General paper

NUMERICAL PREDICTION OF STRENGTH OF NOTCHED UD LAMINATES BY ANALYZING THE PROPAGATION OF INTRA- AND INTER-LAMINAR DAMAGE

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Abstract: To predict the strength of notched unidirectional (UD) fiber reinforced laminates, computational modeling method is shown as well as its verification. Both intra-laminar damage such as fiber breaking, matrix cracking and fiber-matrix interface debonding and inter-laminar damage such as delamination occur and propagate up to the final breaking. In the proposed numerical method, the occurrence of the intra-laminar damage is judged by Hoffman's failure criterion and its propagating behavior is theoretically formulated by anisotropic damage mechanics. The delamination is numerically treated by introducing inter-laminar element (IE) in the finite element analysis. Applying the proposed method to CFRP laminates with a hole under tensile load, the damage mode in each lamina and the notched strength have been predicted. By comparison with the experimental results, the validity and the reliability of our simulation have been recognized.

Key words: *Laminate, CFRP, Notched strength, Damage mechanics, Three-dimensional FEM, Inter-laminar element*

1 INTRODUCTION

In recent years, laminated composite materials have been used as primary structures in many industrial applications. Subsequently, the evaluation of their safety and reliability is becoming more important. However, we have generally no other method to predict the strength than classical failure criterion of Tsai-Wu[1] or Hoffman[2]. The problem is the lacking of reliable computational modeling method that can analyze the propagation of damage in unidirectional (UD) fiber reinforced laminates. Here, the damage that occurs in UD laminates can be categorized into intra-laminar damage and inter-laminar damage. The intra-laminar damage includes fiber breaking, matrix cracking and fiber-matrix interface debonding. The inter-laminar damage is known as delamination. It is necessary to consider all these damage for the strength evaluation.

The three-dimensional finite element method seems to be suitable to treat the above complicated phenomena. Conventionally, the laminated plate has been regarded as an equivalent homogeneous plate by the classical laminate theory. However, above mentioned many kinds of damage occur in each lamina. This implies that the stresses in each lamina have to be calculated. In other words, the stress distribution in the lamination direction is important, whereas the Kirchhoff-Love assumption in the classical laminate theory is insufficient even for a thin laminate. Finally, we reached the conclusion that three-dimensional finite element model is necessary and is the most efficient to calculate the stresses in each lamina.

In this framework of using three-dimensional finite element method for the modeling of laminates, it

seems to be adequate to use different modeling methods for intra-laminar and inter-laminar damage.

The occurrence of intra-laminar damage can be judged by Hoffman's failure criterion. The consideration of the damage mode is very important because it affects considerably the behaviors of damaged lamina. The mechanical properties of the damaged laminates must be evaluated considering the damage mode. For this aim, we have proposed to adopt the concept of damage mechanics in the two-dimensional finite element analysis[3] and the three-dimensional one[4,5]. In this paper, the modified damage mechanics based formulation to calculate the propagation of anisotropic damage is presented. By comparison with other researches[6-8], it is clearly found that our developed method is superior in evaluating the damaged state.

Contrary to the theoretical consideration of the propagation of intra-laminar damage, we employed the numerical treatment of the inter-laminar damage. Inter-laminar element (IE) has been introduced in the three-dimensional finite element model.

As an example, notched strength of CFRP laminates has been calculated by the developed method. The notch sensitivity of composite laminates is still an important unsolved problem. Many efforts have been devoted to solve this problem, but we can find no research work that considered above all damage mode in the numerical simulation. In this paper, the effects of hole diameter, fiber orientation and stacking sequence on the strength are investigated. Calculated damage mode and the strength are compared with experimental results by Lagace et al.[9,10].

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2 NUMERICAL MODEL OF UD LAMINATES

2.1 General Remarks

As mentioned in the introduction, we use three-dimensional finite element model for UD laminates. That is, every lamina is divided into solid elements. Because lamina is supposed to be a CFRP prepreg sheet, it can be regarded as an equivalent homogeneous material. However, the inter-laminar region should be taken into account in another way. Therefore, inter-laminar element (IE) is introduced.

