Review paper

STRENGTH CHARACTERISTICS OF STRUCTURAL CERAMICS

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Abstract: In structural ceramics, a remarkable scatter of strength is observed due to brittle behavior in deformation and flaws unavoidably formed during material processing. Therefore, information on strength characteristics of ceramics is required for the mechanical design in their applications to high-performance components. In the present paper, the strength properties of typical ceramics are reviewed from statistical aspects, especially. As for the static strength, general features of ceramic strength are first outlined, and then several factors affecting strength, such as environments, specimen geometry, stress state and machining, are described. The fatigue strength is reviewed in two categories of the static and cyclic fatigue. As one of screening procedures to exclude components with a lower strength, the proof testing in ceramics is described and the problems in the testing are also mentioned.

Key words: *Ceramics, Strength, Statistics, Static strength, Fatigue, Proof testing, Weibull distribution*

1 INTRODUCTION

Two decades have almost passed since ceramic materials attracted attention as candidate materials for high-performance structural components. It has been recognized, however, that a large scatter is found in strength characteristics of ceramics compared with those of metallic materials. For instance, strength variations of $\pm 25\%$ from the average values are often observed in ceramics, while a scatter in yield or tensile strength of metallic materials is within a few percentages [1]. Such a marked variation in strength is mainly attributed to the sensitivity of strength to pre-existing flaws of various types in ceramics, which are inevitably generated during processes of sintering and machining after sintering. The sensitivity of strength to defects is caused by difficulty in motion of dislocation due to ion and/or covalent atomic bonding in ceramics [1]. This results in brittle fracture even in an ambient atmosphere, because the temperature, at which the primary slip system is activated, is much higher than metals; e.g. 1473K in a hexagonal alumina, higher than 2073K in a hexagonal β -silicon nitride, and higher than 2273K in cubic β -silicon carbide [1]. Consequently, strength properties of ceramics, which are sensitive to inherent flaws as described above, should be discussed from statistical aspects.

In this review, investigations reporting strength characteristics of structural ceramics from statistical aspects are mainly surveyed. The static strength is first summarized in Chapter 2, and then the fatigue strength is reviewed in Chapter 3. Finally, the proof testing, one of important design concepts for ceramics, is described in Chapter 4.

2 STATIC STRENGTH

Tensile strength is usually adopted as a representa-

tive static strength in metals, though bending strength is very often used in ceramic materials. Because of a remarkable brittle behavior in ceramics, ceramic strength is sensitive to an eccentric stress easily generated in tensile tests especially. This is the reason why it is difficult to evaluate an accurate strength of ceramics under axial loading. Although the procedure of tensile testing has been improved to decrease such an eccentric stress, data obtained in tensile tests are not enough compared with those in bending tests. Consequently, in this chapter, experimental results reporting bending strength are first outlined to describe general characteristics of static strength. In addition, influences of environments, specimen geometry and machining on strength properties are also reviewed.

2.1 General Characteristics

2.1.1 Statistical aspects

The Weibull theory [2], which is based on the weakest link model, has been applied to present statistics of strength in ceramics. Actually, distribution functions $F(\sigma)$ of strength σ in ceramics are most frequently fitted by the Weibull distribution function as

$$F(\sigma) = 1 - \exp \left[- \left(\frac{\sigma - \gamma}{\alpha} \right)^m \right], \quad (1)$$

where m , γ and α are the shape, location and scale parameters, respectively. Figure 1 shows an example [3] adequately expressed by the three-parameter Weibull distribution function of Eq.(1). However, in fitting by the two-parameter Weibull distribution function of Eq.(1) with $\gamma=0$, which is most widely applied, it is pointed out that a strength distribution is not always represented by a single line in a Weibull probability paper [4], or that a fitted curve becomes convex downward [4, 5]. The former may be able to be described by using the three-parameter Weibull distribution func-

tion. If the latter is fitted by Eq.(1), the location parameter γ becomes negative that implies the loss of its physical meaning. In this case, it may be better to adopt other kinds of distribution. Some reports show that a strength distribution in ceramics is more appropriately presented by the doubly exponential distribution [7] or by the log normal distribution [8] rather than the Weibull distribution.

Fig.1. Bending strength distribution fitted by three-parameter Weibull distribution function [3].

