高温における二相系の変形について*

On the Deformation of 2-Phase Systems at High Temperature

by

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This paper is concerned with the isothermal deformation of 2-phase structures subjected to a constant input of stress. Materials of this type can be regarded as essentially a dispersion of fine particles (hard phase), surrounded by a soft matrix (soft phase). An approach on basis of irreversible thermodynamics leads to an overall strain-time function in a macrorheological sense. It is evident that a more adequate description of the deformational behaviour in particular during the transient stage, will be obtained, if microstructural conditions are taken into account. An attempt is therefore made to include in the rheological formulation of the strain-time function such effects as volume fraction of the two phases, particle size etc. Introducing a stiffness parameter, which is related to the deviatoric stress tensor and a structure function, the total strain-time function for the material is obtained.

1. Introduction

It is well known that the strengthening effect in two-phase materials arising from the dispersion of the hard particles depends upon the distribution of this phase in the matrix. The distribution can be described by either specifying particle size and mean spacing or by volume fractions. These factors however are inter-related, i.e. one cannot be altered without changing the others. Quantitative relations between strength and geometrical factors have not been determined to any extent for such materials, although considerations of this nature are given in Ref. 1 and 2. A theory based on dislocation models which has been verified approximately by experiments, shows the deformational behaviour of such structures to depend largely on the degree of dispersion.³⁾ It is apparent that the volume fraction of the two-phases will be a significant parameter in the formulation of the strain-time function.

The isothermal mechanical response of a single phase material is usually described as follows:⁴⁾⁵⁾

$$\left. \begin{aligned} \sigma &= E_t \cdot \varepsilon - E_t \int_{-\infty}^t \dot{\varepsilon}(\tau) \Phi(t-\tau) d\tau \\ \varepsilon &= E_t^{-1} \cdot \sigma + E_t^{-1} \int_{-\infty}^t \dot{\sigma}(\tau) \Psi(t-\tau) d\tau \end{aligned} \right\} \quad (1.1)$$

in which E_t denotes the instantaneous dynamic modulus of the material, Φ , Ψ the normalised relaxation and creep functions, respectively, and ε the strain measured from a reference equilibrium state at a fixed temperature T . Internal consistency of equation (1.1) is given by an auxiliary relation in the form of:⁶⁾

$$\{1 - s\bar{\Phi}(s)\} \{1 + s\bar{\Psi}(s)\} = 1 \quad (1.2)$$

where the bar sign denotes the Laplace transform, e.g.

$$\bar{\Psi}(s) = \int_0^{\infty} e^{-st} \Psi(t) dt$$

If it is assumed that the response of the material can be represented analytically by a model employing a continuous distribution of elastic and dissipative elements, the functions Φ , Ψ can be written in integral form:

$$\left. \begin{aligned} \Phi(t) &= \int_0^{\infty} F(\tau) [1 - e^{-t/\tau}] d\tau \\ \Psi(t) &= \int_0^{\infty} G(\tau) [1 - e^{-t/\tau}] d\tau \end{aligned} \right\} \quad (1.3)$$

where the creep function (second eq. 1.3) also admits the inclusion of a linear term.⁷⁾

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2. Rheological Model

In investigating the high temperature deformation of materials, the temperature range considered is usually between 0.5–1 of absolute melting point T_m . In this region the deformation of most materials is essentially a steady flow. The relatively short transient stage occurring after the application of a constant load is characterised by a decreasing creep rate. The latter is ascribed to the higher mobility of the dislocation field and the formation of low angle grain boundaries. It is generally assumed that the steady state, which predominates above 0.5 T_m , arises from a balance between strain hardening and recovery. It has been shown that the thermally activated process of recovery may be expressed by:

$$ce^{-(H-q\sigma)/kT} = r = -\frac{\partial\sigma}{\partial t} \quad (2.1)$$

in which H is an activation energy,

q an activation volume related to atomic stress concentration,

c a structure constant,

k Boltzmann's constant, and

r the rate of recovery of flow stress on annealing, which is proportional to the applied stress σ . Since the steady state requires the flow stress to remain constant it follows that:

$$d\sigma = 0 = \frac{\partial\sigma}{\partial t}dt + \frac{\partial\sigma}{\partial\varepsilon}d\varepsilon \quad (2.2)$$

$$\text{and the creep rate becomes } \dot{\varepsilon} = -\frac{\partial\sigma/\partial t}{\partial\sigma/\partial\varepsilon} = \frac{r}{p} \quad (2.3)$$

where p denotes a strain hardening coefficient or the stiffness of the material as represented by the slope of the curve at the value of the applied stress. It is assumed in this model that p is independent of strain. Experimental evidence however shows the steady state deformation to be strongly dependent on strain and thermal history.

Hence it is suggested by the writer of this paper to use an "instantaneous" stiffness parameter of the form:

$$\Phi_s = \frac{\partial\sigma_{ij}'}{\partial\varepsilon_{ij}'} \quad (2.4)$$

in which σ_{ij}' , ε_{ij}' denote the deviatoric components of stress and strain, e.g.

$$\left. \begin{aligned} \sigma_{ji}' &= \sigma_{ij} - \frac{1}{3}\sigma_{ii}\delta_{ij} \\ \dot{\varepsilon}_{ij}' &= \dot{\varepsilon}_{ij} - \frac{1}{3}\varepsilon_{ii}\dot{\delta}_{ij} \end{aligned} \right\} \quad (2.5)$$

δ_{ij} being the Kronecker symbol, i.e. $\delta_{ij}=0$, $i \neq j$, $\delta_{ij}=1$, $i=j$. An analogous expression to the flow equation (2.2) can be obtained, if it is assumed that the stress is linearly related to the strain by the stiffness parameter:

$$\sigma_{ij}' = \Phi_s \cdot \varepsilon_{ij}' \quad (2.6)$$

and that at each volume element of the material the stress is related to the microstructure by a structure function (isothermal) of the general type:

$$\sigma_{ij}' = \Psi_s(v_r, D, d, \rho_r, \varepsilon_{ij}', \dots, t) \quad (2.7)$$

where v_r denotes the volume fraction of the phases, D the particle size, d the mean spacing and ρ_r the relative dislocation density. Simplifying (2.7), since Ψ_s is essentially a function of strain and time:

$$\sigma_{ij}' = \Psi_s(\varepsilon_{ij}', t) \quad (2.8)$$

From which the instantaneous stiffness parameter and the rate of recovery of flow stress are obtained as:

$$\left. \begin{aligned} \frac{\partial\sigma_{ij}'}{\partial\varepsilon_{ij}'} &= \frac{\partial\Psi_s}{\partial\varepsilon_{ij}'} = \Phi_s \\ r^* &= -\frac{\partial\sigma_{ij}'}{\partial t} = -\frac{\partial\Psi_s}{\partial t} \end{aligned} \right\} \quad (2.9)$$

Hence using (2.8) and (2.9) yields the relation:

$$d\sigma_{ij}' = 0 = \frac{\partial\Psi_s}{\partial t}dt + \frac{\partial\Psi_s}{\partial\varepsilon_{ij}'} \cdot d\varepsilon_{ij}'$$

$$\dot{\varepsilon}_{ij}' = \frac{\dot{\gamma}^*}{\phi_s} \quad (2.10)$$

in which $\dot{\gamma}^*$ is now related to the structure function of the material. A rheological model for the 2-phase system can be established, if it is assumed that both phases are not of comparable strength. Hence whilst the hard phase has a stiffness parameter ϕ_H , the soft phase is considered to have a stiffness ϕ_M and a viscosity λ_M . Since the applied stress σ_{ij} will be balanced by the resistance of both phases the model will be as indicated in Fig. 1 (a). In this figure and in the following considerations compressive stresses are taken as positive. This model is equivalent to the 3-element model of Fig. 1 (b) with the transformed parameters as shown in (c). If the latter relations are satisfied both models behave identically.

