EXAMINATION OF A WAKE-DILATATION MODEL OF MICROCRACK TOUGHENING IN CERAMICS
Study on a Model Composite System with Low-Expansion Dispersed Particles

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Abstract: An attempt was made to examine the validity of a wake-dilatation model of microcrack toughening in ceramic matrix composites. Model two-phase particulate composite system was chosen for study. Spheroidized alumina particles with average size 25 and 12 µm, respectively, were dispersed in a soda lime silica glass having thermal expansion coefficient greater than the expansion coefficient of alumina. The composite containing 25-µm alumina particles was formulated by selecting a combination of differential thermal expansion and particle size, whereby microcrack toughening is expected to occur most effectively. Another composite containing 12-µm alumina particles was designed as a reference material, in which the condition for the occurrence of microcracking is not met. For both composites, specimens containing 30vol% of alumina particles were prepared by hot-pressing technique. Fracture toughness and R-curve measurements were carried out on each of the composites. Glass-alumina(25µm) composite exhibited a rising R-curve behavior. The experimental results were interpreted on the basis of a wake-dilatation model of microcracking. By evaluating physical parameters involved in the model, its validity was discussed.

Key words: Ceramics, Particulate composites, Brittle matrix composites, Microcrack toughening, Crack-tip shielding, Zone shielding, Wake-dilatation model, R-curve behavior, Fracture toughness.

1 INTRODUCTION

Stress-induced microcracking is one of the most commonly accepted toughening mechanisms in ceramic matrix, particulate composites[1-3]. This mechanism is primarily due to thermal expansion mismatch between the matrix and the dispersed phase, which causes residual stresses within and around the dispersed particles when the composite cools down from its fabrication temperature. Microcracks may be generated due to superposition of the high tensile stresses concentrated near the crack tip and the residual mismatch stresses. It has long been considered that the microcrack toughening results primarily from energy dissipation due to the elastic modulus diminution in an extended microcrack zone in front of crack tip(frontal process zone)[1,2]. However, recent studies suggest that the main source of the microcrack toughening is the crack-tip shielding caused by the process zone wake. On the basis of the concept of zone shielding by the wake, originally developed for transformation toughening[4], Evans and Faber[5] proposed a wake-dilatation model of microcrack-induced toughening in brittle solids. Later, detailed analyses of this model were made by Hutchinson[6]. On the other hand, Lawn et al.[7] and Padture et al.[8] proposed different models of microcrack toughening based on the concept of contact shielding(bridging). There exist little experimental data that enable us to judge which of the models explains really microcrack toughening phenomena; it may also be possible that both models are reasonable in different circumstances. The validity of these models, therefore, remains to be basically examined by systematic experiments.

The purpose of the present study is to examine the validity of the wake-dilatation model for microcrack toughening. The study was focused on a model composite system exhibiting radial microcracking around dispersed particles. To accomplish this, experimental studies were undertaken on two-phase glass matrix composites with low-expansion dispersed particles.

2 EXPERIMENTAL PROCEDURE

2.1 Design of Microcracking Two-Phase Composite System

Two-phase composite system consisting of spherical alumina particles dispersed in a glass matrix was designed. As the matrix, a soda lime silica glass was formulated to have thermal expansion coefficient greater than that of alumina. In subsequent description, this glass will be referred to as G4. As the dispersed phase, spherical alumina particles were selected; a fused alumina powder spheroidized in an oxy-propane gas flame (HARIMIC AX25, Micron Co., Ltd., Himeji, Japan). Hereinafter, this alumina powder will be referred to as A. In Table 1, chemical composition, linear coefficient of

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Table 1. Materials used for study.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Composition (wt%)</th>
<th>Linear Coef.Therm. Expansion $\alpha$ ($10^{-6}$K$^{-1}$)</th>
<th>Young's Modulus $E$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
<td>$65SiO_2$</td>
<td>14.5</td>
<td>64.2</td>
</tr>
<tr>
<td>Glass</td>
<td>$30Na_2O$</td>
<td>5CaO</td>
<td></td>
</tr>
<tr>
<td>G4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispersed Particles</td>
<td>Fused Alumina</td>
<td>7.8</td>
<td>313</td>
</tr>
<tr>
<td>A</td>
<td>(flame-sprayed)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

thermal expansion and Young's modulus of individual constituent are listed.

