_{y}^{(k)} \\ \tau_{xy}^{(k)} \end{array} \right] dy \]  

(A4)

Bending moments per unit width, so-called moment resultants, acting on a unit composite can also be estimated as follows.

\[
(M_x, M_y, M_m) = \sum_{k=1}^{n} \int_{-h/2}^{h/2} \left[ \begin{array}{c} e_x^{(k)} \\ e_y^{(k)} \\ e_{xy}^{(k)} \end{array} \right] dy \]  

(A5)

Equations (A4) and (A5) are rewritten in matrix form as

\[
\begin{bmatrix}
N_x \\
N_y \\
N_m
\end{bmatrix} =
\begin{bmatrix}
A_y & B_y & D_y
\end{bmatrix}
\begin{bmatrix}
e_x^0 \\
e_y^0 \\
e_{xy}^0
\end{bmatrix}
\]  

(A6)

Where

\[
A_y = \sum_{k=1}^{n} (Q_{ij}) (y_k - y_{k-1}) \\
B_y = \frac{1}{2} \sum_{k=1}^{n} (Q_{ij}) (y_k^2 - y_{k-1}^2) \\
D_y = \frac{1}{3} \sum_{k=1}^{n} (Q_{ij}) (y_k^3 - y_{k-1}^3) \quad (i, j) = 1, 2, 6
\]  

(A7)

Here, \(e_i^0\) and \(k_i\) are the strain and the curvature, respectively. \(A_i\), \(B_i\), and \(D_i\) are the in-plane, coupling, and bending stiffness, respectively. Equation (A6) can be changed as shown below.

\[
\begin{bmatrix}
e_x^0 \\
e_y^0 \\
e_{xy}^0
\end{bmatrix} =
\begin{bmatrix}
A^{-1} - A^{-1} B & A^{-1} B & D
\end{bmatrix}
\begin{bmatrix}
N_x \\
N_y \\
N_m
\end{bmatrix}
\]  

(A8)

As described above, stress resultants are a boundary condition. In this study, only the tensile stress resultant along the \(x\)-axis was given as the condition. In addition, assuming that no bending deformation caused by in-plane loading occurs, another boundary condition, i.e. the components of curvature, were all set to zero. Thus, strain components \(\{\varepsilon\}\) can be given through only the matrix \(A^{-1}\) in Eq. (A8).