2.2 Intra-Laminar Behaviors

To describe the intra-laminar behaviors, we focus on the following two issues. The first is the calculational method to predict the damage occurrence and its mode. The next is the damage mechanics based formulation for the analysis of damage propagation.

Because we treat the intra-laminar damage macroscopically, its occurrence can be predicted in a conventional way by Hoffman's criterion[2]. While this kind of macroscopic failure criterion has been shown to give good estimation to the damage occurrence, damage mode is not addressed. Therefore, we have proposed to classify the intra-laminar damage into four modes as shown in Fig.1[4,5] because crack parallel and/or perpendicular to the fiber occurs in each lamina[11]. 'Mode L', 'Mode T and TL', 'Mode Z and ZL' and 'Mode TZ' represent schematically the fiber breaking, transverse matrix cracking, intra-laminar cracking and fracture by shearing stress, respectively. When intra-laminar damage is predicted by Hoffman's criterion, we have proposed to determine the damage mode by watching the maximum value among the following ratios of stress to strength[4,5];

$$\frac{\sigma_L^2}{X'X^c}, \frac{\sigma_T^2}{Y'Y^c}, \frac{\sigma_Z^2}{Z'Z^c}, \left(\frac{\tau_{TZ}}{S_{TZ}}\right)^2, \left(\frac{\tau_{ZL}}{S_{ZL}}\right)^2, \left(\frac{\tau_{LT}}{S_{LT}}\right)^2 \quad (1)$$

The correlation of these stress-to-strength ratios with damage mode is summarized in Table 1.

Once the intra-laminar damage occurs, the behavior of damaged lamina is strongly dependent on the damage mode. The determination of the damage mode and the description of the dependencies of post-damage behaviors on it are essential. For the latter

Table 1. Determination of intra-laminar damage mode.

Maximum stress-to-strength ratio	Damage mode
$\frac{\sigma_L^2}{X'X^c}$	Mode L
$\frac{\sigma_T^2}{Y'Y^c}$ or $\left(\frac{\tau_{LT}}{S_{LT}}\right)^2$	Mode T and TL
$\frac{\sigma_Z^2}{Z'Z^c}$ or $\left(\frac{\tau_{ZL}}{S_{ZL}}\right)^2$	Mode Z and ZL
$\left(\frac{\tau_{TZ}}{S_{TZ}}\right)^2$	Mode TZ

aim, anisotropic damage mechanics is introduced in the following.

To characterize the damage mode, we utilize Murakami-Ohno's damage tensor $\mathbf{D}$ [12].

$$\mathbf{D} = \sum_{i=1}^3 D_i \mathbf{n}_i \otimes \mathbf{n}_i, \text{ or } [\mathbf{D}] = \begin{bmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{bmatrix}, \quad (2)$$

where D_i and $\mathbf{n}_i$ are the principal value and the principal unit vector of damage tensor $\mathbf{D}$, respectively. D_i takes the value from zero (undamaged initial state) to one (final damaged state). Then, the damage tensor for the assumed four modes of intra-laminar damage can be expressed as follows;

$$\left. \begin{aligned} [\mathbf{D}] &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & (\text{Mode L}) \\ [\mathbf{D}] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & (\text{Mode T and TL}) \\ [\mathbf{D}] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} & (\text{Mode Z and ZL}) \\ [\mathbf{D}] &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & (\text{Mode TZ}) \end{aligned} \right\} \quad (3)$$

Fig.1. Intra-laminar damage modes.