Table 1. List of m values obtained in various ceramics.

Silicon nitride	hot-pressed	7 ~ 16
	pressure-less sintered	7 ~ 11
	reaction bonded	6 ~ 13
Silicon carbide	pressure-less sintered	5 ~ 10
	reaction bonded	10
	CVD	6
Alumina	hot-pressed	17
	pressure-less sintered	11 ~ 34
Sialon	pressure-less sintered	7
Glass materials	soda lime silica	4 ~ 8
	glass ceramics	1 ~ 21
	slip cast silica	6
Other materials	mullite	12
	porcelain	9 ~ 12
	alumina abrasives	15 ~ 24

The shape parameter m , which is obtained in fitting a strength distribution by the two-parameter Weibull model, is sometimes used as a parameter indicating a scatter of strength as well as the coefficient of variation. Values of m in 3-point and 4-point bending tests using square bar specimens of various ceramics are summarized in Table 1. In general, the m value in ceramics seldom exceeds twenty, which is considerably small compared with that obtained for yield stress or tensile strength of metallic materials. This implies that a strength dispersion in ceramics is relatively large. It is also found that the m value has a scatter in the same kind of material. This is ascribed

to the variation of strength distribution due to differences in additives, sintering process, specimen size, machining procedure after sintering and so on, even if the same kind of material is tested.

It is revealed that the m value is dependent on the sample size n [5, 6]. Figure 2 presents the dependency of m value on the sample size in 4-point bending test using 113 specimens of a sintered α -silicon carbide [6]. In the figure, the m values calculated for small groups of strength data, which are arbitrarily sampled from the total data, are plotted against the sample sizes of the corresponding small groups. It is found that a scatter of m value drastically decreases when the sample size is more than thirty. Another investigation on the coefficient of variation for a hot-pressed silicon nitride reports that, in the case of the sample size of 15~20, the coefficient of variation of the strength is around 0.02, while the coefficient of variation of the m value is 0.3 which is larger in one order compared with the case of strength [6]. JIS (Japanese Industrial Standard) R 1601, however, specifies that the sample size should be more than ten. According to the result mentioned above, the m value obtained by following JIS using the least samples does not always give us a valid information on the dispersion of strength data.

Fig.2. Effect of sample size on estimation of shape parameter [6].

2.1.2 Fracture mechanics aspect

As mentioned in Introduction, the inert strength in particular is dominated by inherent flaws in a material. A predominant flaw observed on a fracture surface is identified by the typical feature with mirror, mist and hackle patterns around a fracture origin [9]. The size of flaw identified as the fracture origin has been correlated with the fracture strength in the framework of fracture mechanics [10-17]. In the case, however, it is reported that fracture originating from a small flaw occurs at a lower stress than that expected from the fracture criterion using the fracture toughness K_{IC} . An example is shown in Fig.3 [15], where

the strength σ_f and the flaw size a are normalized by using the average grain size d_o of each material. The anomalous behavior observed for small flaws has been analyzed by several models [12-17]. The simplest and efficient procedure to explain the behavior is the method using the next relation;

$$\sigma_f = \frac{K_{IC}}{\sqrt{\pi(a + a_o)}} \quad (2)$$

where a_o is a material constant [12-14]. In the figure, the curve indicates Eq.(2) with $a_o = 2d_o$, which well represents the central relation in the scatter of data. An empirical relation to estimate a_o by using mechanical properties of a material was proposed, and such a dispersion of data as seen in Fig.3 was also successfully simulated [17]. A similar treatment may be applicable to the fatigue strength to be mentioned later by using a crack growth equation instead of the static fracture criterion.

Fig.3. Relation of flaw size versus strength [15].

2.2 Environmental Effects

Figure 4 shows the strength distribution depending on temperature in 3-point bending test of a sintered silicon nitride [18]. In the figure, the result obtained at 1073K is omitted because the distribution almost coincides with that at room temperature. As seen in Fig.4, the strength tends to decrease with elevating temperature [19-22]. The m value indicated in Fig.4 presents the shape parameter in fitting each strength distribution by the two-parameter Weibull distribution. It is reported elsewhere [19, 22, 23] that the m value is larger at a higher temperature; i.e. a scatter of strength decreases with increasing temperature. At elevated temperature, the strength becomes insensitive to inherent flaws, especially subsurface flaws, because of viscoelastic or plastic behavior, and the oxidation of specimen surface progresses. The strength may be determined by the thickness of oxide layer rather than inherent flaws at high temperature. The oxidation thickness is considered to be almost constant under testing with the same stress rate, so the decrease in strength variation is observed at high temperature. This interpretation is also supported by some observations of fracture surface [14, 24].