In brittle matrix composites containing low-expansion particles, radial microcracking at particles can occur either spontaneously (in the absence of an applied stress) or in subsequent stressing, due to residual thermal stresses. It has been recognized that the occurrence of microcracking in particulate composites depends strongly on both the magnitude of residual stress due to thermal expansion mismatch and the dispersed particle size\[1, 9\]. It has also been suggested that microcracking is expected to yield effective toughness increase of composites, as the particle size approaches the critical size for spontaneous cracking \[1, 9, 10\]. An expression for this critical particle size $D_c$ below which such spontaneous microcracking does not occur was derived by Lange\[11\] for the case of spherical particle dispersion;

$$D_c = \frac{k}{\beta^2}$$  \hspace{1cm} (1)

with

$$\beta = \frac{\left(\alpha_m - \alpha_p\right) \Delta T}{1 + \frac{1}{E_m} + \frac{1 - 2\nu_p}{2E_m E_p}}$$  \hspace{1cm} (2)

where $\beta$ is the residual internal pressure induced at the matrix-particle interface due to thermal expansion mismatch, $k$ is a semi-empirical constant for a given matrix-particle pair, $\alpha$ is the linear coefficient of thermal expansion, $\Delta T$ is the temperature cooling range over which the matrix plasticity is negligible, $E$ is the Young's modulus, $\nu$ is the Poisson's ratio: the subscripts $m$ and $p$ refer to the matrix and particle, respectively. As preliminary experiments, alumina particles with different size classes were dispersed in G4 glass matrix by hot-pressing, and microscopic observation and mechanical testing as well as AE(acoustic emission) activity measurements during bending test were performed. Such systematic experiments revealed that the critical size of alumina particles for spontaneous microcracking in G4 matrix is nearly 30$\mu$m. On the basis of estimated value of $D_c$ in G4-A system, two glass-alumina composites were formulated;

1. G4-A(25): composite containing 30vol% of alumina particles sized to 25$\mu$m in average diameter. In this composite, microcrack toughening is expected to occur at favorable conditions.

2. G4-A(12): composite containing 30vol% of alumina particles sized to 12$\mu$m in average diameter. This composite is designed as reference material where no effective toughening by microcracking occurs.

2.2 Preparation of Composite Specimens

The matrix glass was prepared by conventional melting procedure. The glass batches of reagent grade powders were melted at 1450 to 1500$^\circ$C in platinum crucibles to produce homogeneous bubble-free glasses. The melts were air-quenched and then ground in a vibratory mill. The A(25) powder was obtained by screening as-received alumina powder, HARMIC AX25 into a narrow range of size, 20 to 32$\mu$m. The A(12) powder was prepared by passing finer alumina particles through 20$\mu$m-sieve to separate them from coarser ones.

Alumina powders, A(25) and A(12), respectively were thoroughly mixed with the powdered matrix glass G4 to form mixtures containing 30vol% of alumina. The mixtures were then hot-pressed in 30 mm diameter stainless steel dies coated with graphite lubricant, heated by an electric tube furnace. For both G4-A(25) and G4-A(12) composites, disk specimens 30 mm in diameter and 6 or 10 mm in thickness were fabricated. The details of specimen fabrication technique were described elsewhere\[12\].

2.3 Microstructure Characterization

Polished surfaces of as-fabricated specimens were observed using optical and scanning electron microscopy. Densities of the prepared composites were determined by the Archimedes immersion technique in kerosene relative to a fused silica standard.