The constitutive equation for damaged lamina can be derived from Cordebois and Sidoroff[13]. They suppose that the mechanical state in the damaged configuration can be stated in the fictitious undamaged configuration. Therefore, we employ the expression of the effective stress tensor $\bar{\boldsymbol{\sigma}}$ in the fictitious undamaged configuration by means of the actual stress tensor $\boldsymbol{\sigma}$ in the damaged configuration proposed by Murakami[12].

$$\boldsymbol{\sigma}^* = \frac{1}{2}[(\mathbf{I} - \mathbf{D})^{-1} \boldsymbol{\sigma} + \boldsymbol{\sigma}(\mathbf{I} - \mathbf{D})^{-1}], \quad (4)$$

where $\mathbf{I}$ is a unit tensor of second order, and the effective stress tensor $\boldsymbol{\sigma}^*$ is symmetric. Equation (4) can be rewritten in the following form,

$$\boldsymbol{\sigma}^* = \mathbf{M}(\mathbf{D}) \boldsymbol{\sigma}, \quad (5)$$

where $\mathbf{M}(\mathbf{D})$ is a linear transformation [13].

Now we define the constitutive equation in the damaged configuration by

$$\boldsymbol{\varepsilon} = \mathbf{C}(\mathbf{D}) \boldsymbol{\sigma}, \quad (6)$$

where $\boldsymbol{\varepsilon}$ is the actual strain tensor in the damaged configuration and $\mathbf{C}(\mathbf{D})$ is the elastic tensor to obtain.

According to Cordebois and Sidoroff's notion [13], the constitutive equation in the fictitious undamaged configuration can be written by

$$\boldsymbol{\varepsilon}^* = \mathbf{C}_0 \boldsymbol{\sigma}^*, \quad (7)$$

where $\mathbf{C}_0$ is the elastic tensor for the fictitious undamaged configuration and $\boldsymbol{\varepsilon}^*$ is the effective strain tensor which is defined by

$$\boldsymbol{\varepsilon}^* = (\mathbf{M}(\mathbf{D}))^T \boldsymbol{\varepsilon}. \quad (8)$$

Substituting Eq.(5) and Eq.(8) into Eq.(7), we obtain the elastic tensor for the damaged configuration $\mathbf{C}(\mathbf{D})$ in Eq.(6).

$$\mathbf{C}(\mathbf{D}) = (\mathbf{M}(\mathbf{D}))^T \mathbf{C}_0 \mathbf{M}(\mathbf{D}) \quad (9)$$

Derived constitutive equations for damaged UD lamina, Eq.(6) and Eq.(9), can be rewritten in matrix/vector form as follows.

$$\{\boldsymbol{\sigma}_{LTZ}\} = [\mathbf{Q}_{LTZ}^*] \{\boldsymbol{\varepsilon}_{LTZ}\} \quad (10)$$

Equivalent stress-strain matrix $[\mathbf{Q}_{LTZ}^*]$ is represented by that for the initial state $[\mathbf{Q}_{LTZ}]$ and the principal value of damage tensor D_i as follows.

$$[\mathbf{Q}_{LTZ}^*] = \begin{bmatrix} d_L^2 Q_{11} & d_L d_T Q_{12} & d_L d_L Q_{13} & 0 & 0 & 0 \\ d_L d_T Q_{12} & d_T^2 Q_{22} & d_T d_L Q_{23} & 0 & 0 & 0 \\ d_L d_L Q_{13} & d_T d_L Q_{23} & d_L^2 Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66}^* \end{bmatrix}$$

$$Q_{44}^* = \frac{4d_T^2 d_L^2 Q_{44}}{\{d_T + d_L\}^2}, \quad Q_{55}^* = \frac{4d_T^2 d_L^2 Q_{55}}{\{d_T + d_L\}^2}, \quad Q_{66}^* = \frac{4d_T^2 d_L^2 Q_{66}}{\{d_T + d_L\}^2} \quad (11)$$

where $d_i = (1 - D_i)$ and Q_{ij} shows the component of $[\mathbf{Q}_{LTZ}]$. For each case of four modes of damage, the stress-strain matrices after the occurrence of damage are summarized in Table 2. In the numerical simulation, we let $D_i = 0.99$ for damaged state. Comparing with other methods [6-8], it is clearly found that our damage mechanics based formulation is superior in describing the complicated damaged state.

This formulation makes it possible to simulate the

Table 2. Components of damaged stress-strain matrices.