It should be noted that the distribution characteristics of strength at elevated temperature depends

on the stress rate in testing. For a lower stress rate, the distribution shifts toward the lower strength region [20, 22]. Since an inelastic stress-strain behavior in ceramics appears at high temperature as described in Introduction, such an inelasticity should be taken into account in evaluating stress under bending with stress gradient. Consequently, the result in Fig.4 must be interpreted as a relative comparison. In a work [22], the bending strength at elevated temperature was analyzed in consideration of the viscosity effect.

Fig.4. Temperature dependence of bending strength distribution [18].

Fig.5. Bending strength distribution in corrosive environment [26].

Strength of oxide ceramics in a corrosive environment, particularly in water, is observed to reduce. This is attributed to the water-induced bond rupture [26] of glassy phase in grain boundary as

For example, 3-point bending test of an alumina was carried out in dry nitrogen gas and in water [26]. The

result is shown in Fig.5, where the strength distribution in water is found to shift toward the lower strength side. A similar result was also observed in other alumina [6, 27] and glass ceramics [28]. These degradation behaviors are caused by a slow or sub-critical crack growth (SCG) enhanced in water, as expressed by Eq.(3), during loading process [29]. It is also revealed that the strength distribution depends on the stress rate in corrosive environments as well as at high temperature [28].

Fig.6. Strength distribution in ring compression test [3].

Fig.7. Strength distribution in ring expansion test [36].

2.3 Effects of Specimen Geometry and Stress State on Strength

Tensile test [30-34], rotating disc test [31], ring compression test [3, 32, 35] and ring expansion test [36-38] of ceramics were carried out to generate uni-axial stress or nearly tensile stress state like as bending tests. Some examples in these tests are presented in Figs.6 and 7; Fig.6 shows the result in a ring compression test [3], and Fig.7 that in a ring expansion test [36]. For comparison, the dashed straight lines as

seen in Figs.6 and 7 are the distributions of bending strength of the same materials, which are expressed by the two-parameter Weibull distribution. There is no difference between results under ring compression and bending, though the strength under ring expansion is found to be lower than that under bending. This is associated with the size effect as mentioned in the following.

The effect of specimen size on strength was investigated by bending test of various materials [4, 6, 26, 31, 32, 37, 39], ring expansion test of a reaction-bonded silicon nitride [36], and sphere compression test of a soda lime silica [40]. A typical result in 4-point bending test of an alumina is shown in Fig.8 [26]. As in Fig.8, the strength of ceramics generally reduces with increasing specimen size.

Fig.8. Effect of specimen size on strength [26].

The effective volume and surface area were proposed by Davies [40] to explain the size effect on strength [1, 19]. These parameters are given by converting an actual volume and surface area of a specimen with stress distribution into an equivalent volume and surface area subjected to a uniform stress. For instance, Davies derived the effective volume V_E of a specimen with volume V from the Weibull theory [2] as

$$V_E = \int_V \left(\frac{\sigma}{\sigma_R} \right)^m dv, \quad (4)$$

where m is the shape parameter in the two-parameter Weibull distribution, and σ_R is a representative stress in the specimen, usually the maximum stress. When the average values of strength of specimens with distinct effective volumes V_{E1} and V_{E2} are denoted as μ_1 and μ_2 respectively, the following relation is introduced [41].

$$\frac{\mu_1}{\mu_2} = \left(\frac{V_{E2}}{V_{E1}} \right)^{1/m} \quad (5)$$

The above equation implies that the average strength of larger components becomes less than that of smaller ones. A successful application of Eq.(5) is presented in Fig.9 [42], where circular marks and error bars present the average values and scatters of strength respectively. In the application of Eq.(4) or Eq.(5), however,

it is noted that the shape parameter m should be a material constant; m does not change for any specimen size and/or loading mode in the same material. It is pointed out that the m value depends on specimen size [26] and Eq.(4) itself is not applicable to a very small specimen or a specimen with a very sharp notch [43].