2.4 Fracture Toughness($K_c$) Evaluation

Fracture toughness($K_c$) was determined using the three-point bend test of a single-edge-precracked-beam(SEPB) \[13\]. Specimens were prepared in the form of rectangular bars 2.5 by 6 by 28 mm by cutting from the hot-pressed disks. Pre-cracking was accomplished by bridge-indentation(BI) technique\[13\]. Pre-crack lengths were measured by optical microscopy prior to testing and these were also confirmed by post-fracture observation of the fracture surface. Pre-crack length/specimen height ratio was chosen to be 0.35 to 0.45. Specimens were tested in three-point bending over a 24 mm span at a cross-head speed of 0.05 mm min$^{-1}$. All measurements were made at room temperature. To minimize the effect of moisture-assisted slow crack growth, a dry nitrogen gas was continuously blown at a constant rate on the stressed surface of the specimen during testing. Seven to ten specimens were fractured for each of the composites. $K_c$ values were determined from specimen dimensions and fracture load using the equation given by Brown and

214
2.5 Evaluation of R-Curve Behavior

R-curve behavior of each of the composites was evaluated using indentation-strength-in-bending (ISB) method. This method was based on the work of Chantikul et al. [15] who showed that bending strength of a specimen pre-cracked by Vickers indentation can be related to the fracture toughness of a material. One of the present authors [16] extended this theory to the case where process zone wake induces crack-tip shielding contributions, and showed that ISB technique is a useful tool for evaluation of crack-size dependent fracture resistance (R-curve).

2.5.1 Theoretical basis

When process zone wake shields the crack-tip from the applied stress, the stress intensity factor just ahead of the tip, \( K_{tip} \), may be characterized by

\[
K_{tip} = K_{ap} - \Delta K
\]

where \( K_{ap} \) is the applied or remote stress intensity factor determined by the applied stress and crack size, \( \Delta K \) (>0) is the stress intensity change due to crack-tip shielding. Suppose the case where an indentation crack with a process zone wake which induces a shielding effect at the crack-tip is subsequently subjected to a bending stress. As a first approximation, shielding effect induced by the wake is assumed to be represented in terms of crack-surface closure stress distributed over a whole length of the crack. The near-tip stress intensity factor \( K_{tip} \) can be expressed as

\[
K_{tip} = \chi_R \frac{P}{a^{3/2}} + \eta \sigma a^{1/2} - \gamma' \sigma a^{1/2}
\]

where \( a \) is the size of indentation crack, \( P \) is the indentation load, \( \chi_R \) is a dimensionless constant for Vickers-produced median/radial crack, depending on indentation geometry and material properties, \( \sigma \) is the applied stress, \( \eta \) is the shielding stress (either compressive stress on crack surfaces in the case of zone shielding or crack-surface traction stress in the case of contact shielding), and \( \gamma \) and \( \gamma' \) are the crack-geometry factors which embrace various effects such as free surface effects and crack-interaction terms other than purely geometrical factor. The first two terms of the right-hand side of Eq. (4) correspond to \( K_{ap} \), originally proposed by Chantikul et al. [15] and the last term to \( \Delta K \) in Eq. (3). It is the near-tip field stress intensity that provides the crack extension criterion:

\[
K_{tip} = K_0
\]

where \( K_0 \) is the fracture resistance of the material immediately ahead of the crack-tip.

By inserting the equilibrium crack-growth condition \( K_{tip} = K_0 \) into Eq. (4) and requiring the equilibrium to be critical \((d\sigma/d\alpha = 0)\), the critical stress \( \sigma_f \) and critical crack size \( a_f \) are defined as follows;

\[
\sigma_f = \left( \frac{K_0}{\eta} \right)^{4/3} \left( \frac{Y'}{Y} \right) \sigma
\]

\[
a_f = \frac{4\chi_R^{2/3} K_0^{1/3}}{p^{2/3}}
\]

\[
\eta = \left( \frac{256}{27} Y^3 \chi_R \right)^{1/4}
\]

The crack extension resistance \( K_R \) is given by

\[
K_R = K_0 + \gamma \sigma a^{1/2}
\]

\[
= \eta \left[ \sigma_f \frac{p^{1/3}}{4} \left( 1 - \frac{1}{4} \left( \frac{Y'}{Y} \frac{\sigma}{\sigma_f} \right)^2 \right) \left( 1 - \frac{\gamma \sigma}{\gamma_f} \right) \right]^{3/4}
\]

