General paper

FINITE ELEMENT ANALYSIS OF OUT-OF-PLANE DEFORMATION IN LAMINATED SHEET METALS BASED ON AN ANISOTROPIC PLASTICITY MODEL

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Abstract: An anisotropic plastic constitutive model is applied to a three-dimensional finite element method based on the up-dated Lagrangian type formulation, and out-of-plane deformation of the laminated sheet metal which comprises different materials of stainless steel sheet and aluminum sheet is analyzed. The constitutive model involves a fourth order tensor in its representation, by which not only the initial orthotropy but also the subsequent skewed-anisotropy can be predicted. The variation of anisotropic axis is taken into account with reference to the kinematic relationship of the axis before and after the deformation. The conventional laws of both a power formula for work-hardening and a normality rule for plastic flow are employed in the model. Before the analysis, simple tension tests in various directions are performed to identify the anisotropic material parameters of each component layer. Since the mechanical properties vary in the thickness direction, a fully three-dimensional scheme is adopted in the numerical analysis. Here, we examine the following three kinds of loading patterns; (a) curl in transverse direction under a simple tension, (b) deflection at longitudinal direction in unloaded state following the tension, and (c) strain localization behavior in large deformation regime. The effect of thickness ratio is discussed from the viewpoint of the reduction of out-of-plane deformation, and the difference due to lamination angle is also examined to clarify the combination effect of planer anisotropy.

Key words: *Plasticity, Anisotropy, Constitutive equation, Finite element method, Three-dimensional analysis, Laminated sheet metals, Out-of-plane deformation*

1 INTRODUCTION

Material anisotropy is one of the most characteristic properties not only in usual sheet metals but also in advanced materials such as fibre-reinforced plastics, unidirectionally solidified super-alloys and so on. In order to evaluate the formability of the materials, initial anisotropy as well as subsequent (skewed-)anisotropy must be taken into account in the constitutive model since the anisotropy has a predominant influence on the deformation behavior. As for yield function, Hill [1] proposed a simplified quadratic form capable of describing orthotropy, and a finite element analysis based on the original Hill model was carried out by Tomita and Shindo [2] in order to predict wrinkling behavior in a thin plate subjected to tension. However, it is necessary to manipulate orthogonal transformation for tensor variables even under the plane stress condition because Hill's parameters are not invariant under the change of observer. The authors have proposed a generalized anisotropic yield function whose parameters are exactly invariant [3] and have examined the validity of the model in terms of the description of strain localization in initially anisotropic sheet metals [4].

Recent development in material design technology enables us to predict and also to improve the mechanical and/or physical properties of materials, and laminated sheet metals are being focused because various desirable properties such as elastic moduli, weight, and stiff-

ness can be obtained by the optimal design on material combination, lamination angle and so on. The laminated sheet metals, however, exhibit so-called "out-of-plane deformation" even in a simple tension since mechanical properties vary in the thickness. Also the material anisotropy is inevitably induced in the materials through the production process of rolling.

This paper discusses the mechanical response of laminated sheets. In the first part of the paper, the anisotropic plasticity model is briefly summarized. The concept of material anisotropy in terms of the invariance requirement is presented, and then an anisotropic yield function is formulated with reference to the representation theorem given by Spencer [5]. The finite element equation based on the up-dated Lagrangian type (U-L) formulation is obtained next, by which a three-dimensional simulation can be performed. We examine three patterns of loading process on the laminated sheet which is composed of an aluminum and a stainless steel. They are (a) curl in tension, (b) deflection in unloaded state, and (c) strain localization behavior.

2 AN ANISOTROPIC PLASTICITY MODEL

2.1 Material Anisotropy

Material anisotropy is characterized by the existence of so-called preferred orientation and/or anisotropic axis. We assume that there are anisotropic vectors, $m_i^{(1)}$, $\dots$, $m_i^{(n)}$, lying along the axis in a body and that they are

embedded in the body throughout deformation. The norm of the vectors is unity in the initial state. When these vectors are orthonormal, as is schematically illustrated in Fig. 1, the relationship

$$m_i^{(1)} m_j^{(1)} + m_i^{(2)} m_j^{(2)} + m_i^{(3)} m_j^{(3)} = \delta_{ij} \quad (1)$$

holds [5]. When the body undergoes a deformation, the orientation is no longer orthotropic, and so we must specify the rule to transform the vectors.