Fig.9. Relation between mean strength and effective volume [42].

2.4 Effect of Machining on Strength

A remarkable shrinkage after sintering is observed in most ceramics. This behavior poses a problem in application of ceramics to actual mechanical components. Although a near-net shape technique has been improved to avoid such a shrinkage, it is still difficult to control the rate of shrinkage in components with complex geometry especially. Consequently, machining is required to get the prescribed size accuracy for a ceramic component. Grinding in particular has a possibility that scratches or flaws on surface of a component may be induced. Since ceramic materials are sensitive to such flaws as mentioned previously, it is important to understand the relation between machining and strength properties. Related literature reported the bending strength affected by factors as follows; machining procedures [44], difference in grinding direction [19, 45, 46], removal of machined surface layer [19], mesh size of grinding wheel [47], and surface roughness after grinding [19, 35]. Manifest influence of the grinding direction on strength is generally recognized, though effects of other factors on strength appear inconsistent depending on a grinding-induced residual stress and interactions with other processing procedures. The bending strength of specimens ground in the transverse direction, in which grinding grooves are generated perpendicularly to the nominal tensile stress, is lower than the strength of those ground longitudinally [19, 45, 46].

3 FATIGUE STRENGTH

Delayed fracture of ceramics has been called "fa-

tigue" by ceramists in particular. They classified phenomena of delayed fracture into "dynamic fatigue" under a constant loading or stroke rate, "static fatigue" under a constant load, and "cyclic fatigue" under a cyclic loading. The dynamic fatigue strength is equivalent to the static strength, which has been reviewed in Chapter 2. In this chapter, the strength of static and cyclic fatigue is described.

3.1 Static Fatigue

Fracture under a static load in ceramics, similar to stress corrosion cracking in metals, is anticipated to occur in water environment. The mechanism is primarily the same as mentioned in §2.2 [29]. Figure 10 shows the life distribution of a soda lime silica in water, plotted on a Weibull probability paper [48]. The stress value in the figure means the stress level kept constant during the test. As the stress level is increased, the life distribution shifts toward the lower life region. In each case, a scatter in life distribution is found to be very large. A similar result has also reported in another soda lime silica in water [49,50] and a vitreous-bonded alumina abrasive in a commercial aqueous coolant [51]. In these materials, the life ratio of the maximum to the minimum is observed to be in the range of 10^2 to 10^3 . Since the life distribution of a silica fiber in water is found to be lower than that in heavy water (deuterium oxide, D_2O) [52], the life is suggested to be dominated by SCG from an inherent flaw due to the water-induced bond rupture as Eq.(2). A large dispersion of life observed in delayed fracture of ceramics is attributed to a scatter of size and location of inherent flaw, just like the case of static strength. This is supported by the fact that the life variation became extremely low when soda lime silica specimens with cracks introduced under the same indentation condition were tested [53].

Fig.10. Life distribution of soda lime silica in water [48].

The creep characteristics is important in the mechanical design of ceramics to be expected as candidate materials used at a higher temperature. Figure 11 indicates the life distribution of a hot-pressed silicon nitride under 4-point bending at 1273K and 1473K [22]. At 1273K, the range of life scatter is about 10^4

for each stress level. On the other hand, at 1473K, the life scatter appears to decrease, while the dispersion is still large for the minimum stress level.

Fig.11. Characteristics of creep strength [22].

3.2 Cyclic Fatigue

The relations between applied stress and number of cycles to failure in several materials under 3-point bending fatigue are depicted in Fig.12 [54, 55]. The stress plotted on the ordinate is normalized by the average of static bending strength in each material. The number attached to the arrow in the figure denotes the number of unbroken specimens at 10^7 cycles. The data plotted at $N=1$ imply those of specimens broken during the loading process at the first cycle. This behavior is understood to the static fracture due to large flaws dominating fracture at the critical stress below the prescribed stress level. As seen in Fig.12, the fatigue strength at truncated number of cycles 10^7 appears in the stress range of 50~70% of the average of static bending strength. A typical example of statistical analyses on fatigue life properties [56-58] is shown in Fig.13 [56]. The figure indicates the result of an alumina under rotating bending, and the dependence of life distribution on stress. Each solid curve in the figure presents the life distribution fitted by the three-parameter Weibull distribution function.