Equation (6) suggests that a plot of \( \sigma_f \) against \( P^{-1/3} \) should give a straight line with a slope of \((K_0/\eta)^{4/3}\) and an intercept of \((Y'/Y)\sigma_f\). It is also found from Eq. (7) that a plot of \( a_f \) against \( P^{2/3} \) should give a straight line that coincides with the origin and its slope should be \((4\chi_R/K_0)^{2/3}\). These relations enable the values for \( K_0 \), \( \eta \) (and hence \( \chi_R \)) and \( \sigma_f \) to be estimated by measuring the bending strength \( \sigma_f \) as well as critical crack size \( a_f \) of indented specimens with varied loads. Once the values for \( K_0 \), \( \eta \) and \( \sigma_f \) have been reasonably determined, \( K_R \) can be calculated from indentation-strength data using Eq. (9) and hence R curve can be evaluated.

2.5.2 Technique for R-curve evaluation

Flexural specimens were prepared in the form of rectangular bars 4 by 5 by 24 mm by cutting from the hot-pressed disks. Three Vickers indentations were made at the center of the prospective tensile surface of each specimen in such a way that the pyramidal edges are aligned with respect to the longitudinal axis for the specimen. The indentation loads were varied from 49 to 147 N (5 to 15 kgf). To minimize the effect of moisture-assisted slow crack growth, a drop of liquid paraffin was placed on the pre-selected site prior to indentation experiments. Bending strength of the indented specimens were measured with a testing machine using four-point loading over an inner span of 10 mm and an outer span of 20 mm at a cross-head speed of 0.05 mm min\(^{-1}\). During fracture testing, a dry nitrogen gas was continuously blown on the tensile surface of the specimen. Failure occurred from one of the three indentations while at the remaining two intact indentations, cracks perpendicular to the applied stress had grown to critical size. The sizes of two extended cracks remaining intact were measured by optical microscopy, and their average was taken as critical.
crack size \( a_f \). Six to eight specimens were tested for each indentation load. After determining \( K_0 \), \( \eta \) and \( \sigma_s \) values through \( \sigma_f \) vs. \( P^{-1/3} \) and \( a_f \) vs. \( P^{2/3} \) plots, \( K_R \) values were calculated from indentation-strength data. \( R \) curve was then evaluated by plotting \( K_R \) against measured \( a_f \).

3 RESULTS

3.1 Microstructure Characteristics of Hot-Pressed Specimens

Figure 1 shows polished surface of as-fabricated G4-A(25) composite (scanning electron microscope, back scattering electron mode). It was found that spherical alumina particles are uniformly distributed and the porosity is nearly zero. Similar features in microstructure were observed for G4-A(12) composite. In fact, for both G4-A(25) and G4-A(12) composites, the measured densities were in fair agreement with those predicted from the densities of matrix glass and alumina constituents. Even in G4-A(25) composite, no spontaneous microcracking during fabrication process could be detected by scanning electron microscope observation.

3.2 Fracture Toughness and R-Curve Behavior

Results of fracture toughness \( K_c \) measurements are listed in Table 2. Note that the measured \( K_c \) value for G4 glass is 0.77 MPa m\(^{1/2}\) and the dispersion of 30 vol% alumina particles in G4 matrix produces an increase in \( K_c \). It is found that G4-A(25) composite, where effective microcrack toughening is expected to occur, exhibits higher value of fracture toughness than G4-A(12) composite.

<table>
<thead>
<tr>
<th>Composite</th>
<th>Fracture Toughness ( K_c ) (MPa m(^{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>G4-A(25)</td>
<td>2.33 ± 0.08</td>
</tr>
<tr>
<td>G4-A(12)</td>
<td>1.89 ± 0.04</td>
</tr>
</tbody>
</table>