The velocity gradient L_{ij} transforms an infinitesimal vector dx_i in space into its rate $\dot{dx}_i$, so that this concept may be applicable to the evolution law. Taking account of the objectivity, one of the possible evolutions for the vector takes the form

$$\dot{m}_i^{(n)} = D_{ij} m_j^{(n)} - W_{ij} m_j^{(n)}, \quad (2)$$

where $\dot{m}_i^{(n)}$ denotes the Jaumann derivative of the vector, while D_{ij} and W_{ij} stand for the stretching tensor and the spin tensor, respectively.

2.2 Anisotropic Yield Function

A quadratic yield function F is expressed as

$$F = \frac{1}{2} N_{ijkl} \sigma_{ij} \sigma_{kl} - \frac{1}{3} \bar{\sigma}_y^2 = 0, \quad (3)$$

where σ_{ij} and $\bar{\sigma}_y$ denote the Cauchy stress and the uniaxial yield stress, respectively, and N_{ijkl} is a fourth order tensor reflecting the material anisotropy.

Any function in the constitutive equation must satisfy the objectivity, and so the yield function must be invariant under orthogonal transformation. If the material is purely isotropic, we have a well-known result for N_{ijkl} as

$$N_{ijkl} = \lambda_1 (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) + \lambda_2 \delta_{ij} \delta_{kl}, \quad (4)$$

where λ_1 and λ_2 are scalar coefficients. In particular, we put $\lambda_1 = 1/2$ and $\lambda_2 = -1/3$ to satisfy the incompressibility of plastic strain. When the fourth order tensor involves the anisotropic vectors, it is expressible in a multi-linear representation with the arguments [5]. Provided that two kinds of vectors $m_i^{(1)}$ and $m_i^{(2)}$, which are mutually orthonormal, are involved in the tensor, we obtain the full representation of N_{ijkl} with totally six material parameters. And it was shown that the yield function is then completely identical to Hill's anisotropic yield function [4]. Since the anisotropic vectors do not maintain any particular symmetry in subsequent state, we must generalize the representation so that it should be available for a skewed-anisotropy. As a simple extension of the representation, we put

$$\begin{aligned} N_{ijkl} = & \lambda_1 (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) + \lambda_2 \delta_{ij} \delta_{kl} + \frac{1}{2} \beta_1 (\delta_{ij} G_{kl} + G_{ij} \delta_{kl}) + \beta_2 G_{ij} G_{kl} - \frac{3}{8} \beta_1 (\delta_{ik} G_{jl} + \delta_{il} G_{jk} + G_{ik} \delta_{jl} + G_{il} \delta_{jk}) \\ & + \frac{1}{2} \beta_3 (\delta_{ij} H_{kl} + H_{ij} \delta_{kl}) + \beta_4 H_{ij} H_{kl} - \frac{3}{8} \beta_3 (\delta_{ik} H_{jl} + \delta_{il} H_{jk} + H_{ik} \delta_{jl} + H_{il} \delta_{jk}) + \frac{1}{2} \beta_5 (\delta_{ij} L_{kl} + L_{ij} \delta_{kl}) + \beta_6 L_{ij} L_{kl} \\ & - \frac{3}{8} \beta_5 (\delta_{ik} L_{jl} + \delta_{il} L_{jk} + L_{ik} \delta_{jl} + L_{il} \delta_{jk}) + \frac{1}{2} \beta_7 (G_{ij} H_{kl} + H_{ij} G_{kl}) + \frac{1}{2} \beta_8 (H_{ij} L_{kl} + L_{ij} H_{kl}) + \frac{1}{2} \beta_9 (L_{ij} G_{kl} + G_{ij} L_{kl}), \end{aligned} \quad (5)$$

Fig. 1. Fundamental concept of material anisotropy.

in the case where three kinds of vectors are involved. Here, G_{ij} , H_{ij} , and L_{ij} are given as follows:

$$\begin{aligned} G_{ij} &= m_i^{(1)} m_j^{(1)} - \frac{1}{3} m_k^{(1)} m_k^{(1)} \delta_{ij} \\ H_{ij} &= m_i^{(2)} m_j^{(2)} - \frac{1}{3} m_k^{(2)} m_k^{(2)} \delta_{ij} \\ L_{ij} &= m_i^{(3)} m_j^{(3)} - \frac{1}{3} m_k^{(3)} m_k^{(3)} \delta_{ij} \end{aligned} \quad (6)$$

$m_i^{(1)}$ is chosen to be a vector in the thickness (normal) direction, $m_i^{(2)}$ the rolling direction, and $m_i^{(3)}$ represents the direction normal to these two vectors.