Fig.12. S-N relations under 3-point bending fatigue [54, 55].

As in Fig.13, the minimum life is sometimes confirmed [59, 60], and may be determined by the correlation of the applied stress level and the size and location of a predominant flaw in a material. Anyway, the life

is also suggested to be dominated by inherent flaws in cyclic fatigue as well as the cases of static strength and static fatigue [5, 21, 56-61].

Next, effects of temperature and fatigue condition on life properties are outlined. The temperature dependence of life distribution of an alumina was investigated under 4-point bending with trapezoidal stress waves [62]. The life is defined as the summation of holding time at the peak stress. Although the life tends to decrease with increasing temperature up to 479K, the life distribution shifts toward the higher region above the temperature. This behavior is explained by being associated with the softening temperature of glassy phase in grain boundary [62].

Fig.13. Distribution characteristics of fatigue life [56].

A thermal fatigue using a soda lime silica was investigated [63]. In the test, temperature variation ΔT by heating and cooling in water of temperature 303K was repeated under a constant stress of 5.4MPa by mechanical bending. The life distribution in thermal fatigue is also found to be fitted by the two-parameter Weibull distribution function. In this case, it is clarified that the slope of fitted line, i.e. the shape parameter, becomes larger as ΔT is increased.

In addition, fatigue tests with variable stress amplitudes were conducted using a sintered silicon nitride and a partially stabilized zirconia [64]. The loading pattern was a repeated two-step type under 3-point bending with sinusoidal wave. For changing the combination of the low stress with N_L cycles and the high stress with N_H cycles, it is revealed that there is no variation in the life distribution in the silicon nitride, though in the zirconia the distribution shifts toward the lower regime with increasing the fraction of N_H occupied in one block loading. In the zirconia, the stress-induced transformation may influence the fatigue property.

4 PROOF TESTING AND SUBSEQUENT STRENGTH DISTRIBUTION

As noted in the previous chapters, a large scatter is observed in the static strength, the static and cyclic fatigue strength in ceramic materials. It is also pointed out that such scatter can be explained by the geometric and spatial distributions of inherent flaws in a material. Therefore, some ceramic components

have rather low strength or short lives due to large flaws included in a material. Proof testing for ceramic materials has been proposed as an efficient procedure to exclude such components with a lower strength, which are not permissible in the structural design [65]. Although the application of proof testing is expected to yield an improvement in the reliability of ceramic components, it is necessary to take account of strength degradation due to SCG during loading and/or unloading processes in the testing. In this chapter, examples of proof testing applied to ceramic strength are reviewed and the problem in proof testing is also cited.

The most successful example of proof testing for the bending strength of a hot-pressed silicon nitride is shown in Fig.14 [66]. The distribution after proof testing is found to have such a shape as the initial distribution truncated below the proof stress level. Another application was investigated in rotating strength test of disc specimens [51].

Fig.14. Strength distribution of specimens surviving after proof testing [66].

In the works mentioned above, both of proof testing and strength test were conducted at room temperature. In some investigations, a strength test at elevated temperature was followed by proof testing at room temperature [20, 35, 36]. For instance, the truncation in the lower strength region was confirmed in the strength distribution of an alumina at 1273K [20]. On the other hands, no difference between the initial and subsequent distributions was observed for the strength characteristics of a hot-pressed silicon nitride at 1473K [66] and of pre-oxidized hot-pressed silicon nitrides at 1273K and 1373K [67]. These results suggest that the type of flaw dominating fracture may change depending on tested temperature and/or material. The temperature dependence is associated with the sensitivity to the flaw type at room and elevated temperatures; the strength at room temperature is sensitive to inherent flaws, while the high temperature

strength is controlled by SCG from surface oxide layer rather than embedded inherent flaws as described in §2.2.