Damage mode	Damaged stress-strain matrix
Mode L	$\begin{bmatrix} \xi Q_{11} & \psi Q_{12} & \psi Q_{13} & 0 \\ \psi Q_{12} & Q_{22} & Q_{23} & \\ \psi Q_{13} & Q_{23} & Q_{33} & \\ 0 & & & Q_{44} \\ & & & \xi Q_{55} \\ & & & \xi Q_{66} \end{bmatrix}$
Mode T and TL	$\begin{bmatrix} Q_{11} & \psi Q_{12} & Q_{13} & 0 \\ \psi Q_{12} & \xi Q_{22} & \psi Q_{23} & \\ Q_{13} & \psi Q_{23} & Q_{33} & \\ 0 & & & \xi Q_{44} \\ & & & Q_{55} \\ & & & \xi Q_{66} \end{bmatrix}$
Mode Z and ZL	$\begin{bmatrix} Q_{11} & Q_{12} & \psi Q_{13} & 0 \\ Q_{12} & Q_{22} & \psi Q_{23} & \\ \psi Q_{13} & \psi Q_{23} & \xi Q_{33} & \\ 0 & & & \xi Q_{44} \\ & & & \xi Q_{55} \\ & & & Q_{66} \end{bmatrix}$
Mode TZ	$\begin{bmatrix} Q_{11} & \psi Q_{12} & \psi Q_{13} & 0 \\ \psi Q_{12} & \xi Q_{22} & \xi Q_{23} & \\ \psi Q_{13} & \xi Q_{23} & \xi Q_{33} & \\ 0 & & & \xi Q_{44} \\ & & & \xi Q_{55} \\ & & & \xi Q_{66} \end{bmatrix}$

$$\xi = 1/10000, \quad \psi = 1/100, \quad \xi = 3.92/10000$$

mechanical behaviors after the occurrence of damage depending on both damage mode and loading conditions. Obviously, obtained stress-strain matrix for the damaged state can be implemented in FEA. Incremental linearized analysis procedure is utilized to simulate the damage propagation. See reference [5] for the detail of the incremental analysis procedure as well as the calculational method of stress release for damaged element.

2.3 Numerical Treatment of Inter-Laminar Damage

There are some important points to solve in the inter-laminar behaviors. They are delamination and residual thermal stress caused by forming process.

Microscopic observations of laminates by Killas et al. [14] and Swain et al. [15] show that there are resin-rich region between two plies with different fiber orientations. Since delamination is detected almost in this region, the inter-laminar element (IE) is introduced in FEA to consider the delamination. We assume that IE consists of resin and has the thickness of 0.01mm. Note that the thickness of common commercial prepreg sheet is 0.125mm.

To consider the residual thermal stress due to the

mismatch of anisotropic thermal expansion of each lamina, we assume the temperature change from 180 °C in the lamination process to room temperature 20 °C. Thermal stress is calculated before the simulation of damage propagation, and is introduced as the self-equilibrating initial stress.

3 RESULTS AND DISCUSSION

The damage behavior of a 6-ply graphite/epoxy laminate ($V_f = 60\%$) with hole under tensile load has been analyzed. The thickness (t) and the width (w) of every laminate are 0.81mm and 50mm, respectively. First, calculated damage mode of $[45/-45/0]_s$ laminate with 13-mm-diameter after failure has been compared with the X-ray photograph by Lagace[9]. Second, predicted notched strengths for various laminates were compared with Lagace's experiments[10] to investigate the effects of hole diameter, fiber orientation and stacking sequence on the strength. For this purpose, following laminates with three different stacking sequences, $[\theta/\theta/0]_s$, $[0/\theta/\theta]$, and $[\theta/0/\theta]$, for $\theta = 30^\circ$, 60° and 90° , were examined and three different hole diameters, 4mm, 8mm and 13mm, were considered.

To begin with, we compare the damage mode in $[45/-45/0]_s$ laminate with 13-mm-diameter hole between calculated results and the X-ray photograph by Lagace[9]. Figure 2 illustrates schematically the X-ray photograph at just after the maximum load. The observed damage is like the spokes of a wheel, but we can not distinguish the damage modes in detail. On the other hand, calculated final damage modes in Fig.3 show the complex combination of fiber breaking and transverse cracking. By superimposing every calculated damage mode on the X-ray image, it can be said that both results have a good agreement.