2.3 Work-hardening and Material Flow

In order to complete the anisotropic constitutive model, we apply the conventional laws of both work-hardening and material flow to the yield function. A simple power formula with isotropic hardening is employed in the evolution of the yield stress as

$$\bar{\sigma}_y = a (b + \bar{\epsilon}^p)^n, \quad (7)$$

where the hardening parameter $\bar{\epsilon}^p$ is related to a quadratic of plastic strain. As for the material flow, the normality rule is adopted. Then the stress rate $\dot{\sigma}_{ij}$ is represented, through Prager's consistency condition, as

$$\dot{\sigma}_{ij} = \left\{ E_{ijkl} - \frac{E_{ijpq} s_{pq} E_{klrs} s_{rs}}{E_{\alpha\beta\gamma\delta} s_{\alpha\beta} s_{\gamma\delta} + \frac{4}{9} \bar{\sigma}^2 H'} \right\} \dot{\epsilon}_{kl} = D_{ijkl}^{\text{tan}} \dot{\epsilon}_{kl}, \quad (8)$$

where

$$E_{ijkl} = \frac{E}{1+\nu} \delta_{ik} \delta_{jl} + \frac{\nu E}{(1+\nu)(1-2\nu)} \delta_{ij} \delta_{kl}, \quad (9)$$

$$s_{ij} = N_{ijkl} \sigma_{kl}. \quad (10)$$

E denotes Young's modulus, ν Poisson's ratio, and H' is the plastic tangent modulus derived from eq.(7). In

order to utilize the above equations in the finite element method based on the up-dated Lagrangian type formulation, the stress rate in Eq.(8) is replaced by the Jaumann derivative of the Kirchhoff stress and the strain rate by the stretching tensor [2]. Such a conventional treatment may involve a critical issue on the decomposition of deformation as well as the constitutive modeling. Overall discussions are given elsewhere (e.g. [6].)

2.4 Finite Element Equation

Let a body with volume V be subjected to a velocity constraint on S_v and a surface force rate on S_t . The principle of virtual work is given by

$$\int_V (\dot{S}_{ji} + \sigma_{mj} v_{m,i}) \delta v_{i,j} dV = \int_{S_t} \dot{t}_i \delta v_i dS, \quad (10)$$

where S_{ij} indicates the Kirchhoff stress and the virtual velocity δv_i does not violate the constraint on the surface S_v . The body force is neglected for simplicity. By use of the usual discretization technique, we finally obtain the finite element equation as

$$\sum_e \int_{V_e} \left\{ \mathbf{B}^T (\mathbf{D}^{tan} - \mathbf{F}) \mathbf{B} + \mathbf{Q}^T \mathbf{E} \mathbf{Q} \right\} dV \dot{\mathbf{u}}^{ne} = \sum_e \int_{S_{te}} \mathbf{N}^T \dot{\mathbf{t}} dS. \quad (11)$$

The matrices $\mathbf{B}$ and $\mathbf{D}^{tan}$ are the strain-displacement matrix and tangent modulus matrix, respectively, while $\mathbf{F}$, $\mathbf{Q}$ and $\mathbf{E}$ are conversion terms due to the U-L type formulation [2]. A fully three-dimensional finite element scheme is adopted in the analysis by using a 20-node isoparametric element.

3 ANALYSES AND DISCUSSIONS

3.1 Parameter Identification

The materials employed in the present investigation are a SUS430/pure aluminum(A1100) clad sheet called "clad 1" and a SUS430/aluminum alloy(A3004) sheet named "clad 2." The chemical compositions and the thickness combination of the layers are presented in Table 1. By removing one of the layers on machining, the material properties for each layer are identified from tension tests. Figure 2(a) demonstrates the stress-strain diagrams of each component material. The stress is calculated through the net stress acting on the cross-section, and the rolling direction (R.D.) coincides with the tensile direction, i.e. $\theta = 0^\circ$, in this figure. The SUS430 stainless steel sheet is much hardened because of the prestraining during the production process, and it goes

to rupture only about 3% strain while the aluminum sheets are durable up to 18%. As for the clad sheets, the limits of elongation are about 16% which is nearly equal to that of the aluminum layer. Moreover, we observe that the stainless layer of the clad sheets does not show any clear necking even when it exceeds its own limit of 3%. This implies that the forming limit of laminated sheet can be hardly predicted by the simple combination law of each layer.