As mentioned first in this chapter, we should understand the influence of proof testing conditions, including the prescribed proof stress and stress rates during loading and unloading processes. The effect of stress rate in proof testing on the subsequent strength distribution was investigated [68-70]. The proof testing with changing unloading stress rate was conducted using a soda lime silica in heptane under a constant stress rate of 33MPa/s in loading [68]. The result is shown in Fig.15, where the subsequent strength distribution is found to shift toward the lower strength side with decreasing the rate in unloading. In each case, strength data lower than the proof stress are observed. Since the degradation for the same unloading rate is reported to be larger in water than in air [69], the above behavior may be explained by a reduction in residual strength enhanced by SCG due to the water-induced bond rupture during loading and unloading processes.

Fig.15. Subsequent strength distribution affected by unloading stress rate [68].

Fig.16. Subsequent life distribution depending on proof testing condition [75].

By proof testing, the minimum life was observed in the life characteristics under static fatigue [48] and thermal fatigue [71]. Efficiency of proof testing was also investigated under cyclic fatigue [72-75]. The subsequent life properties in cyclic fatigue of an alumina [75] is indicated in Fig.16. In the figure, solid marks present the original data obtained without proof testing. Figure 16(a) shows the effect of proof stress on the subsequent life. For a larger proof stress, the minimum life becomes longer, but the number of surviving specimens is less. The effect of unloading stress rate on the life characteristics is presented in Fig.16(b), where the minimum life under the condition of very low stress rate is found to decrease compared with that in the original life distribution. This tendency is similar to the static strength as seen in Fig.15. It is noted that the condition of proof testing should be appropriately prescribed for efficient screening in consideration of the fracture mechanism.

5 CONCLUDING REMARKS

Several strength characteristics in structural ceramics were reviewed from statistical aspects, especially. A large scatter in each strength property was reported in ceramics, and most of strength distributions were found to be fitted by using the Weibull distribution function. When any type of distribution is applied, however, it is necessary to pay attention enough to the fitness of the distribution type because the strength distribution in the lower probability region is important in the reliability design of ceramic

components. In applying the two-parameter Weibull distribution in particular, it should be noted that the minimum strength or life cannot be determined in the mechanical design of ceramic components. Although crack growth behavior in ceramics is not described in this paper, the exponent in the equation of crack growth rate expressed by the stress intensity factor is found to be very large as 50~100. Consequently, it may be difficult to use the relation for the life assessment by taking account of a scatter of growth rate in addition to the high exponent. The proof testing will be applicable in the above situations, but the testing condition should be adequately controlled so that the screening may be efficient in correlation with its cost performance. Construction of strength databases is also required for the statistical design of ceramic components. Although two databases [76, 77] have recently been published, the contents should be enriched with an improvement in the material processing in future.

REFERENCES

1. R.W. Davidge, *Mechanical Behaviour of Ceramics*, Cambridge University Press, Cambridge (1979) p.55.
2. W. Weibull, Ing. Vetenskaps Akad. Handl. (Proc. Roy. Swedish Inst. Eng. Res., Stockholm), No.151 (1939).
3. T. Yamada and T. Hoshide, Proc. ICOSAR'85, **II** (1985) 441.
4. G.K. Bansal and W.H. Duckworth, *Fract. Mech. Ceram.*, **3** (1979) 189.
5. S. Ito, S. Sakai and M. Ito, Proc. 25th Japan Congr. Mater. Res., (1982) 218.
6. S.G. Seshadri and M. Srinivasan, *Trans. ASME, J. Eng. Mater. Tech.*, **105** (1983) 219.
7. S. Kase, *Ceramics*, **18** (1983) 320.
8. A.V. Matveev, V.A. Karpov and V.I. Konstantinov, *Sov. J. Opt. Technol.*, **47** (1980) 154.
9. R.W. Rice, ASTM STP 827 (1984) 5.
10. A.G. Evans and T.G. Langdon, *Prog. Mater. Sci.*, **21** (1976) 191.
11. T. Hoshide, H. Furuya, Y. Nagase and T. Yamada, *Int. J. Fract.*, **26** (1984).
12. S. Usami, H. Kimoto, I. Takahashi and S. Shida, *Eng. Fract. Mech.*, **23** (1986) 745.
13. T. Yamada, Proc. 31st Symp. Mater. Strength Fract., **31** (1986).
14. J. Kitazumi, Y. Taniguchi, T. Hoshide and T. Yamada, *J. Soc. Mater. Sci., Japan*, **38** (1989) 1254.
15. T. Hoshide and T. Inoue, *Eng. Fract. Mech.*, **38** (1991) 307.
16. T. Hoshide, *Eng. Fract. Mech.*, **44** (1993) 403.
17. T. Hoshide and M. Masuda, *Mater. Sci. Res. Int.*, **1** (1995) 108.
18. M. Kawai and H. Abe, *Rep. Res. Lab. Asahi Glass Co., Ltd.*, **32** (1982) 25.
19. H. Okuda and M. Ito, *J. Soc. Mater. Sci., Japan*, **32** (1983) 823.
20. K. Jakus, T. Service and J.E. Ritter, Jr., *J. Amer. Ceram. Soc.*, **63** (1980) 4.
21. G.G. Trantina, *Ceram. Bull.*, **61** (1982) 555.
22. T. Fett and D. Munz, ASTM STP 844 (1984) 154.