In Fig. 2, \( \sigma_f \) vs. \( P^{-1/3} \) and \( a_f \) vs. \( P^{2/3} \) plots are respectively shown for G4-A(25) composite. In accordance with Eqs. (6) and (7), these two plots are found to be linear. From the slopes, \( K_0/\eta \) and \( \chi_R/\chi_0 \) can be evaluated. Then, if we take an appropriate value for crack-geometry factor \( Y \) through which \( \chi_R \) and \( \eta \) are mutually related, \( K_0 \) and \( \eta \) can be evaluated from these two slopes. The crack-geometry factor \( Y \) was experimentally estimated at 0.9708 for a Vickers-produced median/radial crack in soda-lime-silica glass by Marshall and Lawn[17]. In the present study, the value for \( Y \) was taken as 0.971, and then \( K_0 \) and \( \eta \) were determined by slope data. Thus, all the constants involved in Eq. (9), \( K_0 \), \( \eta \), \( (Y'Y)\sigma_s \) (intercept in \( \sigma_f \) vs. \( P^{-1/3} \) plot) being reasonably estimated, \( K_R \) was calculated from \( \sigma_f \) and plotted against measured value of \( a_f \). \( K_R \) vs. \( a_f \) plot
MODEL OF MICROCRACK TOUGHENING IN CERAMICS

Fig. 3. Crack growth resistance data for G4-A(25) composite plotted against critical crack size $a_f$.

(R curve) thus obtained for G4-A(25) composite is illustrated in Fig. 3. It is found that G4-A(25) composite exhibits a rising $R$-curve behavior and the inherent fracture resistance, $K_0$ is estimated at 1.85 MPa m$^{1/2}$. $R$ curve of reference composite, G4-A(12) was evaluated in much the same way. Plots of $\sigma_f$ vs. $P^{-1/3}$ and $a_f$ vs. $P^{2/3}$ for this composite are shown in Fig. 4(a) and (b), respectively, and resulting $R$ curve is illustrated in Fig. 5. G4-A(12) composite is found to exhibit a flat $R$-curve behavior, and $K_0$ is estimated at 2 MPa m$^{1/2}$.

4 DISCUSSION

4.1 $R$-Curve Behavior

Figure 3 clearly suggests that, in the fracture process of G4-A(25) composite, the process zone wake induces some shielding effects on crack-tip stress intensity; either zone shielding or contact shielding. It is expected that $K_R$ reaches a saturated value with increasing $a_f$. However, since the maximum indentation load set up in the present ISB procedure was 147 N, only a rising region of $R$ curve was obtained. The shaded band on the right-hand side in Fig. 3 represents the fracture toughness $K_c$ measured by bend test of SEPB specimen. In the present study, this $K_c$ value will be taken as the saturated value for $K_R$.

It is found that $K_0$ evaluated for G4-A(25) composite is approximately identical to that for G4-A(12) composite exhibiting a flat $R$-curve. This $K_0$ value is also identical to $K_c$ values measured on other glass-30 vol% alumina particle composites, where no effective microcrack toughening occurs[12]. These facts suggest that the value of $K_0$ can be reasonably interpreted as the inherent crack-growth resistance involving contributions due to direct interactions between propagating crack front and particles. For G4-A(25) composite, $K_0$ may include some contribution due to crack-tip interactions between propagating crack and alumina particles such as crack bowing(pinning) and crack deflection. This interpretation concerning $K_0$ may be supported by the fact that the increase in toughness due to direct crack-particle interaction mechanisms is independent of particle size and depends only on shape and volume fraction of dispersed particles[2,3].

Fig. 4. Plots of (a) $\sigma_f$ vs. $P^{-1/3}$ and (b) $a_f$ vs. $P^{2/3}$ for G4-A(12) composite.

Fig. 5. Crack growth resistance data for G4-A(12) composite plotted against critical crack size $a_f$. 
4.2 Wake-Dilatation Model of Microcrack Toughening

A wake-dilatation model[5,6] assumes that the crack-tip shielding may be induced by both the elastic modulus reduction in the microcracked process zone and the dilatation caused by microcrack opening in the process zone wake(Fig.6). The analysis based on this model predicts that the increase in toughness by microcracking is primarily due to the crack-tip shielding induced by fully developed process zone wake. Such an influence of the wake should necessarily give rise to an R-curve behavior. Hutchinson[6] discussed the effect of profuse microcracking at the tip of a main crack and computed the shielding effect of microcracks to the lowest order in microcrack density, considering a number of possible alternative models. The fracture toughness due to microcrack toughening is given in the general form by

\[ K_c = K_\theta + \frac{E\theta^T h^{1/2}}{(1-v)(1-\delta f)} \]  \hspace{1cm} (10)