The tension tests are repeated for test pieces cut out along several directions to obtain the material parameters which are listed in Table 2. Examples of stress-strain diagram are given in Fig. 2(b) where the pure aluminum layer of clad 1 is examined by the comparison with the constitutive model. The symbol θ indicates the angle between the rolling direction and the tensile direction. The proposed model shows good agreement with the experiments particularly in larger strain region than $\epsilon = 0.1$.

Fig. 2. Stress-strain diagrams of laminated sheets.

Table 1. Chemical compositions of materials in wt %.

Material	Thickness mm	C	Si	Fe	Mn	P	S	Cr	Cu	Nb	Zn	Mg	Al
Clad 1	SUS430	0.56	0.016	0.50	0.45	0.031	0.001	16.11	0.40	0.57	—	—	—
	A1100	1.43	—	0.65	0.01	—	—	—	0.16	—	0.01	—	99.16
Clad 2	SUS430	0.56	0.016	0.53	0.46	0.033	0.001	16.23	0.45	0.59	—	—	—
	A3004	1.47	—	0.12	0.53	0.99	—	—	0.13	—	0.01	0.67	bal.

Table 2. Material parameters of each laminar layer.

Materials		E GPa	ν	a MPa	b	n	β_1	β_2	$\beta_3 = -\beta_5$	$\beta_4 = \beta_6$	$\beta_7 = -\beta_9$	$\beta_8 = -2\beta_4$
Clad 1	SUS430	210	0.30	639	0.0073	0.017	-0.500	-0.675	0.100	0.225	-0.250	-0.450
	A1100	70	0.33	144	0.0154	0.130	-0.200	0.0250	-0.200	-0.125	0.300	0.250
Clad 2	SUS430	210	0.30	718	0.0110	0.017	0.0500	0.0750	0.250	0.175	-0.550	-0.350
	A3004	70	0.33	211	0.0126	0.139	-0.600	-0.525	0.100	-0.0750	-0.300	0.150

3.2 Curl in Tension

When a laminated sheet is subjected to a simple tension, the sheet is distorted in the plane perpendicular to the tensile direction, as is schematically illustrated in Fig. 3(a). This phenomenon is known as "curl." In order to simulate the laminate, a three-dimensional finite element mesh is generated such that each layer has at least two elements in the thickness. The mesh pattern shown in Fig. 3(b) is common for all the simulations throughout this work. The total number of elements is 640, the number of nodes is set to be 3413, and the dimensional scale is $a:b:c = 5:1:10$. The thickness ratio is changed to examine the influence of the layer thickness on the growth of curl.

Figure 4 shows the relationship between the angle of rolling direction (R.D.) from the tensile direction and the conventional parameter $\rho^{-1}t/\varepsilon_z$, which reflects the out-of-plane deformation. Here, ρ and t represent the radius of curvature and the total thickness of the laminated sheet, respectively. We also evaluate the effect of layer thickness by changing the thickness ratio t_A/t , in which t_A indicates the thickness of the aluminum layer. The simulated results sometimes overestimate the curl. This may be due to a fact that the boundary conditions imposed in the analysis are not strictly the same as in real experiment. For example, the grasp of specimen disturbs the boundary condition at the edge. However, the analytical results show qualitative agreement with the experiments.

The degree of curling is restrained more when thinner aluminum is attached because of the release of con-

straint by the aluminum. And the difference in planer anisotropy of the aluminum layer affects the variation of curl. As for the angle dependency, the tension at 45° seems to be the best choice to reduce the out-of-plane deformation for both materials. The planer anisotropy affects the development of transverse strain perpendicular to the tensile direction, and the curl is generated by the difference of this strain in each layer. It follows that the out-of-plane deformation will be reduced when the lamina layers have small difference in the development of transverse strain, or when we choose the tensile direction such that the difference is the smallest.

(a) Curl in Tension (b) Finite element mesh
Fig. 3. Schematic view of curl and mesh pattern.

(a) SUS430/pure-aluminum (clad 1)

(b) SUS430/aluminum-alloy (clad 2)

Fig. 4. The effect of thickness ratio on the out-of-plane deformation.