23. H. Heigl and K. Heckel, *Ber. DKG*, **60** (1983) 69.
24. Y. Nagase, T. Hoshide, H. Furuya and T. Yamada, Preprint of 33rd Meeting of Soc. Mater. Sci., Japan, (1984) 150.
25. B.R. Lawn and T.R. Wilshaw, *Fracture of Brittle Solids*, Cambridge Univ. Press, Cambridge (1975) p.165.
26. G.K. Bansal, W.H. Duckworth and D.E. Niesz, *J. Amer. Ceram. Soc.*, **59** (1976) 472.
27. D.K. Shetty, A.R. Rosenfield and W.H. Duckworth, *ASTM STP 844* (1984) 57.
28. D.K. Shetty, A.R. Rosenfield, G.K. Bansal and W.H. Duckworth, *J. Amer. Ceram. Soc.*, **64** (1981) 1.
29. S.M. Wiederhorn, *Fract. Mech. Ceram.*, **2** (1974) 613.
30. M. Srinivasan and S.G. Seshadri, *Trans. ASME, J. Mech. Design*, **104** (1982) 635.
31. K. Matsusue, K. Takahara and R. Hashimoto, *Yogyo-kyokaishi*, **90** (1982) 168.
32. K. Matsusue, K. Takahara and R. Hashimoto, *Yogyo-kyokaishi*, **90** (1982) 280.
33. T. Tanaka and A. Sakaida, *Proc. 28th Japan Congr. Mater. Res.*, (1985) 177.
34. H. Kishimoto, A. Ueno, H. Kawamoto and Y. Ura, *J. Soc. Mater. Sci., Japan*, **38** (1989) 32.
35. H. Fessler, D.C. Fricker and D.J. Godfrey, *Ceram.-Perform. Appl.*, **3** (1983) 705.
36. R.L. Jones and D.J. Rowcliffe, *Ceram. Bull.*, **58** (1979) 836.
37. Y. Katayama and Y. Hattori, *J. Amer. Ceram. Soc.*, **65** (1982) C-164.
38. D.M. Neal and E.M. Lenoe, *Fract. Mech. Ceram.*, **5** (1983) 387.
39. S. Harada, N. Noda, T. Endo and M. Nagano, *J. Soc. Mater. Sci., Japan*, **37** (1988) 904.
40. B.A. Kschinka, S. Perrella, H. Nguyen and R.C. Bradt, *J. Amer. Ceram. Soc.*, **69** (1986) 467.
41. D.G.S. Davies, *Proc. Br. Ceram. Soc.*, **22** (1973) 429.
42. T. Soma, M. Matsui and I. Oda, *Non-Oxide Technical and Engineering Ceramics* (ed. by S. Hampshire), Elsevier Appl. Sci., London (1986) 361.
43. T. Hoshide, H. Sano and H. Honda, *Mater. Sci. Res. Int.*, to be submitted.
44. D. Lewis, III, *Ceram. Bull.*, **61** (1982) 1208.
45. R.W. Rice and J.J. Mecholsky, *NBS Spec. Publ. 562*, U.S. Government Printing Office, Washington, D.C., (1979) 351.
46. M.W. Hawmann, P.H. Cohen, J.C. Conway and R.N. Pangborn, *J. Mater. Sci.*, **20** (1985) 428.
47. T. Hoshide, K. Okumura and T. Inoue, *J. Soc. Mater. Sci., Japan*, **40** (1991) 217.
48. J.E. Ritter, Jr., *Fract. Mech. Ceram.*, **4** (1978) 667.
49. S. Aoki and M. Sakata, *Int. J. Fract.*, **16** (1980) 459.
50. S. Aoki, I. Ohta, H. Ohnabe and M. Sakata, *Int. J. Fract.*, **21** (1983) 285.
51. J.E. Ritter, Jr. and S.A. Wulf, *Ceram. Bull.*, **57** (1978) 186.