3. The Strain-Time Function

Considering the above model of the 2-phase system acted upon by the effective stress tensor σ_{ij}' , then the strain in the hard phase is expressed from eq. (2.6) by :

$$\varepsilon_{ij}' = \phi_H^{-1} \sigma_{ij}' \quad (3.1)$$

For the Kelvin-Voigt unit representing the matrix and which is in series with element ϕ_H , the strains in ϕ_M and λ_M must be the same, viz :

$$[\varepsilon_{ij}']_{\phi_M, \lambda_M} = \left\{ \frac{1}{1/\phi_M + s/\lambda_M} \right\} [\sigma_{ij}']_{\phi_M, \lambda_M} \quad (3.2)$$

where the bracket expression is an equivalent modulus and s the operator $\partial/\partial t$. Thus expressing the stresses in these elements by the strains yields :

$$\left. \begin{aligned} [\sigma_{ij}']_{\phi_M} &= \phi_M^{-1} \varepsilon_{ij}' \\ [\sigma_{ij}']_{\lambda_M} &= \lambda_M^{-1} \dot{\varepsilon}_{ij}' \end{aligned} \right\} \quad (3.3)$$

since the sum of $[\sigma_{ij}']_{\phi_M}$ and $[\sigma_{ij}']_{\lambda_M}$ must be equal to the effective stress σ_{ij}' the following relation for the deformation of the matrix is obtained :

$$\phi_M^{-1} \varepsilon_{ij}' + \lambda_M^{-1} \dot{\varepsilon}_{ij}' = \sigma_{ij}'(t) \quad (3.4)$$

Denoting as before by $\bar{\sigma}_{ij}'$, $\bar{\varepsilon}_{ij}'$ the Laplace transforms of these quantities and transforming both sides of eq. (3.4) gives

$$\phi_M^{-1} \bar{\varepsilon}_{ij}' + \lambda_M^{-1} s \bar{\varepsilon}_{ij}' = \left\{ \frac{s\phi_M + \lambda_M}{\phi_M \lambda_M} \right\} \bar{\sigma}_{ij}' = \int_0^\infty \bar{\sigma}_{ij}'(t) e^{-st} dt$$

and

$$\begin{aligned} \bar{\varepsilon}_{ij}' &= \lambda_M [s + \lambda_M \phi_M^{-1}]^{-1} \int_0^\infty \bar{\sigma}_{ij}'(t) e^{-st} dt \\ \varepsilon_{ij}' &= L^{-1} [\lambda_M (s + \lambda_M \phi_M^{-1})^{-1}] \int_0^\infty \bar{\sigma}_{ij}'(t) e^{-st} dt \end{aligned} \quad (3.5)$$

Using the convolution theorem, viz :

$$\begin{aligned} \int_0^\infty F_1(\tau) F_2(t-\tau) d\tau &= \int_0^t F_1(\tau) F_2(t-\tau) d\tau \\ &= L^{-1} F_1(s) F_2(s) \quad (F_2(t-\tau) = 0 \quad \tau > t) \end{aligned}$$

yields :

$$\begin{aligned} (c) \quad \phi_H' &= \phi_H + \phi_M \\ \phi_M' &= \frac{\phi_H \phi_M}{\phi_H + \phi_M} \\ \lambda_M' &= \left(\frac{\phi_H + \phi_M}{\phi_M} \right)^2 \lambda_M \end{aligned}$$

Fig. 1

$$\varepsilon_{ij}' = \lambda_M \int_0^t \sigma_{ij}'(\tau) e^{-\lambda_M(t-\tau)/\phi_M} d\tau \quad (3.6)$$

It is easily seen that this solution satisfies the boundary condition $\varepsilon_{ij}'(0)=0$ in (3.4).

Hence using eq. (3.1) the total strain-time function is obtained as follows:

$$\varepsilon_{ij}'(t) = \phi_H^{-1} \sigma_{ij}'(t) + \lambda_M \int_0^t \sigma_{ij}'(\tau) e^{-\lambda_M(t-\tau)/\phi_M} d\tau \quad (3.7)$$

which is of the same mathematical form as eqns. (1.1) (1.3), but including the relations given by (2.6) and (2.9). This solution is only valid for the isothermal case and on the assumption that the parameters ϕ , λ themselves are independent of temperature.

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