3.3 Deflection in Unloading Process

When a laminated sheet is taken out from a testing machine after tension, we observe a deflection at longitudinal direction. This is caused by the difference of spring-back characteristics in each layer.

In order to simulate the unloading process in the finite element analysis, the release of surface force is treated in the way such that the prescribed displacement boundary at the preceding tension is changed into the surface force boundary. Once the tensile loading is terminated, the stress components σ_{ij}^* on the surface are evaluated by extrapolating the values at the integral points which are involved in the related elements. The stress vector t_i^* on the surface can be calculated through the stress principle as

$$t_i^* = \sigma_{ji}^* n_j^* \quad (12)$$

where n_i^* stands for the unit normal. And then a small increment Δt_i^* is successively subtracted (or reversely added) step by step until the norm of the vector t_i^* becomes sufficiently small.

Figure 5(a) shows the profile of the laminated sheet, in which a 3% nominal strain is imposed on the sheet followed by the unloading, and the simulated result in Fig. 5(b) describes this behavior. Since the elastic modulus and the flow stress of the stainless steel are about (but not strictly) three times larger than those of the aluminum sheet, the deflection is generated in the way that the harder layer of the steel sheet is to be, in general, at the inner side of the curvature. As the development of curl in the preceding tension brings about a shell-like structure, it increases the bending stiffness, and so the resultant bending is, on the contrary, less than that in flat plates. It follows that this behavior can not be simulated by the simple assumption of the difference of internal stresses in each layer. Moreover, the deflection changes with respect to the tensile direction

as well as the combination of layers since the flow stresses differ in the tensile direction because of the planar anisotropy.

3.4 Strain Localization

Figure 6 demonstrates the equivalent plastic strain distributions in laminated sheets. The material parameters roughly follows those of steel sheet for both layers, and so the laminate is supposed to comprise the same kind of material. The power factor n in Eq.(7) is set to be 0.09 to avoid the difficulty of step control in numerical computation. Here, we change the thickness ratio as well as the lamination angle in order to clarify the onset and growth of strain localization in the laminated sheets. The nominal strain of 9% is imposed on the sheets. Figure 6(a) indicates the result for the material with the same lamination angle of 45° which corresponds to a single layered sheet, while Fig. 6(c) for an

Fig. 5. Profile of deflection after unloading process.

Fig. 6. Equivalent plastic strain distribution of laminated sheet under strain localization process.

orthogonally laminated material with the thickness ratio of 1:1. Figure 6(b-1) represents the result in the thicker plane of the orthogonally laminated material whose thickness ratio is 3:1, and Fig. 6 (b-2) for the thinner plane.

The strain localization grows in the perpendicular direction to the initial anisotropy in the single layered sheet, and the authors have already reported the validity of this behavior on a steel sheet[4]. In contrast the onset of strain localization is decelerated in the laminated sheets, and the localized direction is not clear as shown in Figs. 6(b) and (c). Moreover, the competing effect of the strain localization in the two layers increases the forming limit. That is to say that the maximum strain in Fig. 6(c) is at most 0.15 (15%) while the plastic strain larger than 0.2 (20%) is observed in Fig. 6(a). When the layers are laminated with different thickness combination, the anisotropy characteristics in thicker layer governs the direction of strain localization as a whole. And the competing effect is not so remarkable. It follows that the forming limit of laminated materials increases when the planer anisotropy cancels with each other and when the load capacity is a similar weight in each layer.

4 CONCLUDING REMARKS

Three-dimensional finite element analyses are carried out in order to evaluate the out-of-plane deformation in laminated sheet metals. The obtained results are as follows:

(1) The curl in tension is reduced when the combination of the layers is chosen such that the difference of the development in transverse strain is to be small, and this difference in transverse strain is caused by the planer

anisotropy.

(2) The out-of-plane deformation in unloading process involves so complicated phenomena that a simple combination law is not available for the analysis of the final dimensional accuracy of laminates.

(3) The numerical results show that the forming limit of laminated materials will increase by choosing not only the proper lamination angle but also the thickness combination.

Although the simulated results show only qualitative agreement with experiments because of some difficulties in computational circumstances, we come to a conclusion that we can evaluate the mechanical response of laminated materials if all the parameters for each constituent are specified based on the anisotropic plasticity model. Further simulation with more elements will enable us to design the optimal material which is required for real machine components.

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