52. W.J. Duncan, *J. Amer. Ceram. Soc.*, **69** (1986) C-132.
53. Y. Kimura, T. Yagasaki and T. Kunio, *J. Soc. Mater. Sci., Japan*, **32** (1983) 315.
54. T. Hoshide, Preprint of 12nd Meeting of Subcommittee No.3, Committee No.136, Japan Soc. Prom. Sci., (1985) 1.
55. T. Hoshide, *J. Soc. Mater. Sci., Japan*, **43** (1994) 902.
56. T. Sakai and K. Fujitani, *J. Soc. Mater. Sci., Japan*, **35** (1986) 1371.
57. T. Tanaka and A. Sakaida, *Trans. Japan Soc. Mech. Eng., Ser.A*, **53** (1987) 1566.
58. T. Tanaka, S. Yamamoto, N. Okabe and H. Nakayama, *J. Soc. Mater. Sci., Japan*, **39** (1990) 318.
59. D.A. Krohn and D.P.H. Hasselman, *J. Amer. Ceram. Soc.*, **55** (1972) 208.
60. S. Ito, Y. Yamauchi, M. Ito and S. Sakai, *Proc. 26th Japan Congr. Mater. Res.*, (1983) 270.
61. T. Yamada, T. Hoshide and H. Furuya, *J. Soc. Mater. Sci., Japan*, **33** (1984) 28.
62. C.P. Chen and W.J. Knapp, *Fract. Mech. Ceram.*, **2** (1974) 691.
63. N. Kamiya and O. Kamigaito, *J. Mater. Sci.*, **17** (1982) 3149.
64. T. Hoshide, T. Ohara and T. Yamada, *Eng. Fract. Mech.*, **31** (1988) 339.
65. A.G. Evans and S.M. Wiederhorn, *Int. J. Fract.*, **10** (1974) 379.
66. S.M. Wiederhorn and N.J. Tighe, *J. Mater. Sci.*, **13** (1978) 1781.
67. K. Jakus, J.P. Fashey and J.E. Ritter, Jr., *Fract. Mech. Ceram.*, **5** (1983) 425.
68. S.M. Wiederhorn, S.W. Freiman, E.R. Fuller, Jr., and H. Richer, *ASTM STP 844* (1984), 95.
69. J.E. Ritter, Jr., P.B. Oates, E.R. Fuller, Jr., and S.M. Wiederhorn, *J. Mater. Sci.*, **15** (1980) 2275.
70. J.E. Ritter, Jr., K. Jakus, G.M. Young and T.H. Service, *J. Amer. Ceram. Soc.*, **65** (1982) C-134.
71. N. Kamiya and O. Kamigaito, *J. Mater. Sci.*, **16** (1981) 828.
72. Y. Yamauchi, S. Sakai, M. Ito and S. Ito, *Proc. 27th Japan Congr. Mater. Res.*, (1984) 235.
73. T. Hoshide, N. Sato, T. Ohara and T. Inoue, *Trans. Japan Soc. Mech. Eng., Ser.A*, **56** (1990) 220.
74. T. Hoshide, N. Sato and T. Inoue, *Trans. Japan Soc. Mech. Eng., Ser.A*, **56** (1990) 212.
75. T. Hoshide, *Cyclic Fatigue in Ceramics* (ed. by H. Kishimoto, T. Hoshide and N. Okabe), Elsevier Sci. B.V., (1993) 207.
76. Database for Ceramic Materials in MSDRD (*Mater. Strength Database Reliab. Design Machines Struct.*), ed. by the Research Group for Statistical Aspects of Materials Strength (1994).
77. *Ceramics Strength Database*, ed. by Subcommittee on Strength of Ceramics, Committee on Fatigue, Soc. Mater. Sci., Japan (1996).