Next, the calculated notched strengths are compared with the experimental results by Lagace[10]. In the calculation, the strength is defined as the maximum stress in the stress-strain diagram. In the case of $[0/60/-60]_s$ and $[60/-60/0]_s$ laminates, for instance, the stress-strain diagram and the strength are shown in Fig.4. Figures 5 to 13 show the effects of hole diameter on the strength as well as the comparison between calculated and experimental results. The calculated strengths show good agreement with experimental ones except for the stacking sequence of $[0/30/-30]_s$, $[90/0/90]_s$, regardless of hole diameters and $[0/60/-60]_s$ with 8-mm-diameter hole ($d/w = 0.16$).

One of the reasons for the disagreement might be that the resin-rich region for these cases is thinner than the other cases because of high pressure applied in the lamination process. Namely, the thinner resin-rich region may cause severe reduction of strength. To examine the above supposition, analysis without IE has been carried out. The mark of white circle in Fig.5, Fig.8 and Fig.12 show the results of $[0/30/-30]_s$, $[0/60/-60]_s$ with 8-mm-diameter hole ($d/w = 0.16$) and $[90/0/90]_s$, respectively. Without IE, calculated

results agree well with experiments. It implies that the thickness of inter-laminar region affects the strength reduction significantly. It is recognized that IE is effective for most of the cases. Therefore, it can be also emphasized that the change of intra-laminar thickness produces the scatter of strength in the experimental results.

Finally, we can evaluate the effects of hole diam-

Fig.2. Schematic diagram of X-ray photograph of $[45/-45/0]_s$ specimen with 12.7-mm-diameter hole at just after the maximum load by Lagace[9].

Fig.3. Calculated final damage modes of $[45/-45/0]_s$ laminate with 13-mm-diameter hole.

Fig.4. Calculated results of stress-strain diagram of laminate with 4-mm-diameter hole.

Fig.5. Comparison of calculated strength of [0/30/-30]_s laminate with experimental results by Lagace[10].

Fig.8. Comparison of calculated strength of [0/60/-60]_s laminate with experimental results by Lagace[10].

Fig.6. Comparison of calculated strength of [30/0/-30]_s laminate with experimental results by Lagace[10].

Fig.9. Comparison of calculated strength of [60/0/-60]_s laminate with experimental results by Lagace[10].

Fig.7. Comparison of calculated strength of [30/-30/0]_s laminate with experimental results by Lagace[10].

Fig.10. Comparison of calculated strength of [60/-60/0]_s laminate with experimental results by Lagace[10].

Fig.11. Comparison of calculated strength of $[0/90_2]_s$ laminate with experimental results by Lagace[10].

Fig.12. Comparison of calculated strength of $[90/0/90]_s$ laminate with experimental results by Lagace[10].

Fig.13. Comparison of calculated strength of $[90_2/0]_s$ laminate with experimental results by Lagace[10].

eter, fiber orientation and stacking sequence on the strength. Clearly the reduction of strength can be seen as the increase of hole diameter. The notch sensitivity becomes large as the angle θ becomes small in case of $[\theta/-\theta/0]_s$ and $[0/\theta/-\theta]_s$. In case of $[\theta/0/-\theta]_s$, however, we can find very small effect of θ on the notch sensitivity. Though the effects of these parameters on the notched strength are very complicated, our computational method can evaluate them accurately enough.

4 CONCLUSION

The damage mode and the strength of CFRP laminates with centered hole under tensile load have been analyzed by using the three-dimensional finite element method based on damage mechanics. The predicted damage mode has been compared with the experimental result. The effects of hole diameter, fiber orientation, stacking sequence and the thickness of interlaminar region on the strength have been investigated and the calculated results have been compared with experiments. Good agreement between the simulation and experiments has been obtained. From above results, we can conclude that the proposed method is able to predict the damage mode and the notched strength with enough accuracy and reliability. Therefore, it is recognized that this method is very useful for the structural design of laminates.

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