where \( K_c \) is the measured fracture toughness, \( K_\theta \) is the fracture resistance of material in the microcracked zone immediately ahead of the tip of main crack, \( E \) is the Young's modulus of uncracked material, \( \theta^T \) is the effective volumetric dilatation of process zone wake, \( f \) is the microcrack density parameter at saturation, \( h \) is the microcracked zone width. \( \xi \) is a constant which depends on microcrack distribution, microcrack-nucleation criterion taken into account, etc. If the orientation of the microcracks is assumed to be randomly distributed within the wake and if a critical mean stress is selected as a microcrack initiation criterion, the constant \( \xi \) is evaluated at 0.214. \( \delta \) is a constant associated with the effect of elastic modulus diminution caused by the formation of microcracks. Its value depends slightly on Poisson's ratio; at the critical mean stress criterion, \( \delta \) is estimated at 1.20 for \( v = 0.25 \). \( K_\theta \) can be related to the intrinsic crack-growth resistance of uncracked material, \( K_\theta^* \);  

\[ K_\theta = 1 - \frac{f}{1 - \delta f} K_\theta^* \] \hspace{1cm} (11)

Since \( \delta \) is close to 1, \( K_\theta \) is nearly equal to \( K_\theta^* \) as long as the microcrack density \( f \) is low. In a particulate composite where thermal expansion coefficient of the particles is lower than that of the matrix, tensile circumferential stresses are developed and radial microcracking in the matrix is expected. Considering an annular microcrack (width \( c \)) outside a spherical particle with radius \( R \) (Fig.7), Hutchinson analyzed the microcrack density parameter, \( f \) and the effective volumetric dilatation of process zone wake, \( \theta^T \), and gave the following equations;

\[ f = \frac{3\pi^2}{16} N R c^2 \left(1 + \frac{2c}{3R}\right) F(c/R) \]  \hspace{1cm} (12)

\[ \theta^T = \pi^2 N (1-v^2) R c^2 \left(1 + \frac{c}{R}\right)^{-1} \frac{\sigma_0}{E} \] \hspace{1cm} (13)

where \( N \) is the number of microcracks per unit volume, \( \sigma_0 \) is the residual tensile circumferential stress just outside the particle (\( \sigma_0 = \beta/2 \) where \( \beta \) is given by Eq.(2)). \( F(c/R) \) is monotonically decreasing function varying from 1 (when \( c/R \) is zero) to 0.81(when \( c/R \to \infty \); for \( c/R \ll 1 \), it is very close to 1.

4.3 Interpretation of Experimental Results

Here, an attempt will be made to interpret the experimental results obtained for G4-A(25) composite on the basis of the wake-dilatation model.

4.3.1 Elastic moduli of uncracked material

It should be noted that Hutchinson's analyses are based on the assumption that both particle and matrix have common Young's modulus \( E \) and Poisson's ratio \( v \). In the present interpretation, effective elastic moduli of composite containing spherical particles[18] were used.

4.3.2 Evaluation of parameters involved in the model

Equations (12) and (13) indicate that both the microcrack density parameter \( f \) and the effective permanent volumetric dilatation within the wake, \( \theta^T \) depend upon the microcrack size to particle radius ratio, \( c/R \). Therefore, in the absence of the knowledge of \( c/R \), it is impossible to evaluate \( f \) and \( \theta^T \) directly from Eqs.(12) and (13). However, another \( \theta^T \)-related value can be
obtained from the experimental results. Recall that the extrapolation of the linear plot of $\sigma_f$ vs. $P^{-1/3}$ to the intercept gives the value for $(Y'/Y)\sigma$. Assuming that a zone shielding is induced by the wake, $\sigma_s$ corresponds to the residual compressive stress over the whole length of the crack. On the basis of this assumption, $\sigma_s$ can be theoretically related to $\theta_T$ as follows[4,5]:

$$\sigma_s = \frac{\bar{B}}{\bar{v}} \theta_T \left( 1 - \frac{2\bar{v}}{1 - \bar{v}} \right)$$  \hspace{1cm} (14)$$

Here, $\bar{B}$ and $\bar{v}$ are, respectively, the bulk modulus and Poisson's ratio of the microcracked material, and obtained from[6]

$$\bar{B} = B \left( 1 + \frac{16(1 - \nu^2)}{9(1 - 2\nu)} f \right)^{-1}$$  \hspace{1cm} (15)$$

$$\bar{v} = \nu \left( 1 - \frac{16(1 - \nu^2)}{15(2 - \nu)} f \right)^{-1}$$  \hspace{1cm} (16)$$

where $B$ and $\nu$ are the bulk modulus and Poisson's ratio of the uncracked material, respectively.

From the intercept in the linear plot of $\sigma_f$ vs. $P^{-1/3}$ (Fig.2(a)), $(Y'/Y)\sigma_s = 12.8$ MPa. Factor $Y'$ which is associated with the crack and residual loading geometry may be somewhat different from $Y$, but its difference is assumed to be small. This assumption may be justified so long as the zone shielding is hypothesized. Hence, by taking $Y'/Y \equiv 1$, $\sigma_s$ is estimated at 12.8 MPa.

Residual mismatch stress $\sigma_s$ was calculated from Eq.(2). Physical parameters involved in the model were estimated by trial-and-error method. If we assume a value for $c/R$, $\theta_T$ value can be calculated independently from Eq.(13) and Eq.(14). The calculations were repeated with varied $c/R$ values until Eqs.(13) and (14) give common value for $\theta_T$. Thus, $c/R$, $f$ and $\theta_T$ values were finally determined. The microcrack-zone width, $h$ was estimated from Eq.(10), using the values $K_c = 2.33$ MPaEm$^{1/2}$ and $K_0 = 1.85$ MPaEm$^{1/2}$. The values of parameters thus estimated are summarized in Table 3. It should be noted that the accuracy of evaluated value for $h$ is not satisfactory, since its value was evaluated from the toughness difference $\Delta K_c = K_c - K_0$; considering the accuracy of the fracture toughness determination, the value for $h$ can be calculated to only one significant figure at most.

4.3.3 Evaluation of the model

It seems, at least numerically, that the toughening behavior of G4-A(25) composite can be interpreted well by the wake-dilatation model. However, in order to judge that this model explains really the microcrack toughening process, evaluated physical parameters should be experimentally verified. For this purpose, two kinds of experiments were attempted. The first attempt was to examine directly whether such microcracked zone really exists. Microscopic observations were made of the tips and flanks of the primary crack extended in G4-A(25) composite.

Table 3. Interpretation of experimental data for G4-A(25) composite using the wake-dilatation model.

<table>
<thead>
<tr>
<th>Composite</th>
<th>$c/R$</th>
<th>$f$</th>
<th>$\theta_T$</th>
<th>$h(\mu m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G4-A(25)</td>
<td>1.0</td>
<td>0.22</td>
<td>4.3×10$^{-4}$</td>
<td>~1,000</td>
</tr>
</tbody>
</table>

Fig.8. Variation of Knoop hardness with distance from fracture surface for (a)G4-A(25) composite, (b)G4-A(12) composite and (c) G4 glass.
composite specimen. But this attempt was unsuccessful; some difficulties arose from unavoidable microcrack formation on specimen surface due to lapping and polishing procedures carried out prior to this observations. Another attempt was to evaluate indirectly the width of microcracked zone by measuring Knoop indentation hardness. 

**CONCLUDING REMARKS**

Toughening phenomena observed in low-expansion particles. Assuming that the second-phase particle are in a state of residual thermal expansion dilatational mismatch relative to the matrix, Lawn et al. [7, 8] proposed a toughening mechanism of bridging due to frictional sliding stresses at the interface. It is then worthwhile to examine whether this sliding friction model explains toughening mechanism of bridging due to frictional stresses at the interphase. It is then worthwhile to examine whether this sliding friction model explains the question was raised whether the microcrack opening effectively results in the wake dilatation and associated crack-tip shielding. So, more comprehensive studies should be undertaken, by considering both the wake-dilatation and sliding friction phenomena, to obtain further understanding of microcrack toughening phenomena in ceramic composites.

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